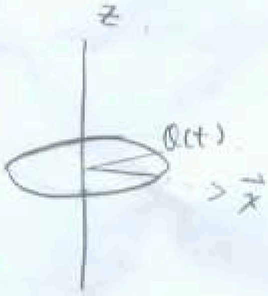
物理

$\vec{F} = m\vec{a}$ $\quad \vec{p} = m\vec{v}$ $\quad E = \frac{1}{2}mv^2$

$\vec{r}(t)$

$\frac{d\vec{r}(t)}{dt} = \vec{v}(t)$ $\Rightarrow \begin{cases} \text{rotational motion} \\ \text{translation motion} \end{cases}$



$$w(t) = \frac{d\theta(t)}{dt}$$

$$\vec{w}(t) = \frac{d\theta(t)}{dt}\hat{z}$$

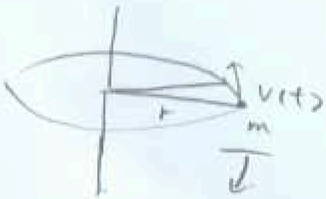
$$\vec{\alpha}(t) = \frac{d\vec{w}(t)}{dt}\hat{z}$$

$\Rightarrow$ $\theta(t) \leftarrow x(t)$ $\quad \vec{w}(t) \leftarrow \vec{v}(t)$ $\quad \vec{\alpha}(t) \leftarrow \vec{a}(t)$

因此 $v(t) = v_0 + at \rightarrow w(t) = w_0 + \alpha t$.

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \rightarrow \theta(t) = \theta_0 + w_0 t + \frac{1}{2}\alpha t^2$.

弧



$$v = r\frac{d\theta}{dt} = rw.$$

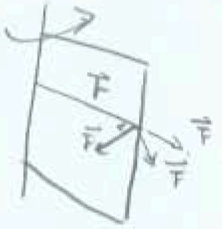
質量 m $\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}m(rw)^2$

$$= \frac{1}{2}(mr^2)w^2$$

$$\Rightarrow \sum_{i=1}^{n}\frac{1}{2}(m_i)v_i^2 = \frac{1}{2}\underbrace{\sum_{i=1}^{n}(m_i r_i^2)}_{I}w^2.$$

$K.E. = \frac{1}{2} I w^2 \qquad K.E = \frac{1}{2} m v^2$

$\Rightarrow \vec{F} = m\vec{a} \longleftrightarrow \vec{\tau} = I\vec{\alpha}$



$\vec{\tau} \propto \vec{F}$ and 施力方向 and $\vec{r}$

$\Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$

$\vec{p} = m\vec{v} \longleftrightarrow \vec{L} = I\vec{w} = \vec{r} \times \vec{p}$

math:

$$\frac{dx^2}{dx} = 2x$$

$$\frac{d\,x \cdot x}{dx} = x\frac{dx}{dx} + \frac{dx}{dx} \cdot x = 2x$$

$$\frac{d(gf)}{dt} = \frac{dg}{dt} f + \frac{df}{dt} g$$

$$\frac{d\vec{r}}{dt} = \vec{v} \qquad \frac{d\vec{L}}{dt} = \vec{\tau} = \underbrace{\frac{d\vec{r}}{dt} \times \vec{p}}_{0} + \vec{r} \times \frac{d\vec{p}}{dt}$$

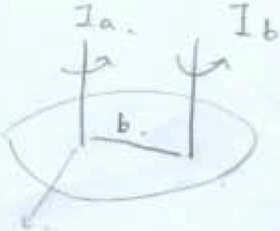
$$= \vec{r} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{out} \qquad \frac{d\vec{p}}{dt} = \vec{F}_{out}$$

$$\vec{L} = \sum_{i=1}^{n} \vec{l}_i \qquad \frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \frac{d\vec{l}_i}{dt} = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \vec{r}_i \times [\vec{F}_{out\,i} + \vec{f}_{ij}] \xrightarrow{} \text{内力}$$

$$= \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_{i\,out} + \sum_{i=1}^{n} \sum_{j=1\, i\neq j}^{n} \vec{r}_i \times \vec{f}_{ij} = \vec{\tau}_{out}$$

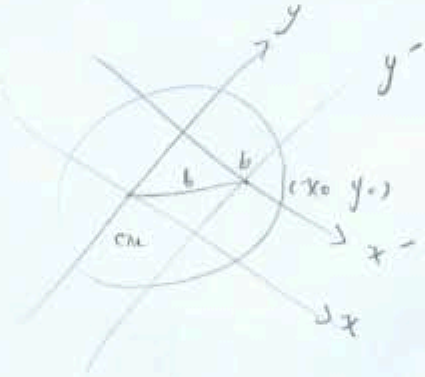


$$I_b = I_{cm} + \underset{\text{總}}{M} b^2$$

物体總質量

質量中心

(平行軸定理)



$$I_b = \sum_{i=1}^{n} (m_i r_i'^2)$$

$$= \sum_{i=1}^{n} m_i (r_i'^2 - r_i^2)$$

$$= \sum_{i=1}^{n} m_i [(x_i - x_0)^2 + (y_i - y_0)^2]$$

$$= \sum_{i=1}^{n} m_i (x_i^2 + y_i^2) + \sum_{i=1}^{n} m_i (x_0^2 + y_0^2)$$

因以點cm的座標，點cm為原點

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = 0$. 故得證

$\Rightarrow I_b = I_{cm} + M b^2$

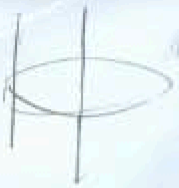
ex:

→ 棍子

$$I = 2\int_0^{l/2} x^2 \, dx = \frac{1}{12} m l^2$$

$$I_b = \int_0^{l} x^2 \, dx = \frac{1}{2} m l^2$$

$$\frac{1}{2} m l^2 = \frac{1}{12} m l^2 + m \left(\frac{l}{2}\right)^2$$



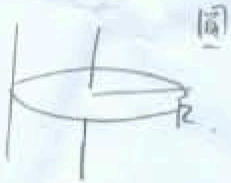
圆环

$$\int_{0,0}^{2\pi} r^2 \cdot \lambda r \, d\theta \qquad \lambda r 2\pi = m$$

$$= \lambda r^2 \cdot 2\pi r = m r^2$$

$$I_a = I_{cm} + m r^2 = 2 m r^2$$

2b



圆盘

面密度

$$\int_0^{2\pi}\int_0^{R} r^2 \sigma \, r \, d\theta \, dr$$

$$= \frac{1}{2} m R^2$$

$$I_b = \frac{3}{2} m R^2$$