1

$N \ (X_{cm} \ Y_{cm} \ Z_{cm})$

$$X_{cm} = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i} \qquad Y_{cm} = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i} \qquad Z_{cm} = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$$

$$M X_{cm} = \sum_{i=1}^{N} m_i x_i$$

$$M V_{cm,x\,cm} = M \frac{dX_{cm}}{dt} \Rightarrow \sum_{i=1}^{N} m_i \frac{dx_i}{dt} = \sum_{i=1}^{N} m_i v_{ix}$$

$$\frac{d\vec{P}}{dt} = \vec{F}_{ext}$$

碰撞

$m_1 \ v_{1i} \qquad m_2 \ v_{2i}=0$ （一維完全彈性碰撞）

$v_{1i} \qquad 0$

碰後 $v_{1f} \qquad v_{2f}$ $\Big\} \Rightarrow$ P守恒 E守恒

$$\Rightarrow m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m v_{1i}^2 = \frac{1}{2} m v_{1f}^2 + \frac{1}{2} m v_{2f}^2$$

$$\Rightarrow m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f}$$

$$m_1 (v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2 v_{2f}^2 \qquad \Rightarrow v_{1i} + v_{1f} = v_{2f}$$

$$\Rightarrow m_1 v_{1i} - m_1 v_{1f} = m_2 v_{1i} + m_2 v_{1f}$$

$$\Rightarrow v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

① $m_1 >> m_2$

$v_{1f} = v_{1i}$ $\quad v_{2f} = 2v_{1i}$

② $m_1 = m_2$

$v_{1f} = 0$ $\quad v_{2f} = v_{1i}$

③ $m_1 << m_2$

$v_{1f} = -v_{1i}$ $\quad v_{2f} = 0$

$\xrightarrow{m_1}$ $\quad \xleftarrow{m_2}$ => 座標轉移即可.

把 m_2 看成靜止 (在 m_2 座標看)

ex.

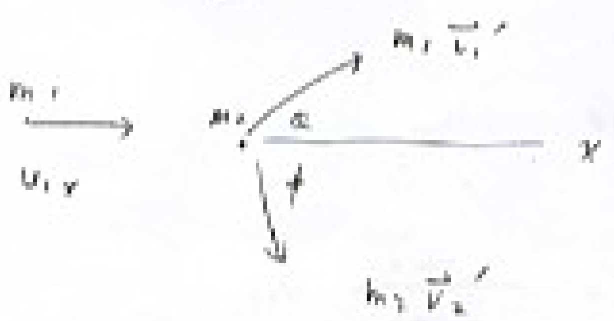
沒摩擦力

$mv = (m+M)\bar{v}$

$\frac{1}{2}(m+M)\bar{v}^2 = \frac{1}{2}kx^2$

$\Rightarrow x^2 = \frac{m^2}{k(m+M)}v^2$

ex: 2 維



$$m_1 v_{1x} = m_1 v_1' \cos\theta + m_2 v_2' \cos\phi$$

$$0 = m_1 v_1' \sin\theta \underset{(-)}{+} m_2 v_2' \sin\phi$$

3 條計算

$$\frac{1}{2} m_1 v_{1x}^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

when $m_1 = m_2$

way 1

$v_{1x} = v_1' \cos\theta + v_2' \cos\phi$ ⇒ 平方 $v_{1x}^2 = v_1'^2 \cos^2\theta + v_2'^2 \cos^2\phi + 2 v_1' v_2' \cos\theta \cos\phi$

$0 = v_1' \sin\theta - v_2' \sin\phi$ ⇒ 〃 $0 = v_1'^2 \sin^2\theta + v_2'^2 \sin^2\phi - 2 v_1' v_2' \sin\theta \sin\phi$

$v_{1x}^2 = v_1'^2 + v_2'^2$

⇓

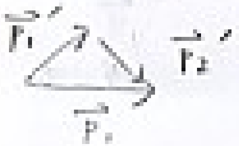
$v_{1x}^2 = v_1'^2 + v_2'^2 + \boxed{\quad}$

因此 $\boxed{\quad} = 0$

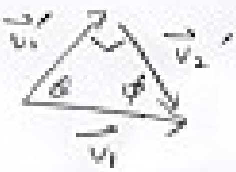
⇓

$2 v_1' v_2' \cos(\theta + \phi) = 0 \qquad \theta + \phi = \frac{\pi}{2}$

way 2

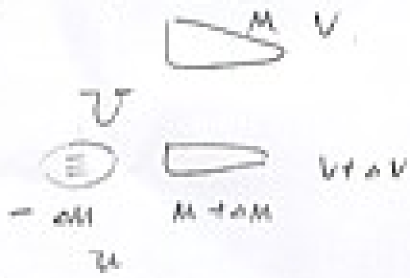


因 $m_1 = m_2$ ⇒



因 $v_1^2 = v_1'^2 + v_2'^2$.

ex. 火箭



$$MV = (M + \Delta M)(V + \Delta V) + (-\Delta M)u$$

$$u = V + \Delta V - \mathcal{V}$$

$\Delta M < 0$

$$\Rightarrow M\Delta V + \Delta M \mathcal{V} = 0$$

$$\Rightarrow M\,dV = -dM\,\mathcal{V}$$

$$\frac{dV}{\mathcal{V}} = -\frac{dM}{M}$$

$$\int_{v_i}^{v_f} \frac{dv}{u} = \int_{M_0}^{M_f} \frac{dM}{M}$$

$$\frac{v_f - v_i}{u} = \ln \frac{M_0}{M_f}$$

ex.

[illegible] spring [illegible] CP $v_1' = v_2'$ [illegible]