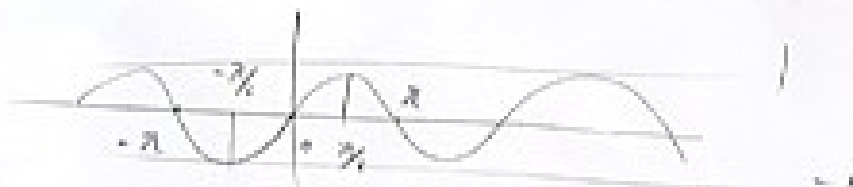


$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

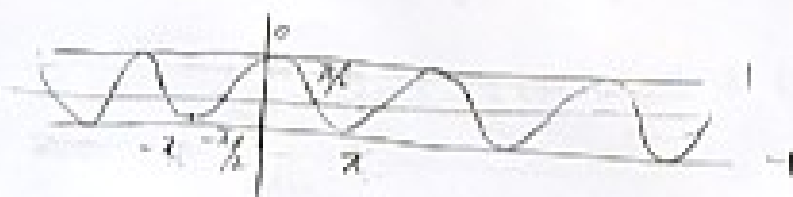
$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

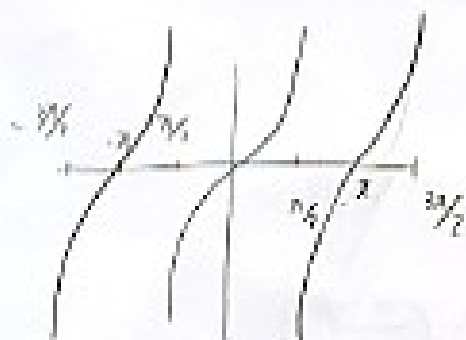
y = sin x



y = cos x



y = tan x



$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1 = 1 - 2\sin^2 \alpha$$

$$0 \leq \alpha \leq \frac{\pi}{2} \quad \cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}} \quad \sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$t = \tan \frac{\alpha}{2}$$



$$\cos \alpha = \frac{1 - t^2}{1 + t^2}$$

$$\sin \alpha = \frac{2t}{1 + t^2} \quad \tan \alpha = \frac{2t}{1 - t^2}$$

$$\sin(3\alpha) = \sin(\alpha + 2\alpha)$$

$$= 3\sin\alpha - 4\sin^3\alpha$$

$$\cos 3\alpha = \cos(\alpha + 2\alpha)$$

$$= 4\cos^3\alpha - 3\cos\alpha$$

$$\sin \alpha \pm \sin \beta$$

$$= \sin\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right) \pm \sin\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right)$$

$$\therefore (\sin \alpha + \cos \beta + \cos \beta \sin \alpha) \pm (\sin \alpha + \cos \beta - \cos \beta \sin \alpha)$$

$$\Rightarrow 2\sin \alpha \cos \beta = 2\sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$$

$$2\cos \alpha \sin \beta = 2\cos \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2}$$

$$\cos \alpha + \cos \beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos \alpha - \cos \beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$2\sin \alpha \cos \beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$

$$2\cos \alpha \sin \beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$

For

$$\sin'x = \frac{d\sin x}{dx} = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$\sin h$ when $h \rightarrow 0$ of $\sin h = h$

$$\frac{d\sin x}{dx} = \frac{1}{1} \cdot \frac{d\sin x}{dx} = \cos x$$

$$\begin{aligned} \cos da &= \sqrt{1 - \sin^2 da} \quad da \rightarrow 0 \\ &= \sqrt{1 - da^2} \\ &= 1 \end{aligned}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

$$\begin{aligned} \sqrt{1-x^2} &= (1-x^2)^{1/2} = 1 + \frac{1}{2}(-x^2) + \dots \quad \text{when } x \rightarrow 0 \\ &\approx 1 \end{aligned}$$

$$(\sin x)' = \frac{h \cos x + \sin x - \sin x}{h} = \cos x$$

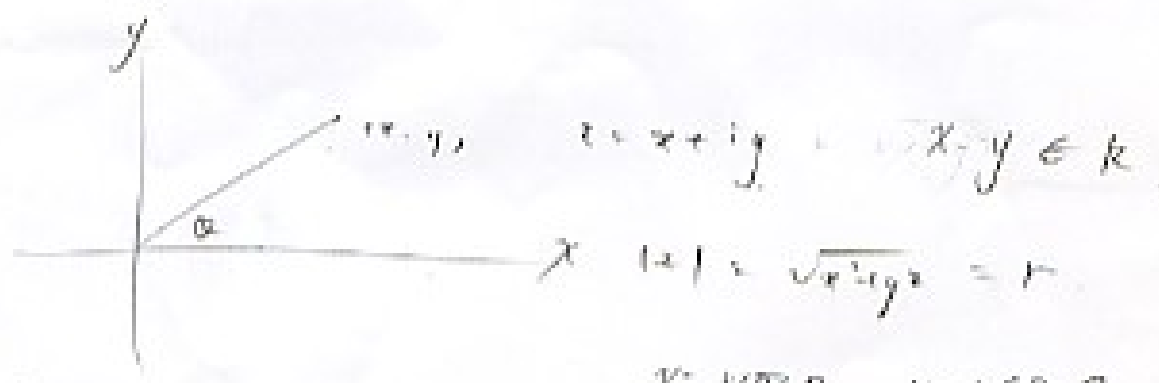
$$(\cos x)' = \frac{\cos x - h \sin x - \cos x}{h} = -\sin x$$

Complex number 複數

$$x^2 + 1 = 0$$

$$\begin{aligned} x^2 &= -1 \quad x = \sqrt{-1} \notin \mathbb{R} \\ &= i \end{aligned}$$

$$i^2 = -1$$



$$x = r \cos \alpha \quad y = r \sin \alpha$$

$$z = r \cos \alpha + i r \sin \alpha = r e^{i\alpha}$$

$$x = \operatorname{Re}\{z\} \quad \bar{z} = z^* = x - iy$$

$$y = \operatorname{Im}\{z\}$$

$$(a + bi)(c + di)$$

$$= (ac - bd) + i(ad + bc)$$

$$\Rightarrow z\bar{z} = \bar{z}z = x^2 + y^2$$

$$|z| \text{ complex } |z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2}$$

$$\text{ex } \frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1}{2} + \frac{3}{2}i$$

$$e^{i\theta} = \cos \theta + i \sin \theta \quad \text{Euler formula}$$

$$(e^{i\theta})^n = (\cos \theta + i \sin \theta)^n$$

$$e^{in\theta} = \cos n\theta + i \sin n\theta$$