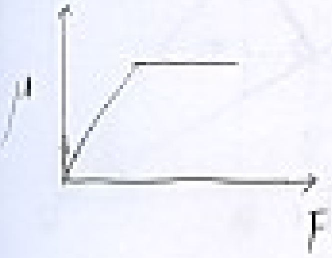
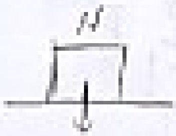
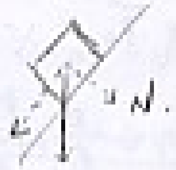
摩擦力



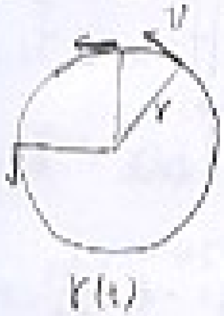
$$f_{s,k} = \mu_s N$$

μ_k ↓ 正向力.





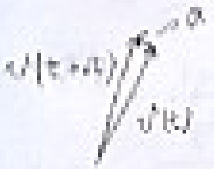
圓周運動

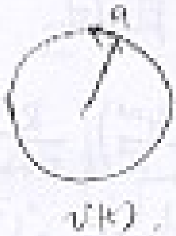


$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi v}{a}$$

$$\Rightarrow \begin{cases} \frac{2\pi r}{v} = \frac{2\pi v}{a} \\ a = \frac{v^2}{r} \end{cases}$$





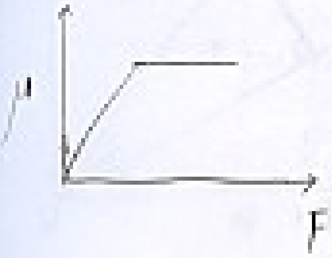
浮力 $F = \frac{1}{2} c \rho A v^2$

$ma = m\frac{dv}{dt}$ $\Rightarrow \frac{2m}{c\rho A}\frac{dv}{v^2} = dt.$

終端速度 $a = 0$ ex: 自由落体 $mg = f = cv^2$

$\Rightarrow \frac{1}{2} c\rho A v^2 = mg$ $\Rightarrow v = \sqrt{\frac{2mg}{c\rho A}}$

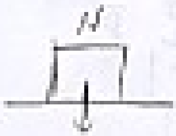
摩擦力

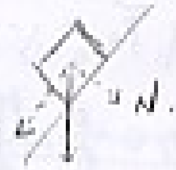


$f_{s,k} = \mu_s N$

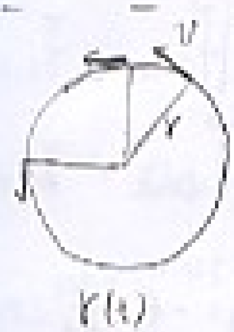
μ_k ↓

正向力.





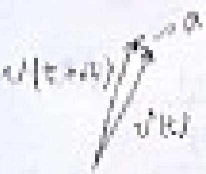
圓周運動

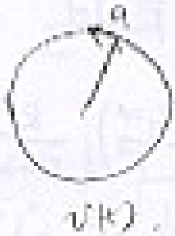


$T = \frac{2\pi r}{v}$

$T = \frac{2\pi v}{a}$

$\Rightarrow \begin{cases} \frac{2\pi r}{v} = \frac{2\pi v}{a} \\ a = \frac{v^2}{r} \end{cases}$





阻力 $F = \frac{1}{2} c\rho A v^2$

$ma = m\frac{dv}{dt}$ $\Rightarrow \frac{2m}{c\rho A}\frac{dv}{v^2} = dt$.

終端速度 $a = 0$ ex: 自由落体 $mg = f = cv^2$

$\Rightarrow \frac{1}{2} c\rho A v^2 = mg \Rightarrow v = \sqrt{\frac{2mg}{c\rho A}}$

$\vec{F} = m\vec{a}$ $\quad \vec{P}$ = momentum $\quad$ W: work

$= m\frac{d\vec{v}}{dt}$

$= \frac{d(m\vec{v})}{dt} = \frac{d(\vec{P})}{dt}$

if $\vec{F} = 0$ $\quad \vec{P}$ = constant 動量守恆

n個物體 $\Rightarrow \vec{P} = (\vec{P}_1 + \vec{P}_2 + \cdots \vec{P}_n) = \sum_{i=1}^{n} \vec{P}_i$

$$\vec{F} = \frac{d\vec{P}}{dt} = \sum_{i=1}^{n} \frac{d\vec{P}_i}{dt} = \sum_{i=1}^{n} \vec{F}_i$$

$$= \sum_{i=1}^{n} \left[\vec{F}_{i,ext} + \vec{F}_{i,int}\right]$$

內力和 = 0.

$$= \sum_{i=1}^{n} \vec{F}_{i,ext} = \vec{F}_{ext}$$

$h \ll R$ $\quad V(R+h) - V(R) = \int \vec{F} \cdot d\vec{r}$ $\quad mg = -G\frac{Mm}{R^2} = m\left(\frac{GM}{R^2}\right)$

$$GMm\left(\frac{1}{R+h} - \frac{1}{R}\right) = -m\left(\frac{GM}{R^2}\right)\left(\frac{R^2}{R+h} - \frac{R^2}{R}\right)$$

$$= -mg\left[R\left(1 - \frac{h}{R}\right) - R\right]$$

$$= mgh$$

※ 當 x 很小.

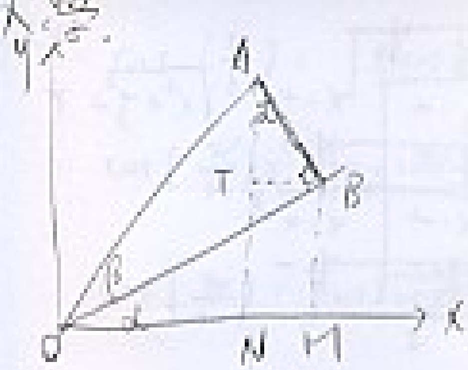
$f(x) = 1 + x + x^2 \cdots$

$\Rightarrow x f(x) = x + x^2 + x^3 + \cdots$

$1 = f(x)(1 - x) \Rightarrow f(x) = \frac{1}{1-x} \approx 1 + x.$

當 $h \ll R$ $\quad \frac{h}{R}$ 相當小.

相 加



$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\triangle OAN$

$$\sin(\alpha+\beta) = \frac{AN}{AO} = \frac{AT+TN}{AO}$$

$$\frac{OB}{OA} = \cos\beta$$

$AN = AT + TN$, $AT = AB\cos\alpha = OA\sin\beta\cos\alpha$

$TN = BM = OB\sin\alpha = AO\cos\beta\sin\alpha$

$$\frac{AN}{AO} = \sin(\alpha+\beta) = \frac{AO(\sin\alpha\cos\beta + \sin\beta\cos\alpha)}{AO}$$

$$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ\cos 30^\circ + \cos 45^\circ\sin 30^\circ$

$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

$\cos 75^\circ = \sqrt{1-(\sin 75^\circ)^2} = \sqrt{1-\frac{4+2\sqrt{3}}{8}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$$\cos(\alpha+\beta) = \frac{ON}{OA} = \frac{OM-NM}{AO} = \frac{OB\cos\alpha - AB\sin\alpha}{AO}$$

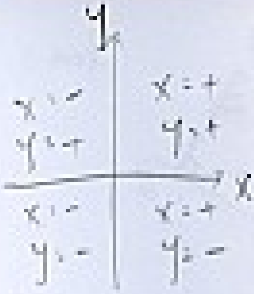
$NM = TB = AB\sin\alpha$

$$= \frac{(AO\cos\beta)\cos\alpha - (AO\sin\beta)\sin\alpha}{AO}$$

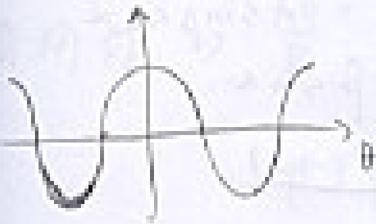
$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$

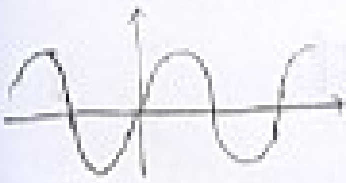
	$\frac{y}{r}$ $\sin\theta$	$\frac{x}{r}$ $\cos\theta$	$\frac{y}{x}$ $\tan\theta$
$\leq\theta\leq\frac{\pi}{2}$	+	+	+
$\leq\theta\leq\pi$	+	−	−
$\leq\theta\leq\frac{3}{2}\pi$	−	−	+
$\leq\theta\leq 2\pi$	−	+	−



$\sqrt{x^2+y^2} = r$



$\cos\theta = \cos(-\theta)$ 為偶函数
(even)



$\sin(-\theta) = -\sin(\theta)$ 為奇函数
(odd)

$$\sin(\alpha-\beta) = \sin(\alpha)\cos(-\beta) + \cos\alpha\sin(-\beta)$$
$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta) = \cos\alpha\cos\beta\ \sin\alpha\sin\beta$$

– – –

$\beta = \alpha$

$$\sin(2\alpha) = \sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha = 2\cos\alpha\sin\alpha$$

$$\cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \cos^2\alpha - \sin^2\alpha$$
$$= 2\cos^2\alpha - 1$$
$$= 1 - 2\sin^2\alpha.$$

$$\Rightarrow \begin{cases} \cos\alpha = \sqrt{\frac{1+\cos 2\alpha}{2}} \\ \sin\alpha = \sqrt{\frac{1-\cos 2\alpha}{2}} \end{cases}$$

Ex:

$$\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

$$\cos\left(\frac{\pi}{16}\right) = \sqrt{\frac{2+2\cos\frac{\pi}{8}}{2}} = \sqrt{\frac{2+\sqrt{2+\sqrt{2}}}{2}}$$

$$\cos\left(\frac{\pi}{32}\right) = \sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}}$$

$$\cos\left(\frac{\pi}{2^n}\right) = \sqrt{\frac{2+\sqrt{2+\cdots}}{2}}$$

当 $n \to \infty$ $\quad \cos\left(\frac{\pi}{2^n}\right) = \cos 0 = 1$