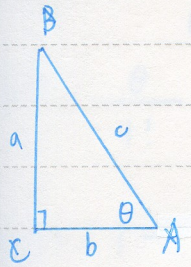
三角

csc



$\sin\theta = \frac{a}{c}$ = 對邊/斜邊　　$\csc\theta = \frac{c}{a}$

$\cos\theta = \frac{b}{c}$ = 鄰邊/斜邊　　$\sec\theta = \frac{c}{b}$

$\tan\theta = \frac{a}{b}$ = 對邊/鄰邊　　$\cot\theta = \frac{b}{a}$

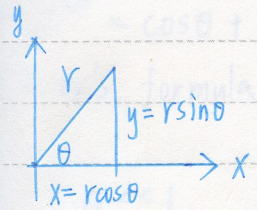
Pythagorean theorem

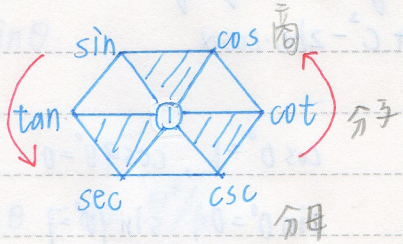
$a^2+b^2=c^2 \Rightarrow (\sin\theta)^2+(\cos\theta)^2 = \frac{a^2}{b^2}+\frac{b^2}{c^2} = 1$

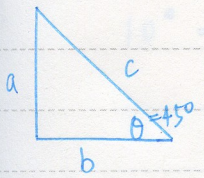
$\sin^2\theta+\cos^2\theta=1$

$\tan^2\theta+1 = a^2/b^2+1 = a^2+b^2/b^2 = c^2/b^2 = \sec^2\theta$

$\Rightarrow \tan^2\theta+1=\sec^2\theta$





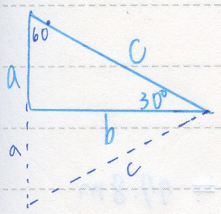


$\theta=45^\circ$, $a=b$, $a^2+b^2=c^2$, $2a^2=c^2$, $c=\sqrt{2}a$

$\sin 45^\circ = a/c = 1/\sqrt{2} \fallingdotseq 0.707$

$\cos 45^\circ = b/c = 1/\sqrt{2}$

$\tan 45^\circ = 1$



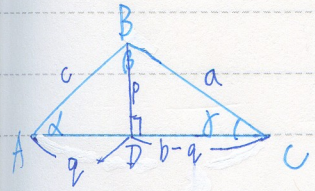
$\theta=30^\circ$, $c=2a$

$\sin 30^\circ = a/c = a/2a = \frac{1}{2}$

$b=\sqrt{3}a$

$\cos 30^\circ = b/c = \sqrt{3}a/2a = \sqrt{3}/2$

$0 \le \theta \le 90^\circ \Rightarrow 0 \le \sin\theta \le 1$, $0 \le \cos\theta \le 1$



$\triangle ABD$ $\sin\alpha = P/c \Rightarrow P = c\sin\alpha$

$\triangle BDC$ $\sin\gamma = P/a \Rightarrow P = a\sin\gamma$

$\Rightarrow c\sin\alpha = a\sin\gamma \Rightarrow a/\sin\alpha = c/\sin\gamma$

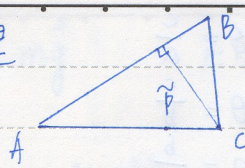
任意三角形

arbitrary triangles

page

date

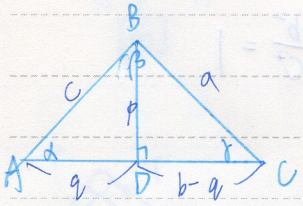
同理



$\tilde{p} = a\sin\beta = b\sin\alpha$

$a/\sin\alpha = b/\sin\beta$

正弦定理 $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$



$\triangle ABD$ $c^2 = p^2 + q^2 \Rightarrow p^2 = c^2 - q^2$

$\triangle BDC$ $a^2 = p^2 + (b-q)^2$

$\Rightarrow a^2 = c^2 - q^2 + (b-q)^2$ $\quad q = c\cdot\cos\alpha$

$a^2 = c^2 - q^2 + b^2 - 2bq + q^2$

$= b^2 + c^2 - 2bc\cdot\cos\alpha$

餘弦定理

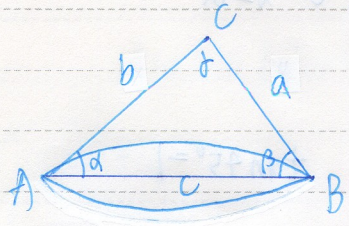
$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = c^2 + a^2 - 2ca\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$\cos 0^\circ = 1$, $\cos 90^\circ = 0$

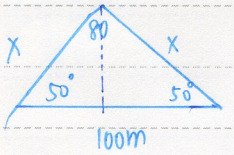
$\sin 0^\circ = 0$, $\sin 90^\circ = 1$



$\gamma = 180^\circ - \alpha - \beta$

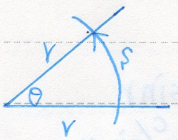
$a \simeq b$

$\alpha \simeq \beta$



$\dfrac{X}{\sin 50^\circ} = \dfrac{100}{\sin 80^\circ}$, $X = 100m \times \dfrac{\sin 50^\circ}{\sin 80^\circ} \fallingdotseq 77.8$

($\sin 50^\circ \approx 0.7666$, $\sin 80^\circ \approx 0.9848$)



$\theta = S/r$ (radian 弧度)

$360^\circ = 2\pi r/r = 2\pi$ (rad)

$90^\circ \Rightarrow \theta = S/r = \pi/2$ (rad)

θ (in rad)

$$\sin\theta = \frac{\theta}{1!} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} \cdots$$

$0! = 1$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \frac{\theta^8}{8!} + \cdots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$x = i\theta$ $\quad -\frac{\theta}{1!} + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!}$

$\Rightarrow e^{i\theta} = \cos\theta + i\sin\theta$

Euler's formula

π (rad) $= 180^\circ$

2π (rad) $= 360^\circ$

$10^\circ = 10^\circ \times \frac{\pi \text{ rad}}{180} = \frac{\pi \text{ rad}}{18}$

$\theta << 1$, $\sin\theta \simeq \theta - \theta^3/6$

$\cos\theta \simeq 1 - \theta^2/2$

$10^\circ = \sin\pi/18 \simeq \frac{1}{6}$

$1 \text{ rad} = 1 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}}$

$\frac{180^\circ}{\pi} \simeq 57^\circ$