历史.

Hans Lippershey (1587–1619)

Galileo Galilei (1564 ~ 1642) 望遠鏡.

Kepler (1571 – 1630) 克普勒.

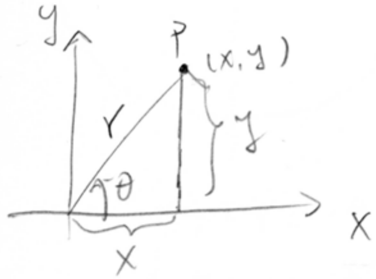
Snell (1591 – 1626) (1621 折射定律)

Fermat (1601 — 1665) (笛卡爾 ↗)

Hurke (1635 — 1705)

Newton (1668 ~ 1707)

Cartesian coordinates (x y) 卡氏坐標

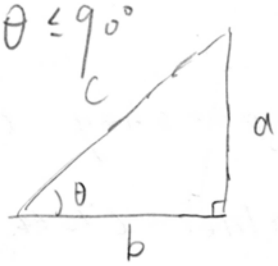


$X = r\cos\theta$

$Y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$

$0^\circ \le \theta \le 90^\circ$



$$\begin{cases} \sin\theta = \dfrac{a}{c} = \dfrac{\text{对边}}{\text{斜边}} \\ \cos\theta = \dfrac{b}{c} = \dfrac{\text{邻边}}{\text{斜边}} \end{cases}$$

$$\tan\theta = \frac{a}{b} = \frac{\text{对边}}{\text{邻边}}$$

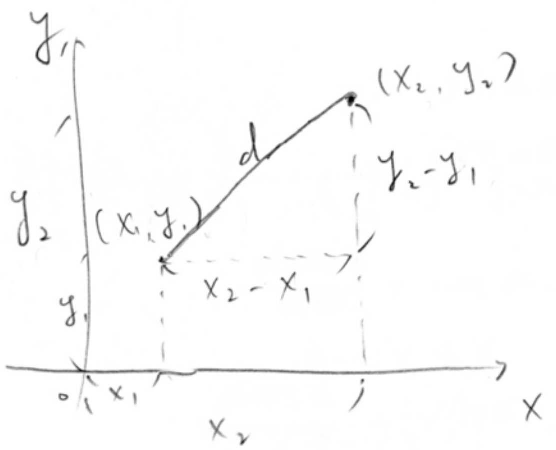
畢式定理

$a^2 + b^2 = c^2$

$\sin^2\theta + \cos^2\theta$

$$= \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1$$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

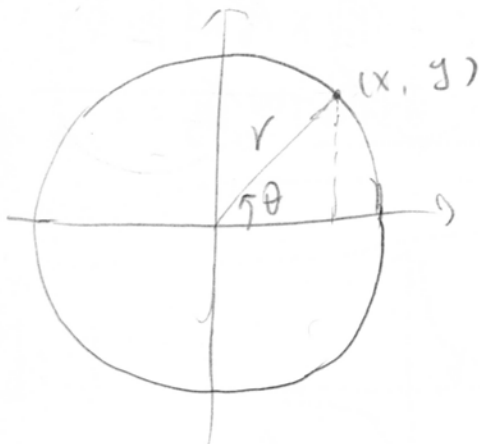


兩点距 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Fermat's conjecture
費瑪 猜想

$a^n + b^n = c^n$ 無整數解 $n > 2$

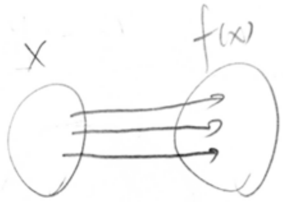
Polar coordinates
極坐標 (r, θ)



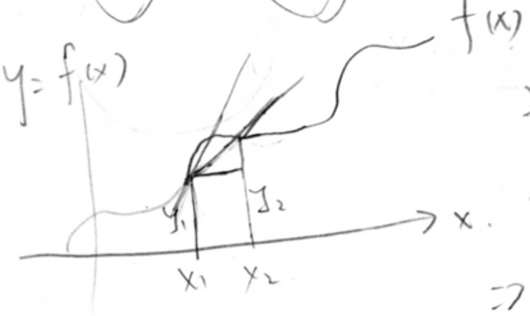
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

当 $r = a$ 時

$$\begin{cases} a = \sqrt{(x-0)^2 + (y-0)^2} \quad (\text{卡氏坐標}) \\ x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta + \sin^2\theta) \\ \qquad = 1 \quad (\text{極坐標}) \end{cases}$$



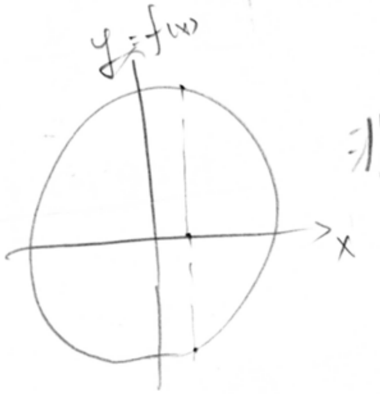
一对一函数.



当取愈靠近两点之 $\frac{\Delta y}{\Delta x}$

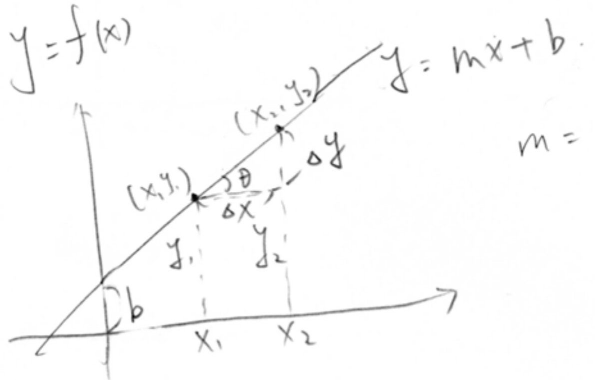
便會愈趋近斜率

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \text{slope}$$



非一对一函数.

∵ 一個 x 會对应到两個 f(x) 值.



m = slope 斜率

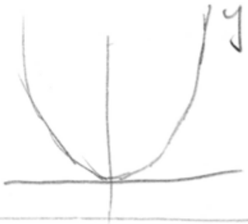
$$\frac{\Delta y}{\Delta x} = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = m$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

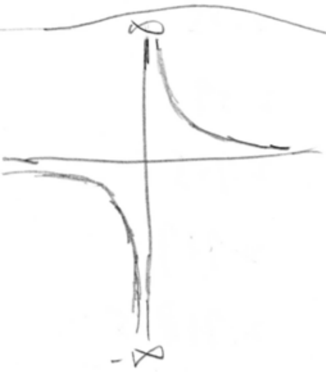
$$= \frac{df(x)}{dx}$$ 導函数

$y = x^2$

$y = cx^2,\ c \geq 0$

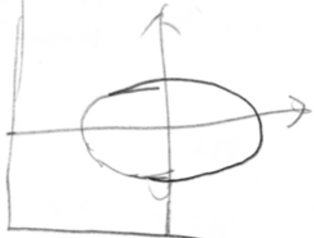


$y = \frac{1}{x}$



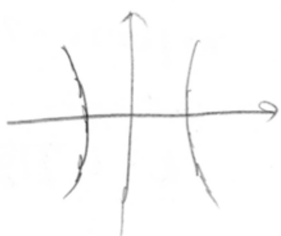
楕圓

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



双曲線

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



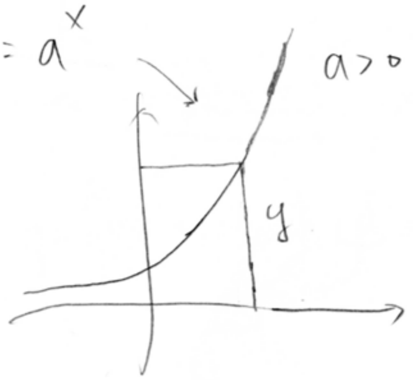
$x \in R$ 實数

$n \in Z$ 整数

$n! = n \cdot (n-1) \cdots 2 \cdot 1$

$0! = 1$

$y=a^x$



当非常巧的 此函数的 $\dfrac{\Delta y}{\Delta x} \xrightarrow[\Delta x \to 0]{} y$.

此函数為 $y=e^x$

$$\Rightarrow \frac{de^x}{dx} = e^x$$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

$n=1 \Rightarrow \left(1+\frac{1}{1}\right)^1 = 2$

$n=2 \Rightarrow$ 2.25

$n=10 \Rightarrow$ 2.59

$n=100 \Rightarrow$ 2.705

$n=1000 \Rightarrow$ 2.717

$n=10000 \Rightarrow$ 2.71815

$e = 2.7182818\ldots$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{x^k}{k!}$$