12/20 級式

$f(x) = \sum_{i=0}^{\infty} a_i x^i = a_0 + a_1 x + a_2 x^2 + \dots + a_N x^N + \dots$ power series expansion.

$f(x+\Delta x) = f(x) + \underbrace{\sum_{i=1}^{\infty} a_i (\Delta x)^i}_{\Delta f(x)}$ if we let $\Delta x \to dx$, $dx < \epsilon$, $\epsilon \ll 1$

忽略

$f(x+dx) = f(x) + [a_1 (dx) + a_2 (dx)^2 + \dots] = f(x) + a_1 dx$

$a_1 = \frac{f(x+dx) - f(x)}{dx} \equiv \frac{df(x)}{dx}$ $= f(x) + dx \cdot \frac{df(x)}{dx}$

Taylor series expansion $= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ $= f(x) + df(x)$

1. $f(b)$ $f(b+x) = f(b) + \sum_{j=1}^{\infty} a_j \cdot x^j = \sum_{j=0}^{\infty} a_j \cdot x^j$

$df(b+x)/dx = 0 + a_1 + 2a_2 x + \dots + N a_N \cdot x^{N-1}$

$a_1 = \frac{df(b+x)}{dx}\Big|_{x=0}$, $a_N = \frac{1}{N!} \frac{d^{(N)} f(x+b)}{dx^N}$

2. $\frac{df(x)}{dx} = k \cdot f(x)$ $f(x) = \sum_{j=0}^{\infty} a_j x^j = a_0 + a_1 x + \dots + a_N x^N + \dots$

$df(x)/dx = a_1 + \dots + N \cdot a_N \cdot x^{N-1} + \dots = \sum_{j=1}^{\infty} a_j \cdot x^{j-1} \cdot j$

$\sum_{m=1}^{\infty} a_m x^{m-1} \cdot m = k \sum_{j=0}^{\infty} a_j x^j \Rightarrow x^N$ term $a_{N+1}(N+1) = k \cdot a_N$

x^0 term $1 \cdot a_1 = k a_0$

$a_{N+1} = \frac{k}{N+1} a_N$

$= \frac{k}{N+1} \cdot \frac{k}{N} a_{N-1}$

$= \frac{k}{N+1} \cdot \frac{k}{N} \cdot \dots \cdot \frac{k}{1} a_N = \frac{k^{N+1}}{(N+1)!} a_0$

$f(x) = \sum_{j=0}^{\infty} a_j x^j = \sum_{j=0}^{\infty} \frac{k^j}{j!} x^j = \sum_{j=0}^{\infty} \frac{(k \cdot x)^j}{j!} = 5$

$f(x) \cdot f(y) = f(x+y)$ 指數律 (you can do it !)

$f(x) = 5^{kx} \Rightarrow 5^x \cdot 5^y = 5^{x+y}$ $5^x / 5^y = 5^{x-y}$

set $k = 1$ $\sum_{j=0}^{\infty} \frac{x^j}{j!} = e^x \Rightarrow e = \sum_{n=0}^{\infty} \frac{1}{n!}$ Base $= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \dots$

※ exponentially decay.

$f(t) = 5 \cdot 2^{-t/T} \to 5 e^{k \cdot t}$

$f(t+T) = f(x) \cdot 2^{-1} = 5 \cdot 2^{-(t+T)/T} = 5 \cdot 2^{-(\frac{t}{T}+1)} = 5 \cdot 2^{-1} \cdot 2^{-t/T}$

$f(x) = 5 \cdot e^{kx}$ k imaginary 虛數 $df(x)/dx = k \cdot f(x)$

$d^2 f(x) / dx^2 = k^2 f(x)$

$d^2 f(x)/dx^2 = -\alpha f(x)$ Ex. SHM Hooke force $F(x) = -kx(t) = m \frac{d^2 x(t)}{dt^2}$ $\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} x(t) = 5 e^{i\sqrt{\alpha} \cdot t}$

$e^{\pm i\sqrt{\alpha}t} = \cos\sqrt{\alpha}t \pm i \sin\sqrt{\alpha}t$ $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $e^{i\theta} = \sum_{n=0}^{\infty} \frac{(i\theta)^n}{n!} = \left(\sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n}}{(2n)!}\right) + i \left(\sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n+1}}{(2n+1)!}\right)$

real part: $\cos\theta$ imaginary part: $\sin\theta$

$f(x) = \sum_{n=0}^{\infty} a_n x^n = f(0) + a_1 x^1 + \dots$ ($a_n = \frac{1}{n!} \frac{d^{(n)} f(x)}{dx}\Big|_{x=0}$) Maclaurin series expansion