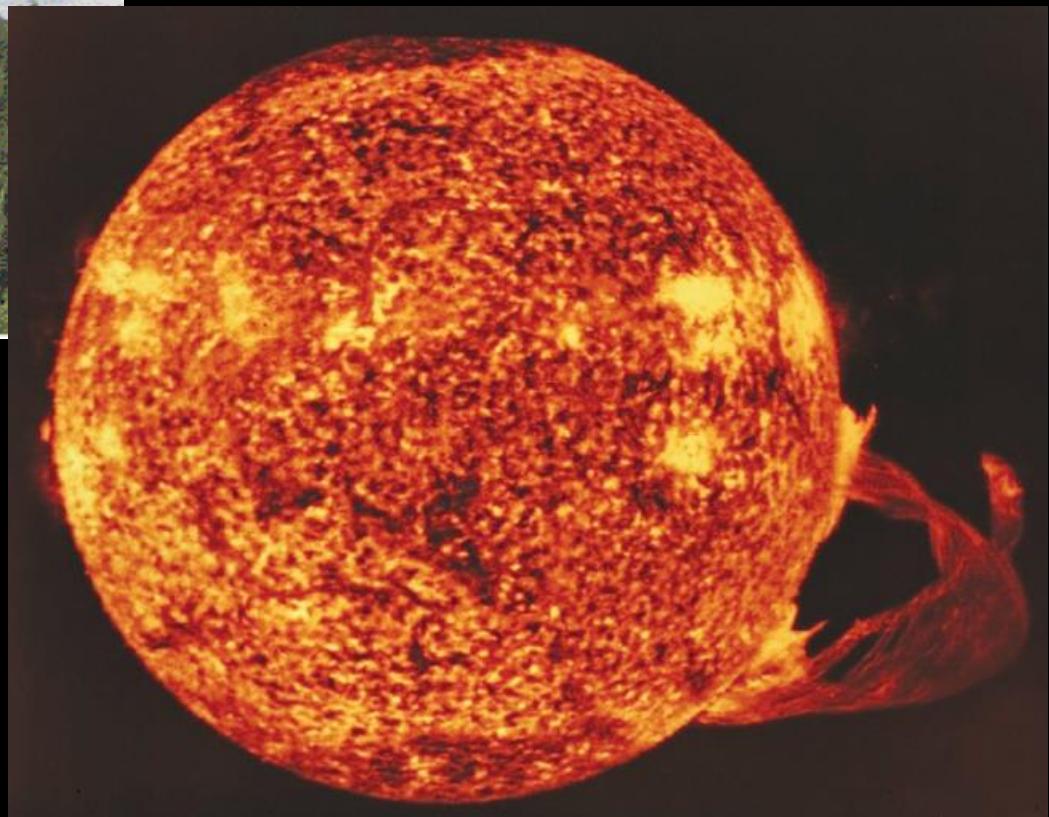
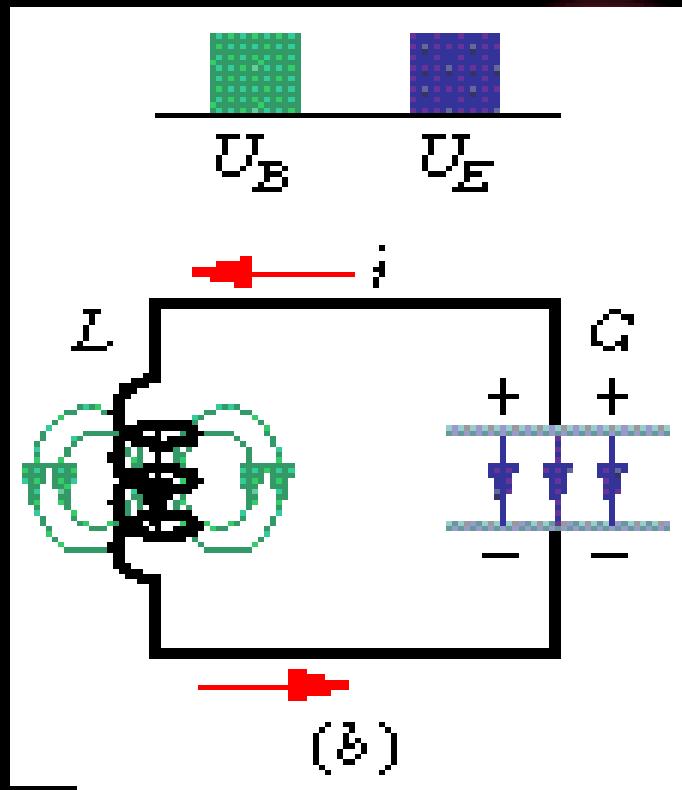
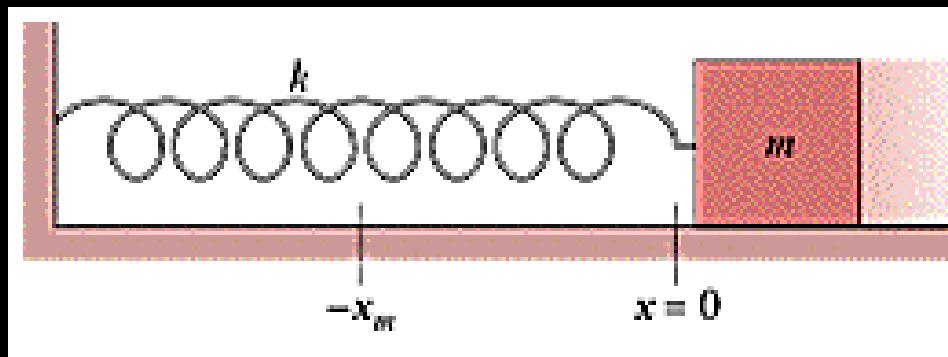


11 Electromagnetic Oscillations and Alternating Current



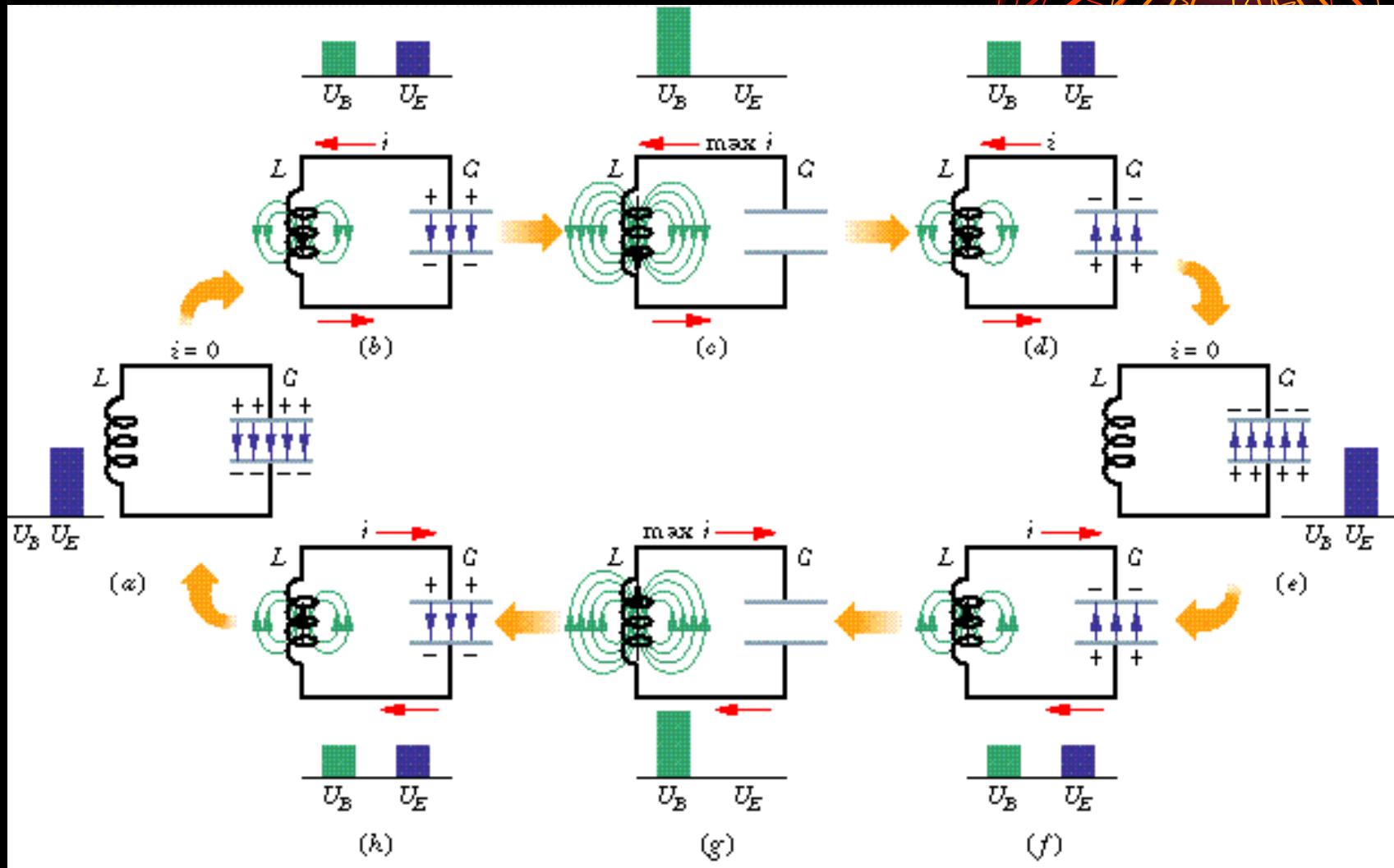
11-1 New Physics - Old Mathematics

- The spring-block system and the LC circuit



(b)

11-2 LC Oscillations, Qualitatively





11-3 The Electrical-Mechanical Analogy

$$U_P = \frac{kx^2}{2}, \quad U_K = \frac{mv^2}{2} \rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$U_E = \frac{q^2}{2C}, \quad U_B = \frac{Li^2}{2} \rightarrow \omega = \sqrt{\frac{1}{LC}}$$

11-4 LC Oscillations, Quantitatively

↓ The Block - Spring Oscillator



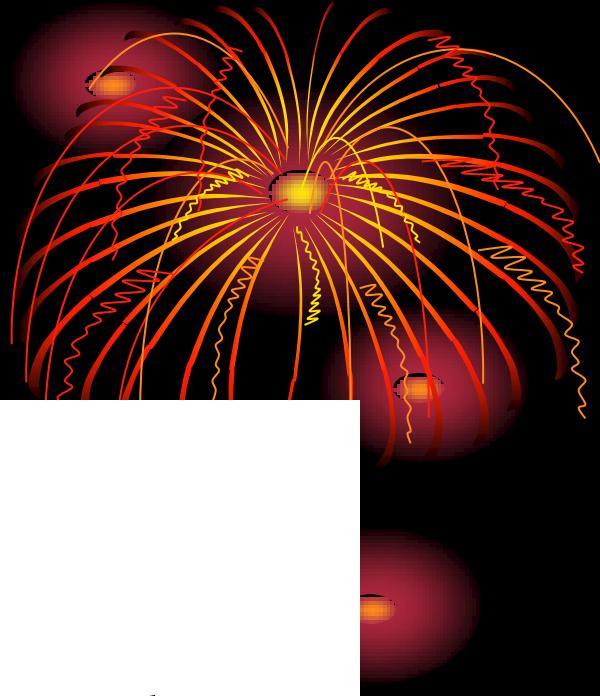
$$U = U_b + U_s = \frac{mv^2}{2} + \frac{kx^2}{2}$$

$$\frac{dU}{dt} = \frac{d}{dt} \left(\frac{mv^2}{2} + \frac{kx^2}{2} \right) = mv \frac{dv}{dt} + kx \frac{dx}{dt} = 0$$

$$m \frac{d^2x}{dt^2} + kx = 0 \quad \omega = \sqrt{\frac{k}{m}}$$

$$x = X \cos(\omega t + \phi) \quad (\text{displacement})$$

↓ The LC Oscillator

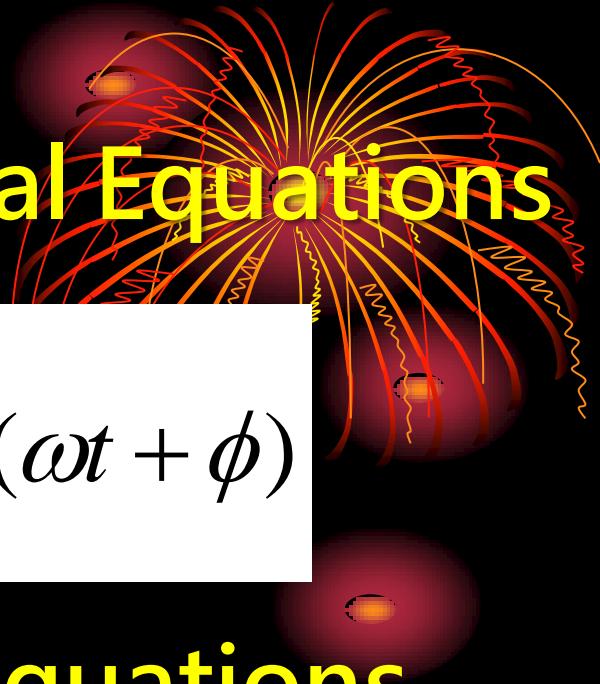


$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

$$\frac{dU}{dt} = \frac{d}{dt} \left(\frac{Li^2}{2} + \frac{q^2}{2C} \right) = Li \frac{di}{dt} + \frac{q}{C} \frac{dq}{dt} = 0$$

$$L \frac{d^2q}{dt^2} + \frac{1}{C} q = 0 \quad \omega = \sqrt{\frac{1}{LC}}$$

$$q = Q \cos(\omega t + \phi) \quad (\text{charge})$$



- Second-order Differential Equations

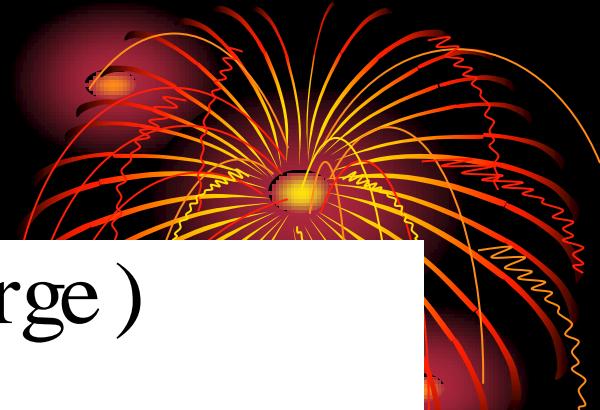
$$L \frac{d^2 q}{dt^2} + \frac{1}{C} q = 0, \quad q = Q \cos(\omega t + \phi)$$

- First-order Differential Equations

$$R \frac{dq}{dt} + \frac{1}{C} q = \mathcal{E}, \quad q = C\mathcal{E}(1 - e^{-t/RC})$$

$$L \frac{di}{dt} + Ri = \mathcal{E}, \quad i = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

• By Substitution



$$q = Q \cos(\omega t + \phi) \quad (\text{charge})$$

$$i = \frac{dq}{dt} = -\omega Q \sin(\omega t + \phi) \quad (\text{current})$$

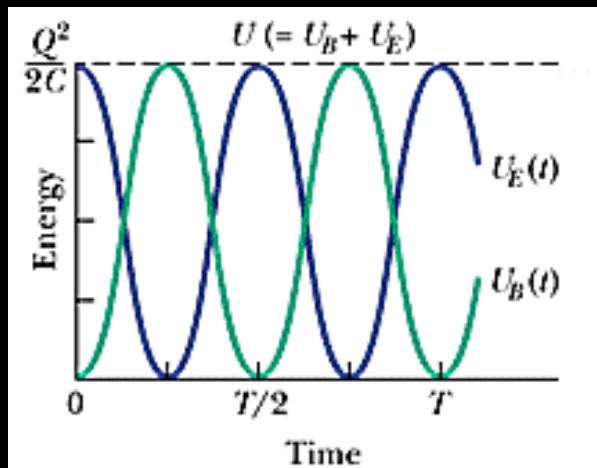
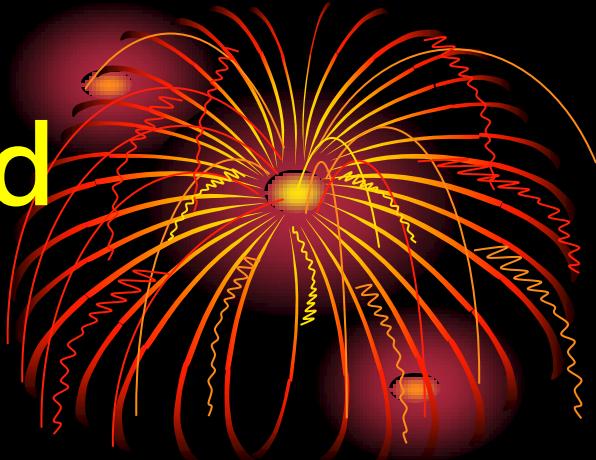
$$i = -I \sin(\omega t + \phi) \quad I = \omega Q$$

$$\frac{d^2q}{dt^2} = -\omega^2 Q \cos(\omega t + \phi)$$

$$-L\omega^2 Q \cos(\omega t + \phi) + \frac{1}{C} Q \cos(\omega t + \phi) = 0$$

$$\rightarrow \omega = \sqrt{1/LC}$$

- The Stored Electric and Magnetic Energy



$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C} \cos^2(\omega t + \phi)$$

$$U_B = \frac{Li^2}{2} = \frac{1}{2} L \omega^2 Q^2 \sin^2(\omega t + \phi)$$

$$U_B = \frac{Q^2}{2C} \sin^2(\omega t + \phi)$$

11-5 Damped Oscillation in an RLC Circuit



$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

$$\frac{dU}{dt} = -i^2 R = Li \frac{di}{dt} + \frac{q}{C} \frac{dq}{dt}$$

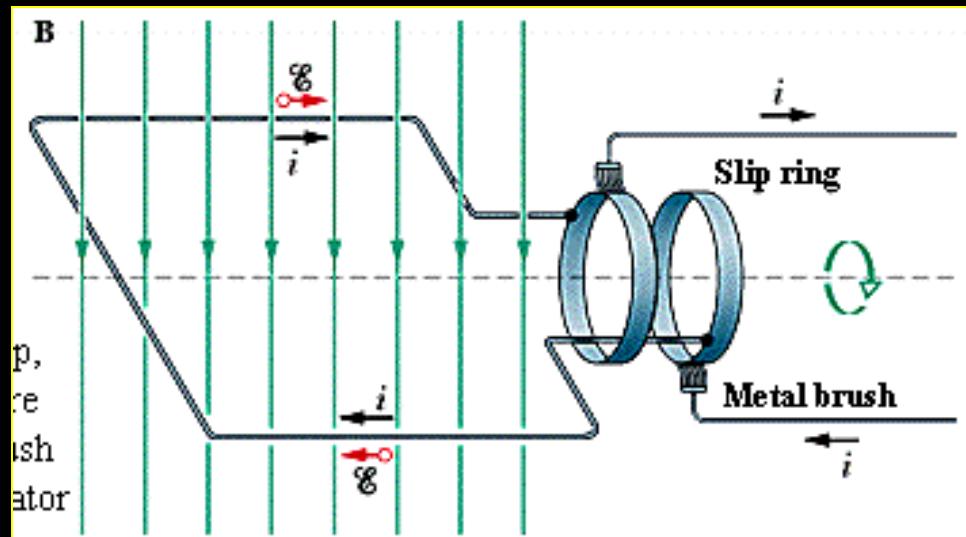
$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = 0$$

$$q = Q e^{-Rt/2L} \cos(\omega't + \phi) \quad \omega' = \sqrt{\omega^2 - (R/2L)^2}$$

11-6 Alternating Current



- A sinusoidally oscillating emf \mathcal{E} and a sinusoidally oscillating current i



$$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$$
$$i = I \sin(\omega_d t - \phi)$$

11-7 Forced Oscillations

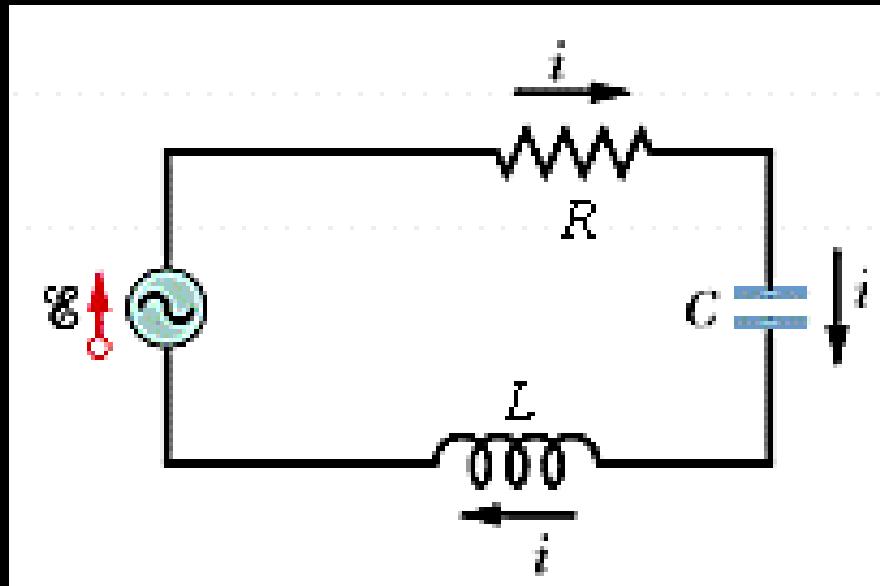


- ω natural angular frequency
- ω_d driving angular frequency
- Forced (driven) oscillations always occur at ω_d
- Resonance: when $\omega = \omega_d$, the amplitude I of the current is maximum

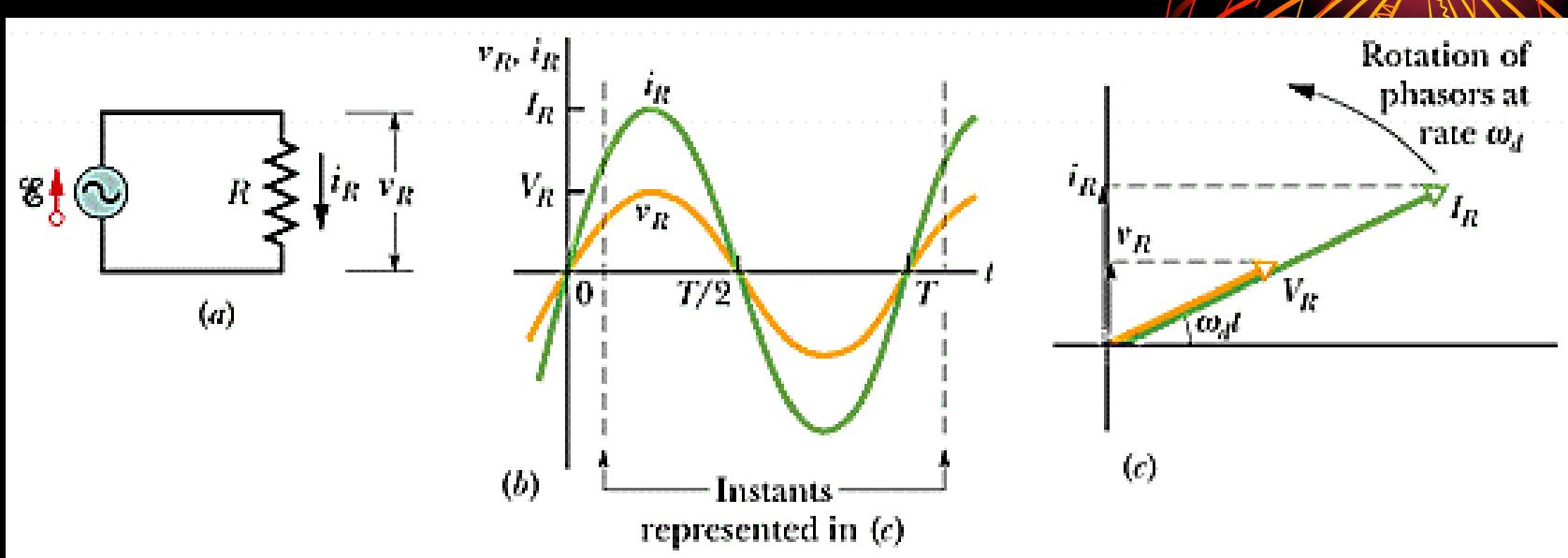
11-8 Three Simple Circuits



■ A series RLC circuit



■ A Resistive Load



$$v_R = \mathcal{E}_m \sin \omega_d t = V_R \sin \omega_d t$$

$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t = I_R \sin(\omega_d t - \phi)$$

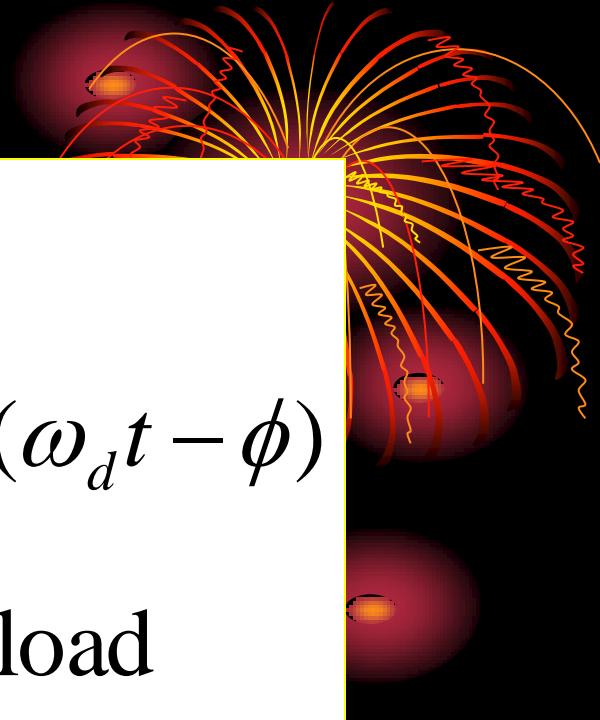
$$v_R = \mathcal{E}_m \sin \omega_d t = V_R \sin \omega_d t$$

$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t = I_R \sin(\omega_d t - \phi)$$

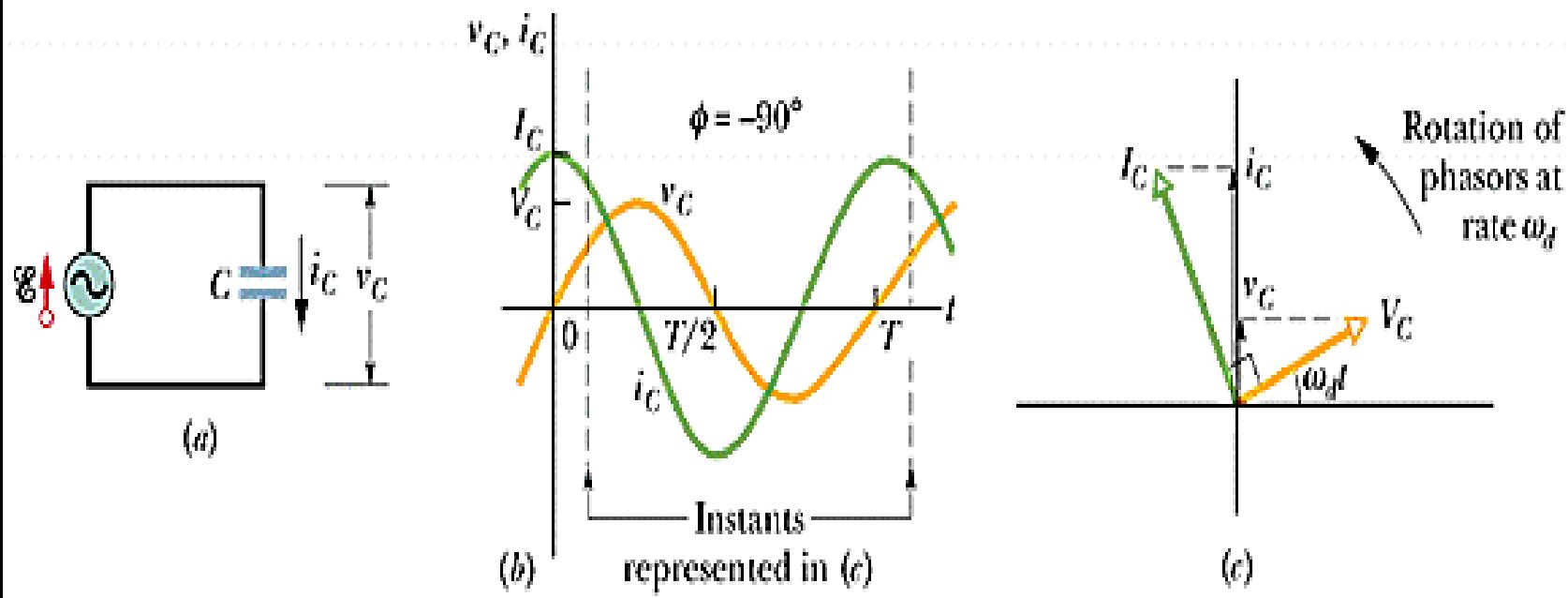
$\phi = 0^\circ$ for a purely resistive load

$$V_R = I_R R$$

- Phasor - a vector rotating around a origin with ω_d

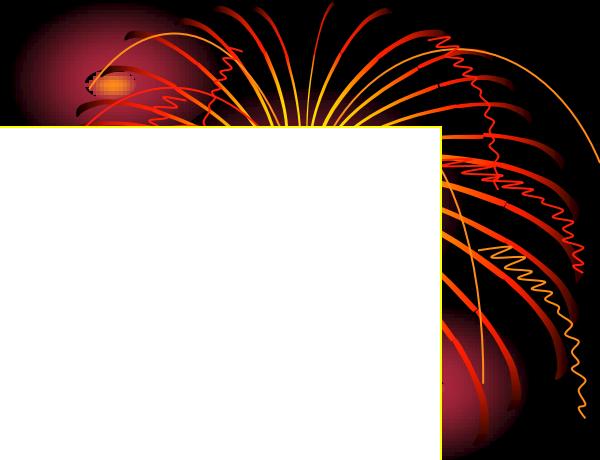


■ A capacitive load



$$v_C = V_C \sin \omega_d t \quad q_C = CV_C = CV_C \sin \omega_d t$$

$$i_C = \frac{dq_C}{dt} = \omega_d CV_C \cos \omega_d t$$



$$i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos \omega_d t$$

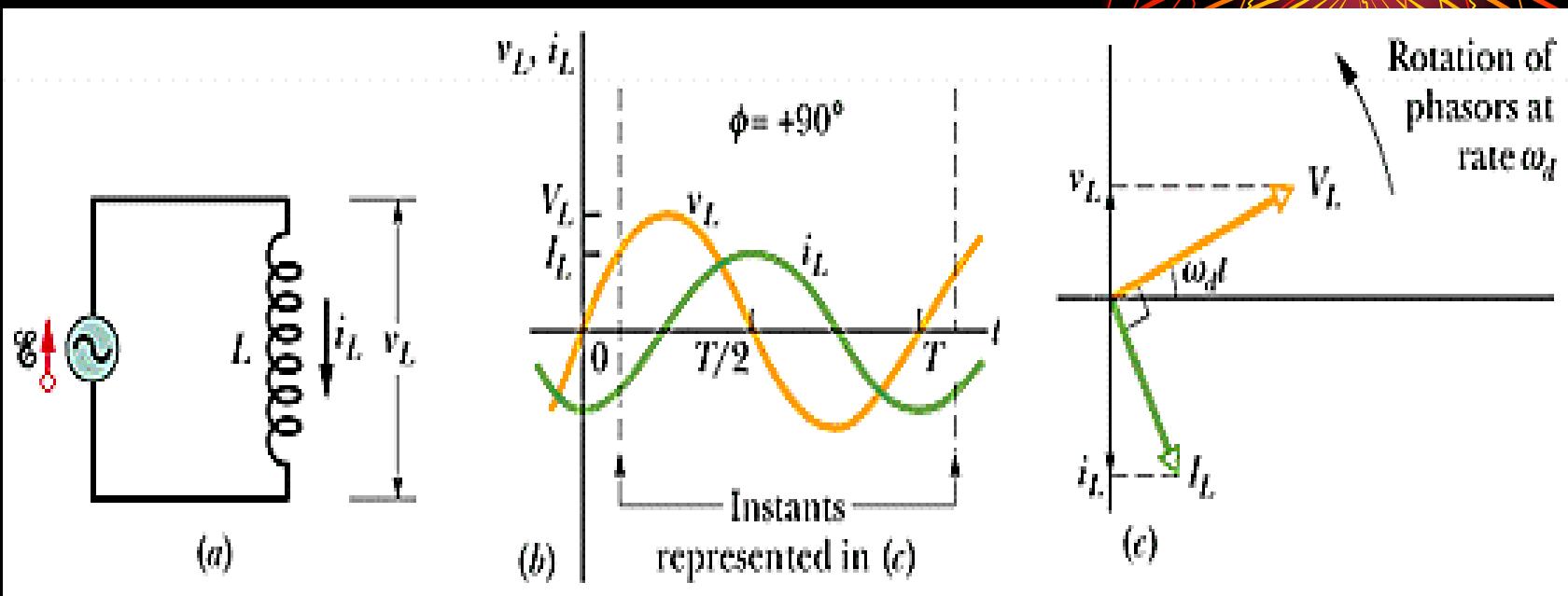
$$X_C \equiv \frac{1}{\omega_d C} \quad (\text{capacitive reactance})$$

$$\cos \omega_d t = \sin(\omega_d t + 90^\circ)$$

$$i_C = \frac{V_C}{X_C} \sin(\omega_d t + 90^\circ) = I_C \sin(\omega_d t - \phi)$$

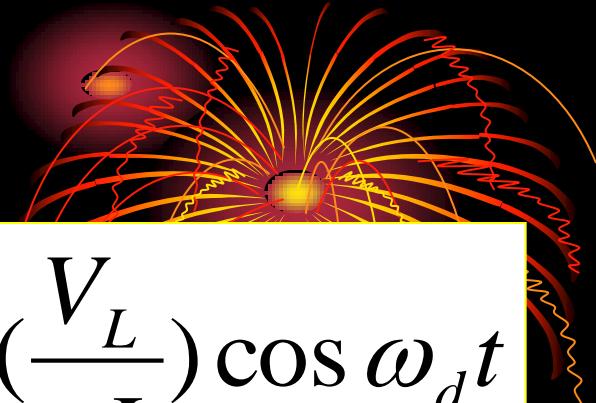
$$\phi = -90^\circ \quad V_C = I_C X_C$$

■ An inductive load



$$v_L = V_L \sin \omega_d t \quad v_L = L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_L}{L} \sin \omega_d t$$



$$i_L = \int di_L = \frac{V_L}{L} \int \sin \omega_d t dt = -\left(\frac{V_L}{\omega L}\right) \cos \omega_d t$$

$X_L \equiv \omega_d L$ (inductive reactance)

$$-\cos \omega_d t = \sin(\omega_d t - 90^\circ)$$

$$i_L = \frac{V_L}{X_L} \sin(\omega_d t - 90^\circ) = I_L \sin(\omega_d t - \phi)$$

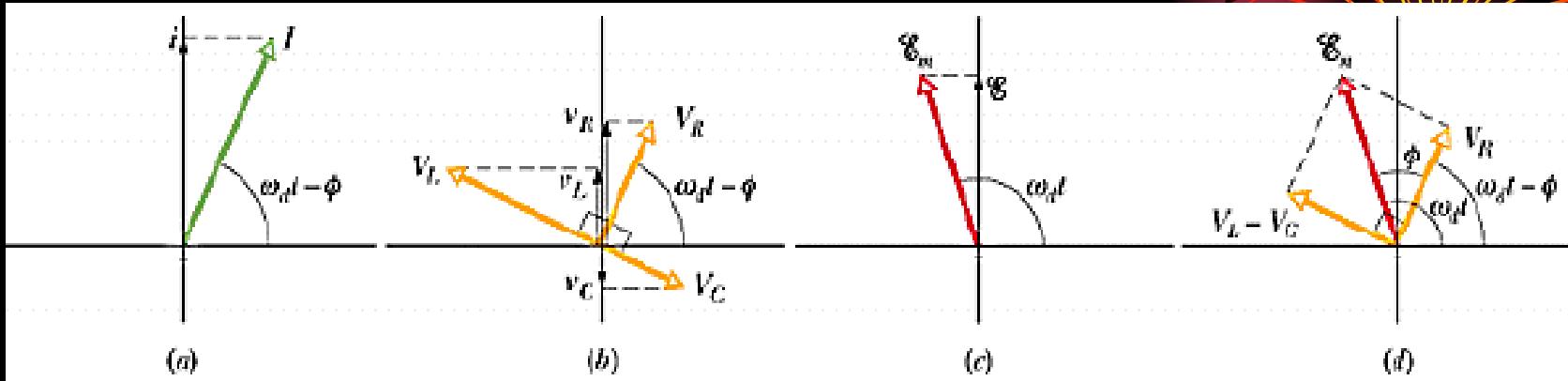
$$\phi = +90^\circ \quad V_L = I_L X_L$$

11-9 The Series RLC Circuits



TABLE 33-2 PHASE AND AMPLITUDE RELATIONS
FOR ALTERNATING CURRENTS AND VOLTAGES

CIRCUIT ELEMENT	SYMBOL	RESISTANCE OR REACTANCE	PHASE OF THE CURRENT	PHASE ANGLE ϕ	AMPLITUDE RELATION
Resistor	R	R	In phase with v_R	0	$V_R = I_R R$
Capacitor	C	$X_C = 1/\omega_d C$	Leads v_C by 90°	- 90°	$V_C = I_C X_C$
Inductor	L	$X_L = \omega_d L$	Lags v_L by 90°	+ 90°	$V_L = I_L X_L$



$$\begin{aligned}\mathcal{E} &= v_R + v_C + v_L \\ \mathcal{E}_m^2 &= V_R^2 + (V_L - V_C)^2 \\ &= (IR)^2 + (IX_L - IX_C)^2\end{aligned}$$

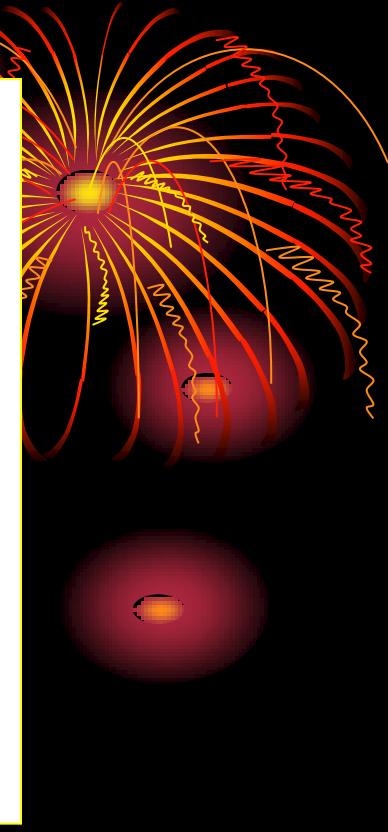
$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} \equiv \frac{\mathcal{E}_m}{Z}$$

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} \equiv \frac{\mathcal{E}_m}{Z}$$

Z = impedance

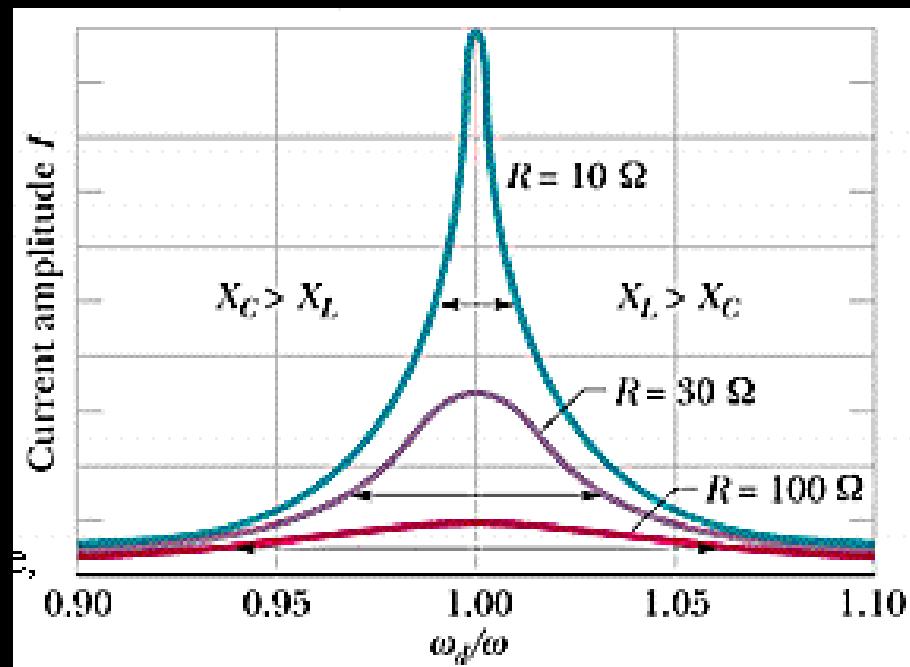
$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

- $X_L > X_C$: inductive load
- $X_L < X_C$: capacitive load
- $X_L = X_C$: resonance

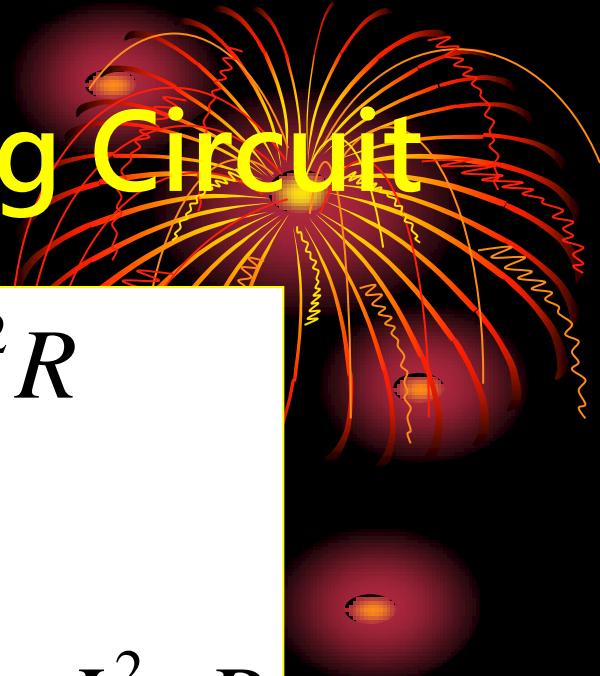


$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

$$X_L = X_C \rightarrow \omega_d L = \frac{1}{\omega_d C} \rightarrow \omega_d = \frac{1}{\sqrt{LC}}$$



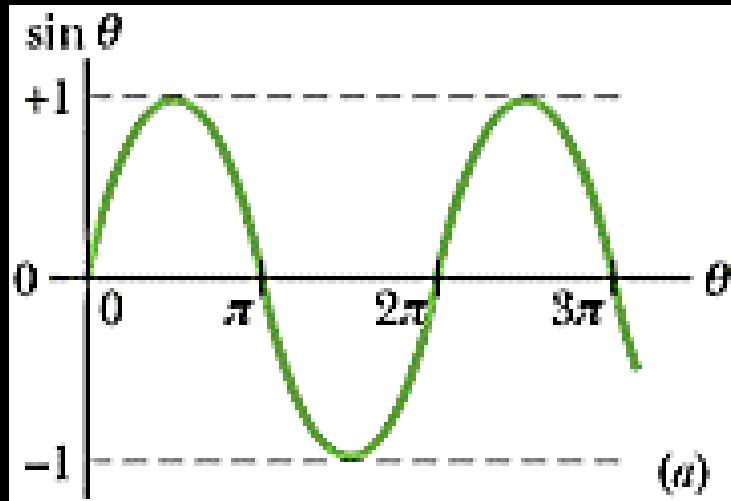
11-10 Power in Alternating Circuit



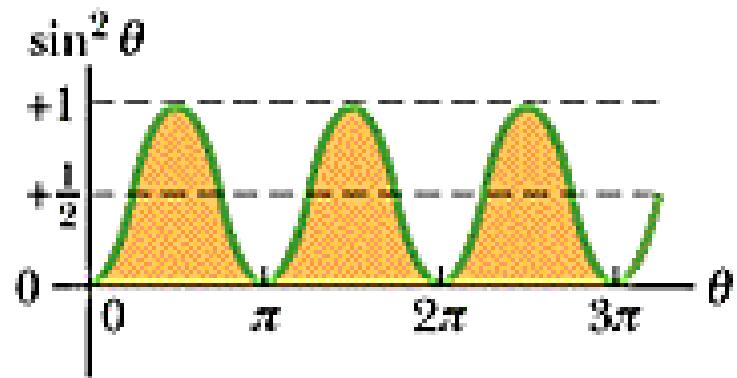
$$P = i^2 R = [I \sin(\omega_d t - \phi)]^2 R$$

$$= I^2 R \sin^2(\omega_d t - \phi)$$

$$P_{av} = I^2 R / 2 = (I / \sqrt{2})^2 R \equiv I_{rms}^2 R$$



(a)



(b)

■ The power factor $\cos\phi$



$$P_{av} = I_{rms}^2 R \quad I_{rms} = \mathcal{E}_{rms} / Z$$



$$P_{av} = (\mathcal{E}_{rms} / Z) \underline{I_{rms} R} = \mathcal{E}_{rms} I_{rms} R / Z$$

$$R/Z = IR/IZ = V_R / \mathcal{E}_m = \cos\phi$$

$$\rightarrow P_{av} = \mathcal{E}_{rms} I_{rms} \cos\phi \text{ (power factor)}$$

11-11 Transformers

- A practical example - Quebec → Montreal



$$P_{av} = \mathcal{E}I$$

$$= (7.35 \times 10^5 \text{ V})(500 \text{ A}) = 368 \text{ MW}$$

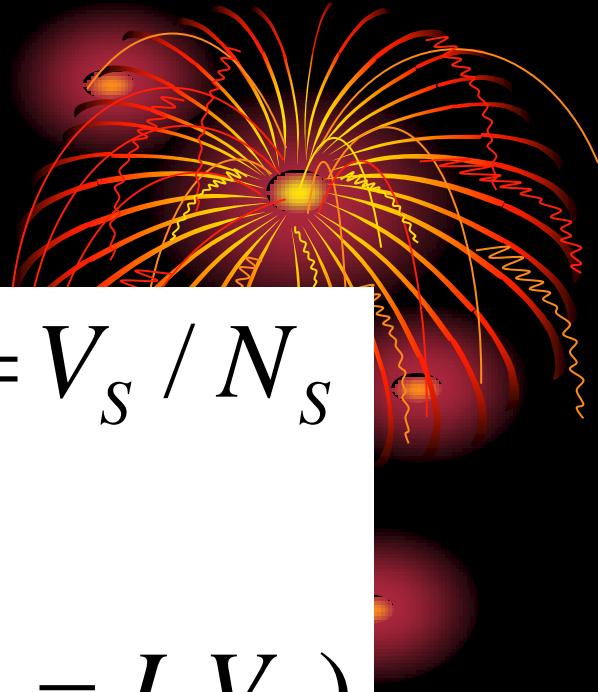
$$P_{av,1} = I^2 R = (500 \text{ A})^2$$

$$\times (0.22 \Omega / m \times 1000 \text{ km}) = 55.0 \text{ MW}$$

$$P_{av,2} = I^2 R = (1000 \text{ A})^2$$

$$\times (0.22 \Omega / m \times 1000 \text{ km}) = 220 \text{ MW}$$

■ The transformer



$$\mathcal{E}_{turn} = d\Phi_B / dt = V_P / N_P = V_S / N_S$$

$$\rightarrow V_S = V_P (N_S / N_P)$$

$$I_S = I_P (N_P / N_S) \quad (I_P V_P = I_S V_S)$$

$$I_S = V_S / R = V_P (N_S / N_P) / R$$

$$\rightarrow I_P = V_P (N_S / N_P)^2 / R$$

$$\rightarrow R_{eq} = (N_P / N_S)^2 R$$

