Home Work 8

8-1 In Fig. 28-34, a conducting rectangular solid of dimensions d_x = 5.00 m, d_y = 3.00 m, and d_z = 2.00 m moves at constant velocity **v** = (20.0m/s)**i** through a uniform magnetic field **B** = (30.0mT)**j**. What are the resulting (a) electric field within the solid, in unit-vector notation, and (b) potential difference across the solid?

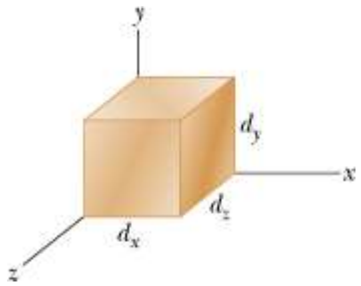


Figure 28-34 Problems 15.

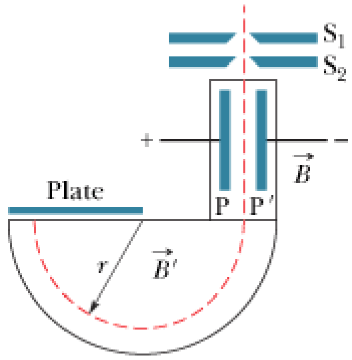


Figure 28-53 Problem 76.

Sol:

(a) We seek the electrostatic field established by the separation of charges (brought on by the magnetic force). With Eq. 28-10, we define the magnitude of the electric field as

$$|\vec E| = v|\vec B| = (20.0\ \text{m/s})(0.030\ \text{T}) = 0.600\ \text{V/m}\,.$$

Its direction may be inferred from Figure 28-8; its direction is opposite to that defined by $\vec v \times \vec B$. In summary,

$$\vec E = -(0.600\,\text{V/m})\hat{\text{k}}$$

which insures that $\vec F = q(\vec E + \vec v \times \vec B)$ vanishes.

(b) Equation 28-9 yields $V = Ed = (0.600\ \text{V/m})(2.00\ \text{m}) = 1.20\ \text{V}$.

8-2 Bainbridge's mass spectrometer, shown in Fig. 28-53, separates ions having the same velocity. The ions, after entering through slits, S_1 and S_2, pass through a velocity selector composed of an electric field produced by the charged plates P and P′, and a magnetic field **B** perpendicular to the electric field and the ion path. The ions that then pass undeviated through the crossed **E** and **B** fields enter into a region where a second magnetic field **B′** exists, where they are made to follow circular paths. A photographic plate (or a modern detector) registers their arrival. Show that, for the ions, $q/m = E/rBB'$, where r is the radius of the circular orbit.

Sol:

Using Eq. 28-16, the charge-to-mass ratio is $\frac{q}{m}=\frac{v}{B'r}$. With the speed of the ion given by $v=E/B$ (using Eq. 28-7), the expression becomes

$$\frac{q}{m}=\frac{E/B}{B'r}=\frac{E}{BB'r}.$$

8-3 A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What are the (a) magnitude and (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding?

Sol: (a) The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; mg, the magnitude of the (downward) force of gravity; $\vec{F}_N$, the normal force exerted by the stationary rails upward on the rod; and $\vec{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that $\vec{f}$ points westward (and is equal to its maximum possible value $\mu_s F_N$). Thus, $\vec{F}$ has an eastward component F_x and an upward component F_y, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the right-hand rule, a downward component (B_d) of $\vec{B}$ will produce the eastward F_x, and a westward component (B_w) will produce the upward F_y. Specifically,

$$F_x=iLB_d,\quad F_y=iLB_w.$$

Considering forces along a vertical axis, we find

$$F_N=mg-F_y=mg-iLB_w$$

so that

$$f=f_{s,\max}=\mu_s\left(mg-iLB_w\right).$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x-f=0\quad\Rightarrow\quad iLB_d=\mu_s\left(mg-iLB_w\right).$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB\cos\theta = \mu_s\left(mg - iLB\sin\theta\right) \;\Rightarrow\; B = \frac{\mu_s mg}{iL\left(\cos\theta + \mu_s\sin\theta\right)}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°.$$

Consequently,

$$B_{\min} = \frac{0.60(1.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(50\,\mathrm{A})(1.0\,\mathrm{m})(\cos 31° + 0.60\sin 31°)} = 0.10\,\mathrm{T}.$$

(b) As shown above, the angle is $\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°$.

8-4 A positron with kinetic energy 2.00 keV is projected into a uniform magnetic field **B** of magnitude 0.100 T, with its velocity vector making an angle of 89.0° with **B**. Find (a) the period, (b) the pitch p, and (c) the radius r of its helical path.

Sol:

(a) If v is the speed of the positron then $v\sin\phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv\sin\phi = m_e(v\sin\phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v/eB)\sin\phi$. The period is given by

$$T = \frac{2\pi r}{v\sin\phi} = \frac{2\pi m_e}{eB} = \frac{2\pi(9.11\times10^{-31}\,\mathrm{kg})}{(1.60\times10^{-19}\,\mathrm{C})(0.100\,\mathrm{T})} = 3.58\times10^{-10}\,\mathrm{s}.$$

The equation for r is substituted to obtain the second expression for T.

(b) The pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT\cos\phi$. We use the kinetic energy to find the speed: $K = \frac{1}{2}m_e v^2$ means

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.00\times10^{3}\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}{9.11\times10^{-31}\,\mathrm{kg}}} = 2.65\times10^{7}\,\mathrm{m/s}.$$

Thus,

$$p = \left(2.65\times10^{7}\ \text{m/s}\right)\left(3.58\times10^{-10}\ \text{s}\right)\cos 89° = 1.66\times10^{-4}\ \text{m}.$$

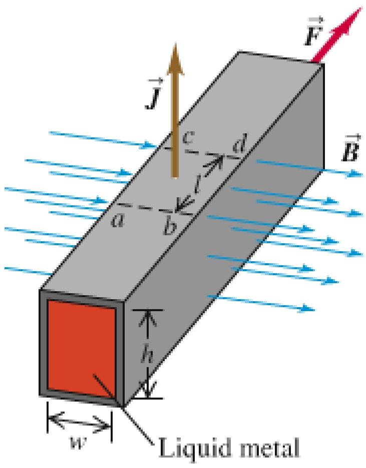
(c) The orbit radius is

$$R = \frac{m_e v \sin\phi}{eB} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.65\times10^{7}\ \text{m/s}\right)\sin 89°}{\left(1.60\times10^{-19}\ \text{C}\right)\left(0.100\ \text{T}\right)} = 1.51\times10^{-3}\ \text{m}.$$

8-5.

27.90 ••• The Electromagnetic Pump. Magnetic forces acting on conducting fluids provide a convenient means of pumping these fluids. For example, this method can be used to pump blood without the damage to the cells that can be caused by a mechanical pump. A horizontal tube with rectangular cross section (height h, width w) is placed at right angles to a uniform magnetic field with magnitude B so that a length l is in the field (Fig. P27.90). The tube is filled with a conducting liquid, and an electric current of density J is maintained in the third mutually perpendicular direction. (a) Show that the difference of pressure between a point in the liquid on a vertical plane through ab and a point in the liquid on another vertical plane through cd, under conditions in which the liquid is prevented from flowing, is $\Delta p = JlB$. (b) What current density is needed to provide a pressure difference of 1.00 atm between these two points if $B = 2.20$ T and $l = 35.0$ mm?

Figure **P27.90**



27.90. IDENTIFY: The current direction is perpendicular to $\vec{B}$, so $F = IlB$. If the liquid doesn't flow, a force $(\Delta p)A$ from the pressure difference must oppose F.

SET UP: $J = I/A$, where $A = hw$.

EXECUTE: **(a)** $\Delta p = F/A = IlB/A = JlB$.

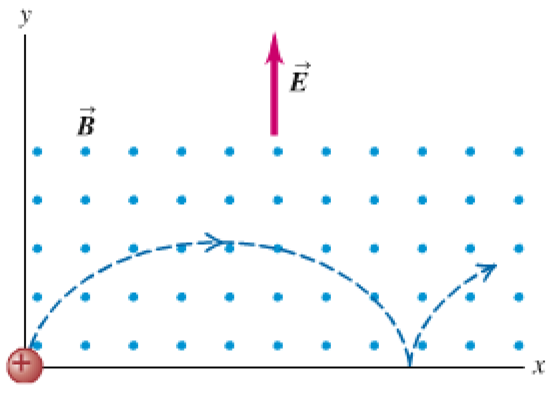
(b) $$J = \frac{\Delta p}{lB} = \frac{(1.00\ \text{atm})(1.013\times10^{5}\ \text{Pa/atm})}{(0.0350\ \text{m})(2.20\ \text{T})} = 1.32\times10^{6}\ \text{A/m}^2.$$

EVALUATE: A current of 1 A in a wire with diameter 1 mm corresponds to a current density of $J = 1.3\times10^{6}\ \text{A/m}^2$, so the current density calculated in part (c) is a typical value for circuits.

8-6.

27.91 ••• CP A Cycloidal Path. A particle with mass m and positive charge q starts from rest at the origin shown in Fig. P27.91. There is a uniform electric field $\vec{E}$ in the $+y$-direction and a uniform magnetic field $\vec{B}$ directed out of the page. It is shown in more advanced books that the path is a *cycloid* whose radius of curvature at the top points is twice the y-coordinate at that level. (a) Explain why the path has this general shape and why it is repetitive. (b) Prove that the speed at any point is equal to $\sqrt{2qEy/m}$. (*Hint:* Use energy conservation.) (c) Applying Newton's second law at the top point and taking as given that the radius of curvature here equals $2y$, prove that the speed at this point is $2E/B$.

Figure **P27.91**



27.91.IDENTIFY: The electric and magnetic fields exert forces on the moving charge. The work done by the electric field equals the change in kinetic energy. At the top point, $a_y = \dfrac{v^2}{R}$ and this acceleration must correspond to the net force.

SET UP: The electric field is uniform so the work it does for a displacement y in the y-direction is $W = Fy = qEy$. At the top point, $\vec{F}_B$ is in the $-y$-direction and $\vec{F}_E$ is in the $+y$-direction.

EXECUTE: **(a)** The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

(b) $W = qEy = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qEy}{m}}$.

(c) At the top, $F_y = qE - qvB = -\frac{mv^2}{R} = -\frac{m}{2y}\frac{2qEy}{m} = -qE$. $2qE = qvB$ and $v = \frac{2E}{B}$.

EVALUATE: The speed at the top depends on B because B determines the y-displacement and the work done by the electric force depends on the y-displacement.