Home Work Solutions 7

1. A 10-km-long underground cable extends east to west and consists of two parallel wires, each of which has resistance 13 *Ω*/km. An electrical short develops at distance *x* from the west end when a conducting path of resistance *R* connects the wires (Fig. 27-31). The resistance of the wires and the short is then 100 *Ω* when measured from the east end and 200 *Ω* when measured from the west end. What are (a) *x* and (b) *R*?

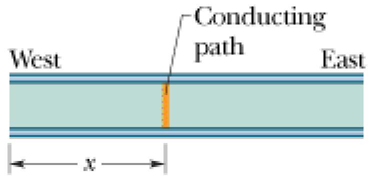

Figure 27-31 Problem 13.

Sol

(a) We denote L = 10 km and α = 13 Ω/km. Measured from the east end we have

$$R_1 = 100\ \Omega = 2\alpha(L - x) + R,$$

and measured from the west end $R_2 = 200\ \Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\,\Omega - 100\,\Omega}{4\left(13\,\Omega/\text{km}\right)} + \frac{10\,\text{km}}{2} = 6.9\,\text{km}.$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\,\Omega + 200\,\Omega}{2} - \left(13\,\Omega/\text{km}\right)\left(10\,\text{km}\right) = 20\,\Omega.$$

2. A solar cell generates a potential difference of 0.10 V when a 500 *Ω* resistor is connected across it, and a potential difference of 0.15 V when a 1000 *Ω* resistor is substituted. What are the (a) internal resistance and (b) emf of the solar cell? (c) The area of the cell is 5.0 cm^2, and the rate per unit area at which it receives energy from light is 2.0 mW/cm^2. What is the efficiency of the cell for converting light energy to thermal energy in the 1000 *Ω* external resistor?

Sol

Let the emf of the solar cell be ε and the output voltage be V. Thus,

$$V = \varepsilon - ir = \varepsilon - \left(\frac{V}{R}\right) r$$

for both cases. Numerically, we get

$$0.10\ \text{V} = \varepsilon - (0.10\ \text{V}/500\ \Omega)r$$
$$0.15\ \text{V} = \varepsilon - (0.15\ \text{V}/1000\ \Omega)r.$$

We solve for ε and r.

(a) $r = 1.0\times10^3\ \Omega$.

(b) $\varepsilon = 0.30$ V.

(c) The efficiency is

$$\frac{V^2/R}{P_{\text{received}}} = \frac{0.15\,\text{V}}{(1000\,\Omega)\left(5.0\,\text{cm}^2\right)\left(2.0\times10^{-3}\,\text{W/cm}^2\right)} = 2.3\times10^{-3} = 0.23\%.$$

3. In Fig. 27-46, $\mathscr{E}$ = 12.0 V, R_1 = 2000 Ω, R_2 = 3000 Ω, and R_3 4000 Ω. What are the potential differences (a) V_A - V_B, (b) V_B - V_C, (c) V_C - V_D, and (d) V_A - V_C?

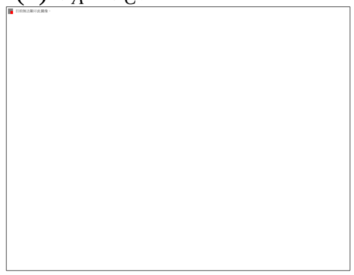

Figure 27-46 Problem 35.

Sol

(a) The symmetry of the problem allows us to use i_2 as the current in *both* of the R_2 resistors and i_1 for the R_1 resistors. We see from the junction rule that $i_3 = i_1 - i_2$. There are only two independent loop rule equations:

$$\varepsilon - i_2 R_2 - i_1 R_1 = 0$$
$$\varepsilon - 2i_1 R_1 - (i_1 - i_2) R_3 = 0$$

where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find i_1 = 0.002625 A, i_2 = 0.00225 A and $i_3 = i_1 - i_2$ = 0.000375 A. Therefore, $V_A - V_B = i_1 R_1$ = 5.25 V.

(b) It follows also that $V_B - V_C = i_3 R_3$ = 1.50 V.

(c) We find $V_C - V_D = i_1 R_1$ = 5.25 V.

(d) Finally, $V_A - V_C = i_2 R_2$ = 6.75 V.

4. In Fig. 27-52, an array of *n* parallel resistors is connected in series to a resistor and an ideal battery. All the resistors have the same resistance. If an identical resistor were added in parallel to the parallel array, the current through the battery would change by 1.25%. What is the value of *n*? *Hint*: 0.0125 (= 1/80)

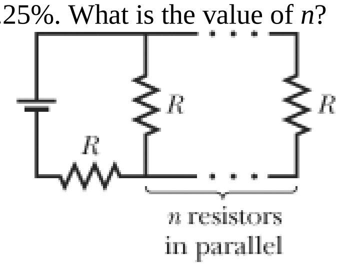

Figure 27-52 Problem 42.

Sol

The equivalent resistance in Fig. 27-52 (with *n* parallel resistors) is

$$R_{eq} = R + \frac{R}{n} = \left(\frac{n+1}{n}\right) R \quad .$$

The current in the battery in this case should be

$$i_n = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n}{n+1}\frac{V_{\text{battery}}}{R}.$$

If there were n +1 parallel resistors, then

$$i_{n+1} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n+1}{n+2}\frac{V_{\text{battery}}}{R}.$$

For the relative increase to be 0.0125 (= 1/80), we require

$$\frac{i_{n+1} - i_n}{i_n} = \frac{i_{n+1}}{i_n} - 1 = \frac{(n+1)/(n+2)}{n/(n+1)} - 1 = \frac{1}{80}.$$

This leads to the second-degree equation

$$n^2 + 2n - 80 = (n + 10)(n - 8) = 0.$$

Clearly the only physically interesting solution to this is $n = 8$. Thus, there are eight resistors in parallel (as well as that resistor in series shown toward the bottom) in Fig. 27-52.

5. In Fig. 27-61, R_s is to be adjusted in value by moving the sliding contact across it until points a and b are brought to the same potential. (One tests for this condition by momentarily connecting a sensitive ammeter between a and b; if these points are at the same potential, the ammeter will not deflect.) Show that when this adjustment is made, the following relation holds: $R_x = R_sR_2/R_1$. An unknown resistance (R_x) can be measured in terms of a standard (R_s) using this device, which is called a Wheatstone bridge.

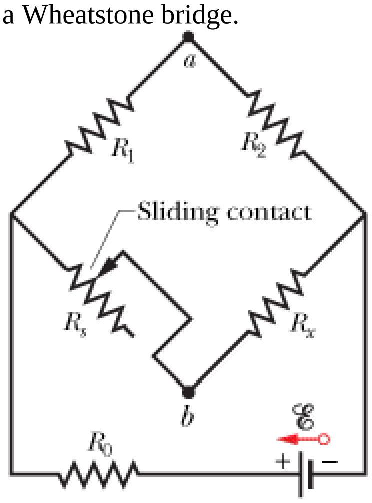

Figure 27-61 Problem 55.

Sol

Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$(R_1 + R_2)i_1 = (R_x + R_s)\frac{R_1}{R_s}i_1 \Rightarrow R_x = \frac{R_2R_s}{R_1}.$$