Home Work Solutions 6

1. How long does it take electrons to get from a car battery to the starting motor? Assume the current is 300 A and the electrons travel through a copper wire with cross-sectional area 0.21 cm^2 and length 0.85 m. The number of charge carriers per unit volume is 8.49 × 10^{28} m^{-3}.

Sol

We use $v_d = J/ne = i/Ane$. Thus,

$$t = \frac{L}{v_d} = \frac{L}{i/Ane} = \frac{LAne}{i} = \frac{(0.85\,\mathrm{m})\left(0.21\times10^{-14}\,\mathrm{m}^2\right)\left(8.47\times10^{28}/\mathrm{m}^3\right)\left(1.60\times10^{-19}\,\mathrm{C}\right)}{300\,\mathrm{A}}$$

$$= 8.1\times10^{2}\,\mathrm{s} = 13\,\mathrm{min}.$$

2. Figure 26-28 shows wire section 1 of diameter D_1 = 4.00R and wire section 2 of diameter D_2 = 2.00R, connected by a tapered section. The wire is copper and carries a current. Assume that the current is uniformly distributed across any cross-sectional area through the wire's width. The electric potential change V along the length L = 2.00 m shown in section 2 is 10.0 mV. The number of charge carriers per unit volume is 8.49 × 10^{28} m^{-3}. What is the drift speed of the conduction electrons in section 1?

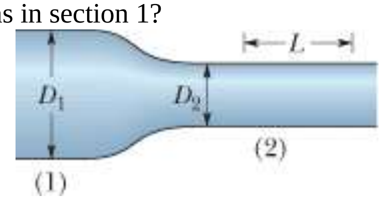

Figure 26-28 Problem 34.

Sol

The number density of conduction electrons in copper is n = 8.49 × 10^{28} /m^3. The electric field in section 2 is (10.0 μV)/(1.75 m) = 5.71 μV/m. Since ρ = 1.69 × 10^{-8} $\Omega\cdot$m for copper (see Table 26-1) then Eq. 26-10 leads to a current density vector of magnitude J_2 = (5.71 μV/m)/(1.69 × 10^{-8} $\Omega\cdot$m) = 338 A/m^2 in section 2. Conservation of electric current from section 1 into section 2 implies

$$J_1 A_1 = J_2 A_2 \quad \Rightarrow \quad J_1(4\pi R^2) = J_2(\pi R^2)$$

(see Eq. 26-5). This leads to J_1 = 84.5 A/m^2. Now, for the drift speed of conduction-electrons in section 1, Eq. 26-7 immediately yields

$$v_d = \frac{J_1}{ne} = 6.22\times10^{-9}\ \mathrm{m/s}$$

3. Earth's lower atmosphere contains negative and positive ions that are produced by radioactive elements in the soil and cosmic rays from space. In a certain region, the atmospheric electric field strength is 120 V/m and the field is directed vertically down. This field causes singly charged positive ions, at a density of 620 cm^{-3}, to drift downward and singly charged negative ions, at a density of 550 cm^{-3}, to drift upward (Fig. 26-27). The measured conductivity of the air in that region is 2.70 × 10^{-14} $(\Omega\cdot\mathrm{m})^{-1}$. Calculate (a) the magnitude of the current density and (b) the ion drift speed, assumed to be the same for positive and negative ions.

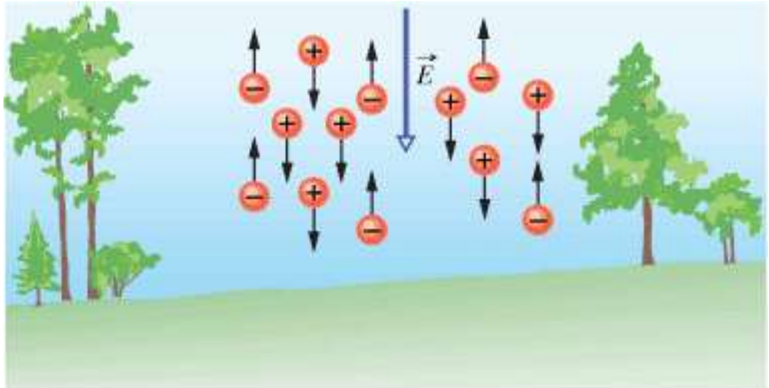

Figure 26-27 Problem 32.

Sol

We use $J = \sigma E = (n_+ + n_-)ev_d$, which combines Eq. 26-13 and Eq. 26-7.

(a) The magnitude of the current density is

$$J = \sigma E = (2.70 \times 10^{-14} / \Omega \cdot \text{m})\ (120 \text{ V/m}) = 3.24 \times 10^{-12} \text{ A/m}^2.$$

(b) The drift velocity is

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{\left(2.70\times10^{-14}/\Omega\cdot\text{m}\right)\left(120 \text{ V/m}\right)}{\left[\left(640+550\right)/\text{cm}^3\right]\left(1.60\times10^{-19} \text{ C}\right)} = 1.70 \text{ cm/s}.$$

4. *Swimming during a storm*. Figure 26-30 shows a swimmer at distance D = 35.0 m from a lightning strike to the water, with current I = 78 kA. The water has resistivity 30$\Omega \cdot$ m, the width of the swimmer along a radial line from the strike is 0.70 m, and his resistance across that width is 4.00 kΩ. Assume that the current spreads through the water over a hemisphere centered on the strike point. What is the current through the swimmer?

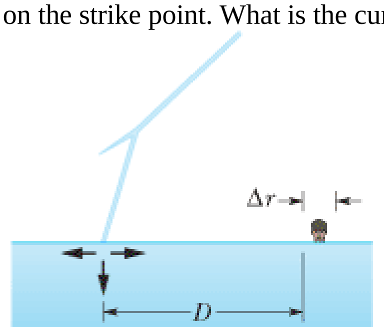

Figure 26-30 Problem 36.

Sol

Since the current spreads uniformly over the hemisphere, the current density at any given radius r from the striking point is $J = I / 2\pi r^2$. From Eq. 26-10, the magnitude of the electric field at a radial distance r is

$$E = \rho_w J = \frac{\rho_w I}{2\pi r^2},$$

where $\rho_w = 30\ \Omega \cdot \text{m}$ is the resistivity of water. The potential difference between a point at radial distance D and a point at $D+\Delta r$ is

$$\Delta V = -\int_D^{D+\Delta r} E dr = -\int_D^{D+\Delta r} \frac{\rho_w I}{2\pi r^2} dr = \frac{\rho_w I}{2\pi}\left(\frac{1}{D+\Delta r} - \frac{1}{D}\right) = -\frac{\rho_w I}{2\pi}\frac{\Delta r}{D(D+\Delta r)},$$

which implies that the current across the swimmer is

$$i = \frac{|\Delta V|}{R} = \frac{\rho_w I}{2\pi R}\frac{\Delta r}{D(D+\Delta r)}.$$

Substituting the values given, we obtain

$$i = \frac{(30.0\ \Omega\cdot\text{m})(7.80\times 10^4\ \text{A})}{2\pi(4.00\times 10^3\ \Omega)}\frac{0.70\ \text{m}}{(38.0\ \text{m})(38.0\ \text{m}+0.70\ \text{m})} = 4.43\times 10^{-2}\ \text{A}.$$