Home Work Solutions 4/5

1. Figure 24-42 shows a thin plastic rod of length L = 12.0 cm and uniform positive charge Q = 56.1 fC lying on an x axis. With V = 0 at infinity, find the electric potential at point P_1 on the axis, at distance d = 2.50 cm from one end of the rod.

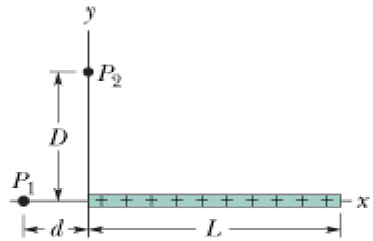


Figure 24-42

Sol

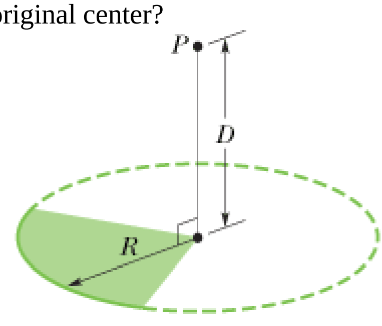
Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right)$$
$$= \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(56.1\times10^{-15}\,\mathrm{C})}{0.12\ \mathrm{m}}\ln\left(1+\frac{0.12\ \mathrm{m}}{0.025\ \mathrm{m}}\right) = 6.31 \times 10^{-3}\ \mathrm{V}$$

2. A plastic disk of radius R = 64.0 cm is charged on one side with a uniform surface charge density σ = 7.73 fC/m^2, and then three quadrants of the disk are removed. The remaining quadrant is shown in Fig. 24-45. With V = 0 at infinity, what is the potential due to the remaining quadrant at point P, which is on the central axis of the original disk at distance D = 25.9 cm from the original center?



Sol

The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P, so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk. We consider a ring of charge with radius r and (infinitesimal) width dr. Its area is $2\pi r\, dr$ and it contains charge $dq = 2\pi\sigma r\, dr$. All the charge in it is a distance $\sqrt{r^2+D^2}$ from P, so the potential it produces at P is

$$dV = \frac{1}{4\text{p}\varepsilon_0} \frac{2\text{p}\sigma r dr}{\sqrt{r^2+D^2}} = \frac{\sigma r dr}{2\varepsilon_0\sqrt{r^2+D^2}}.$$

The total potential at *P* is

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{rdr}{\sqrt{r^2+D^2}} = \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+D^2}\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right].$$

The potential V_{sq} at *P* due to a single quadrant is

$$V_{sq} = \frac{V}{4} = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right] = \frac{(7.73\times10^{-15}\,\text{C/m}^2)}{8(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left[\sqrt{(0.640\ \text{m})^2+(0.259\ \text{m})^2}-0.259\ \text{m}\right]$$
$$= 4.71\times10^{-5}\ \text{V}.$$

Note: Consider the limit D ? R. The potential becomes

$$V_{sq} = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right] \approx \frac{\sigma}{8\varepsilon_0}\left[D\left(1+\frac{1}{2}\frac{R^2}{D^2}+\cdots\right)-D\right] = \frac{\sigma}{8\varepsilon_0}\frac{R^2}{2D} = \frac{\pi R^2\sigma/4}{4\pi\varepsilon_0 D} = \frac{q_{sq}}{4\pi\varepsilon_0 D}$$

where $q_{sq} = \pi R^2\sigma/4$ is the charge on the quadrant. In this limit, we see that the potential resembles that due to a point charge q_{sq}.

3. The thin plastic rod of length L = 10.0 cm in Fig. 24-42 has a nonuniform linear charge density $\lambda = cx$, where c = 49.9 pC/m^2. (a) With V = 0 at infinity, find the electric potential at point P_2 on the y axis at $y = D$ = 3.56 cm. (b) Find the electric field component E_y at P_2. (c) Why cannot the field component E_x at P_2 be found using the result of (a)?

Sol

(a) Consider an infinitesimal segment of the rod from *x* to *x + dx*. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}dx.$$

Thus,

$$V = \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}dx = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2}-y\right)$$
$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\text{C/m}^2)\left(\sqrt{(0.120\ \text{m})^2+(0.0356\ \text{m})^2}-0.0356\ \text{m}\right)$$
$$= 4.02\times10^{-2}\ \text{V}.$$

(b) The *y* component of the field there is

$$E_y = -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}\left(\sqrt{L^2+y^2}-y\right) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right)$$

$$= (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(49.9\times10^{-12}\,\mathrm{C/m^2})\left(1-\frac{0.0356\ \mathrm{m}}{\sqrt{(0.120\ \mathrm{m})^2+(0.0356\ \mathrm{m})^2}}\right)$$

$$= 0.321\ \mathrm{N/C}.$$

(c) We obtained above the value of the potential at any point *P* strictly on the *y*-axis. In order to obtain $E_x(x, y)$ we need to first calculate $V(x, y)$. That is, we must find the potential for an arbitrary point located at (x, y). Then $E_x(x, y)$ can be obtained from $E_x(x,y) = -\partial V(x,y)/\partial x$.

4. Figure 25-43 displays a 12.0 V battery and 3 uncharged capacitors of capacitances $C_1 = 4.00\ \mu$F, $C_2 = 6.00\ \mu$F, and $C_3 = 3.00\ \mu$F. The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right. What is the final charge on (a) capacitor 1, (b) capacitor 2, and (c) capacitor 3?

Sol

The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}.$$

Thus, $C_{\text{eq}} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination, and the potential difference across the equivalent capacitor is given by q_2/C_{eq}. The potential difference across capacitor 1 is q_1/C_1, where q_1 is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{\text{eq}}$. Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where V_0 is the original potential difference across capacitor 1.

(a) Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{\text{eq}}}$$

$$q_1 + q_2 = C_1 V_0$$

for q_1 and q_2, we obtain

$$q_1 = \frac{C_1^2 V_0}{C_{\text{eq}} + C_1} = \frac{C_1^2 V_0}{\dfrac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3}.$$

With $V_0 = 16.0$ V, $C_1 = 4.00\ \mu$F, $C_2 = 6.00\ \mu$F and $C_3 = 3.00\ \mu$F, we find $C_{\text{eq}} = 2.00\ \mu$F and $q_1 = 42.7\ \mu$C.

(b) The charge on capacitors 2 is

$$q_2 = C_1V_0 - q_1 = (4.00\,\mu\text{F})(16.0\text{ V}) - 42.7\ \mu\text{C} = 21.3\ \mu\text{C}.$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3 = C_1V_0 - q_1 = (4.00\ \mu\text{F})(16.0\text{ V}) - 42.7\ \mu\text{C} = 21.3\ \mu\text{C}.$$

5. The parallel plates in a capacitor, with a plate area of 8.50 cm^2 and an air-filled separation of 3.00 mm, are charged by a 6.00 V battery. They are then disconnected from the battery and pulled apart (without discharge) to a separation of 8.00 mm. Neglecting fringing, find (a) the potential difference between the plates, (b) the initial stored energy, (c) the final stored energy, and (d) the work required to separate the plates.

Sol

(a) Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\varepsilon_0 A/d_i$, the charge is $q = CV = \varepsilon_0 AV_i/d_i$. After the plates are pulled apart, their separation is d_f and the potential difference is V_f. Then $q = \varepsilon_0 AV_f/2d_f$ and

$$V_f = \frac{d_f}{\varepsilon_0 A}q = \frac{d_f}{\varepsilon_0 A}\frac{\varepsilon_0 A}{d_i}V_i = \frac{d_f}{d_i}V_i.$$

With $d_i = 3.00\times10^{-3}\text{m}$, $V_i = 6.00\text{ V}$, and $d_f = 8.00\times10^{-3}\text{m}$, we have $V_f = 16.0\text{ V}$.

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV_i^2 = \frac{\varepsilon_0 AV_i^2}{2d_i} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.50\times10^{-4}\ \text{m}^2)(6.00\text{ V})^2}{2(3.00\times10^{-3}\ \text{m})} = 4.51\times10^{-11}\ \text{J}.$$

(c) The final energy stored is

$$U_f = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}\left(\frac{d_f}{d_i}V_i\right)^2 = \frac{d_f}{d_i}\left(\frac{\varepsilon_0 AV_i^2}{d_i}\right) = \frac{d_f}{d_i}U_i.$$

With $d_f/d_i = 8.00/3.00$, we have $U_f = 1.20\times10^{-10}\ \text{J}$.

(d) The work done to pull the plates apart is the difference in the energy:

$W = U_f - U_i = 7.52\times10^{-11}$ J.