Home Work Solutions 10

10-1 Figure A shows two parallel loops of wire having a common axis. The smaller loop (radius r) is above the larger loop (radius R) by a distance x>>R. Consequently, the magnetic field due to the counterclockwise current *i* in the larger loop is nearly uniform throughout the smaller loop. Suppose that x is increasing at the constant rate dx/dt = *v*. (a) Find an expression for the magnetic flux through the area of the smaller loop as a function of x. (Hint: See Eq. 29-27.) In the smaller loop, find (b) an expression for the induced emf and (c) the direction of the induced current.

Sol: (a) In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Eq. 29-27, with $z = x$ (taken to be much greater than R), gives

$$\vec{B} = \frac{\mu_0 i R^2}{2x^3}\,\hat{\mathrm{i}}$$

where the $+x$ direction is upward in Fig. 30-50. The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = \frac{\pi\mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) As the smaller loop moves upward, the flux through it decreases, and we have a situation like that shown in Fig. 30-5(b). The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

10-2 Figure B shows a copper strip of width W = 16.0 cm that has been bent to form a shape that consists of a tube of radius R = 1.8 cm plus two parallel flat extensions. Current *i* = 35 mA is distributed uniformly across the width so that the tube is effectively a one-turn solenoid. Assume that the magnetic field outside the tube is negligible and the field inside the tube is uniform. What are (a) the magnetic field magnitude inside the tube and (b) the inductance of the tube (excluding the flat extensions)?

Sol: (a) We imagine dividing the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is

$$B=\mu_0 n\Delta i=\mu_0\left(\frac{N}{W}\right)\left(\frac{i}{N}\right)=\frac{\mu_0 i}{W}=\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.035\mathrm{A})}{0.16\mathrm{m}}=2.7\times10^{-7}\,\mathrm{T}.$$

(b) Eq. 30-33 leads to

$$L=\frac{\Phi_B}{i}=\frac{\pi R^2 B}{i}=\frac{\pi R^2\left(\mu_0 i/W\right)}{i}=\frac{\pi\mu_0 R^2}{W}=\frac{\pi(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.018\mathrm{m})^2}{0.16\mathrm{m}}=8.0\times10^{-9}\,\mathrm{H}.$$

10-3 A coil C of N turns is placed around a long solenoid S of radius R and n turns per unit length, as in Figure C. (a) Show that the mutual inductance for the coil–solenoid combination is given by $M = \mu_0\pi R^2 nN$. (b) Explain why M does not depend on the shape, size, or possible lack of close packing of the coil.

Sol: (a) The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N(\mu_0 i_s n\pi R^2)}{i_s} = \mu_0\pi R^2 nN\ .$$

(b) As long as the magnetic field of the solenoid is entirely contained within the cross-section of the coil we have $\Phi_{sc} = B_s A_s = B_s\pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

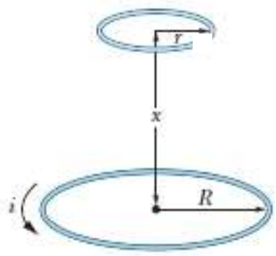

Fig. A

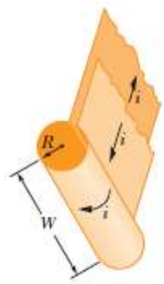

Fig. B

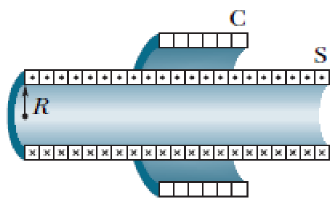

Fig. C

10-4 *Inductors in series*. Two inductors L_1 and L_2 are connected in series and are separated by a large distance so that the magnetic field of one cannot affect the other. (a) Show that the equivalent inductance is given by $L_{eq} = L_1 + L_2$. (*Hint:* Review the derivations for resistors in series and capacitors in series. Which is similar here?) (b) What is the generalization of (a) for N inductors in series?

Sol: (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add ($V_1 + V_2$), then inductances in series must add, $L_{eq} = L_1 + L_2$, just as was the case for resistances.

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that magnetic field lines from one inductor should not have significant presence in any other.

(b) Just as with resistors, $L_{\text{eq}} = \sum_{n=1}^{N} L_n$.

10-5 *Inductors in parallel*. Two inductors L_1 and L_2 are connected in parallel and separated by a large distance so that the magnetic field of one cannot affect the other.

(a) Show that the equivalent inductance is given by $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$.

(*Hint:* Review the derivations for resistors in parallel and capacitors in parallel. Which is similar here?) (b) What is the generalization of (a) for N inductors in parallel?

Sol: (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the Eq. 27-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt}.$$

Thus, although the inductance equation Eq. 30-35 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also apply to inductors. Therefore,

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that the field of one inductor not to have significant influence (or “coupling”) in the next.

(b) Just as with resistors, $\frac{1}{L_{\text{eq}}} = \sum_{n=1}^{N} \frac{1}{L_n}$.