『北區高中物理科學人才培育』計畫高二物理期末考試卷

1 如電位為 $V = 2.00xyz^2$ ，則在($3.00\mathbf{i}-2.00\mathbf{j}+4.00\mathbf{k}$)m 處的電場大小為何？(V is in volts and x, y, and z are in meters) (10%)

2 (a) In Fig. 2, Rs is to be adjusted in value by moving the sliding contact across it until points a and b are brought to the same potential. (One tests for this condition by momentarily connecting a sensitive ammeter between a and b; if these points are at the same potential, the ammeter will not deflect.) Show that when this adjustment is made, the following relation holds: Rx = RsR2/R1. An unknown resistance (Rx) can be measured in terms of a standard (Rs) using this device, which is called a Wheatstone bridge. (10%)

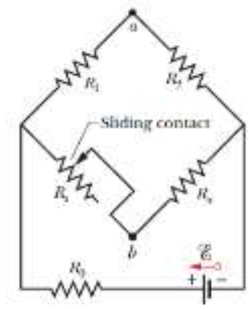

Fig. 2

2 (b) If points a and b in Fig. 2 are connected by a wire of resistance r, show that the current in the wire is

$$i = \frac{\mathscr{E}(R_s - R_x)}{(R + 2r)(R_s + R_x) + 2R_sR_x}$$

Where $\mathscr{E}$ is the emf of the ideal battery and R = R1 = R2. Assume that R0 equals zero. (20%)

3 Figure 3 shows a wood cylinder of mass m = 0.250 kg and length L = 0.100 m, with N = 10.0 turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T, what is the least current *i* through the coil that keeps the cylinder from rolling down the plane? (20%)

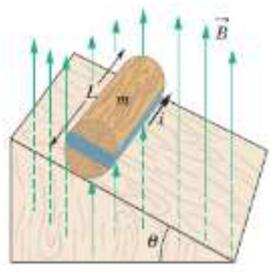
Fig. 3

4 圖四中之 coil 有 N_2 圈、toroid 有 N_1 圈。Toroid 的內半徑為 a、外半徑為 b、高度為 h。試證 toroid–coil 之互感 M 為 (20%)

$$M = \frac{\mu_0 N_1 N_2 h}{2\pi} \ln \frac{b}{a}$$

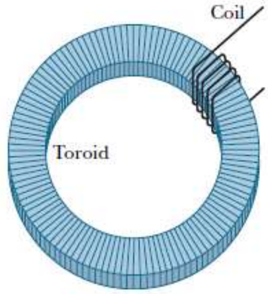

Figure 4

5 (a)請說明 normalization(歸一化)，亦即 $\int_0^\infty P(r)dr = 1$ 的物理意義。(10%)

5 (b)若氫原子在基態時之徑向機率密度(the radial probability density for the ground

state of the hydrogen atom)為 (10%)

$$P(r) = \frac{4}{a^3} r^2 e^{-2r/a}$$

試證其已歸一化(normalized)。本題可使用下列公式：

$$\int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}}$$

1 *Sol*: We apply Eq. 24-41:

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$\rightarrow$

$$E_x = -\frac{\partial V}{\partial x} = -2.00 yz^2$$
$$E_y = -\frac{\partial V}{\partial y} = -2.00 xz^2$$
$$E_z = -\frac{\partial V}{\partial z} = -4.00 xyz$$

which, at (x, y, z) = (3.00 m, –2.00 m, 4.00 m), gives

(E_x, E_y, E_z) = (64.0 V/m, –96.0 V/m, 96.0 V/m).

The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 150\,\text{V/m} = 150\,\text{N/C}.$$

2 (a) *Sol:* Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$\left(R_1 + R_2\right)i_1 = \left(R_x + R_s\right)\frac{R_1}{R_s}i_1 \quad \Rightarrow \quad R_x = \frac{R_2 R_s}{R_1}.$$

2 (b) *Sol:* (a) Placing a wire (of resistance r) with current i running directly from point a to point b in Fig. 27-61 divides the top of the picture into a left and a right triangle. If we label the currents through each resistor with the corresponding subscripts (for instance, i_s goes toward the lower right through R_s and i_x goes toward the upper right through R_x), then the currents must be related as follows:

FIG. 27-61
Problems 55 and 111.

$$i_0 = i_1 + i_s\,,\ \ i_1 = i + i_2$$
$$i_s + i = i_x\ ,\ \ i_2 + i_x = i_0$$

where the last relation is not independent of the previous three. The loop equations for the two triangles and also for the bottom loop (containing the battery and point b) lead to

$$i_s R_s - i_1 R_1 - ir = 0$$
$$i_2 R_s - i_x R_x - ir = 0$$
$$\varepsilon - i_0 R_0 - i_s R_s - i_x R_x = 0.$$

We incorporate the current relations from above into these loop equations in order to obtain three well-posed "simultaneous" equations, for three unknown currents (i_s, i_1 and i):

$$i_s R_s - i_1 R_1 - ir = 0$$
$$i_1 R_2 - i_s R_x - i(r + R_x + R_2) = 0$$
$$\varepsilon - i_s(R_0 + R_s + R_x) - i_1 R_0 - iR_x = 0$$

The problem statement further specifies $R_1 = R_2 = R$ and $R_0 = 0$, which causes our solution for i to simplify significantly. It becomes

$$i = \frac{\varepsilon(R_s - R_x)}{2rR_s + 2R_x R_s + R_s R + 2rR_x + R_x R}$$

which is equivalent to the result shown in the problem statement.

3 *Sol*: We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity *mg*, acting downward from the center of mass, the normal force of the incline F_N, acting perpendicularly to the incline through the center of mass, and the force of friction *f*, acting up the incline at the point of contact. We take the *x* axis to be positive down the incline. Then the *x* component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B \sin\theta$, and the force of friction produces a torque with magnitude *fr*, where *r* is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B\sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate *f* from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = Ni(2rL)$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{2(10.0)(0.100\,\mathrm{m})(0.500\,\mathrm{T})} = 2.45\,\mathrm{A}.$$

4 *Sol*: The flux Φ_B over the toroid cross-section is (see, for example Problem 30-60)

$$\Phi_B = \int_a^b B\,dA = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right) h\,dr = \frac{\mu_0 Nih}{2\pi}\ln\left(\frac{b}{a}\right).$$

Thus, the coil-toroid mutual inductance is

$$M_{ct} = \frac{N_c \Phi_{ct}}{i_t} = \frac{N_c}{i_t}\frac{\mu_0 i_t N_t h}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{\mu_0 N_1 N_2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

where $N_t = N_1$ and $N_c = N_2$.

5 (a) *Sol*:在一氫原子中，電子必然存在於圍繞原子核週圍之空間。

5 (b) *Sol*: The radial probability function for the ground state of hydrogen is

$$P(r) = (4r^2/a^3)e^{-2r/a},$$

where a is the Bohr radius. We want to evaluate the integral $\int_0^\infty P(r)\,dr$. We set $n = 2$ and replace a in the given formula with $2/a$ and x with r. Then

$$\int_0^\infty P(r)dr = \frac{4}{a^3}\int_0^\infty r^2 e^{-2r/a}dr = \frac{4}{a^3}\frac{2}{(2/a)^3} = 1.$$