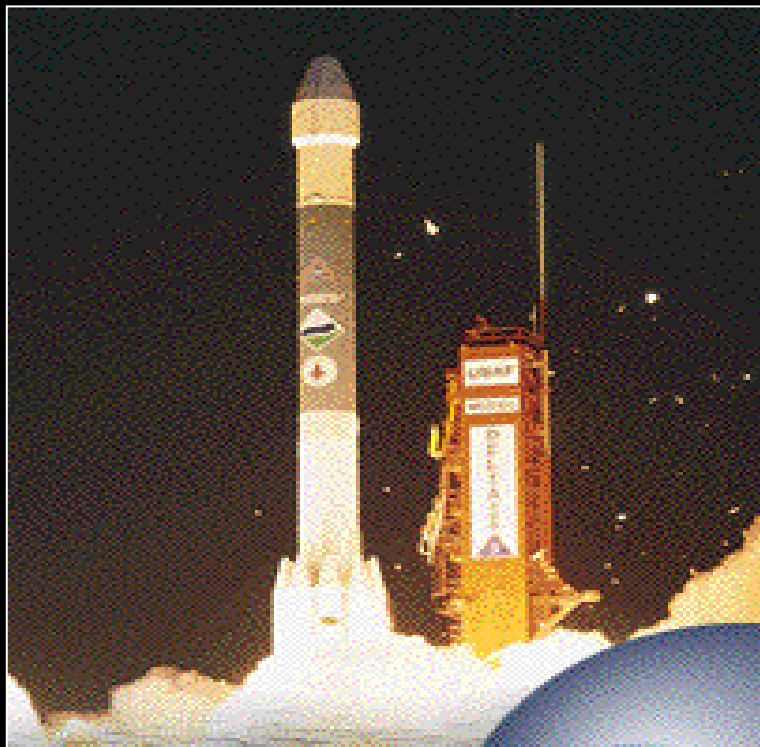
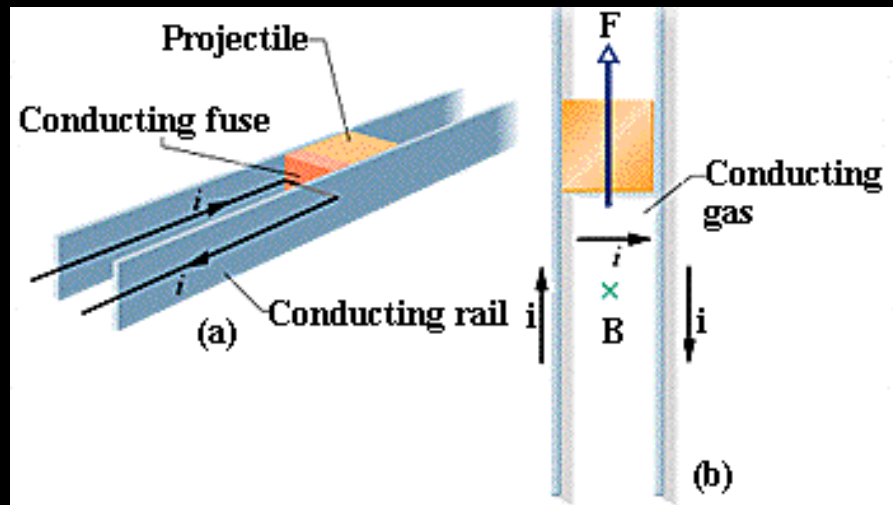


8 電流與磁場、安培定律

Magnetic Fields due to Currents



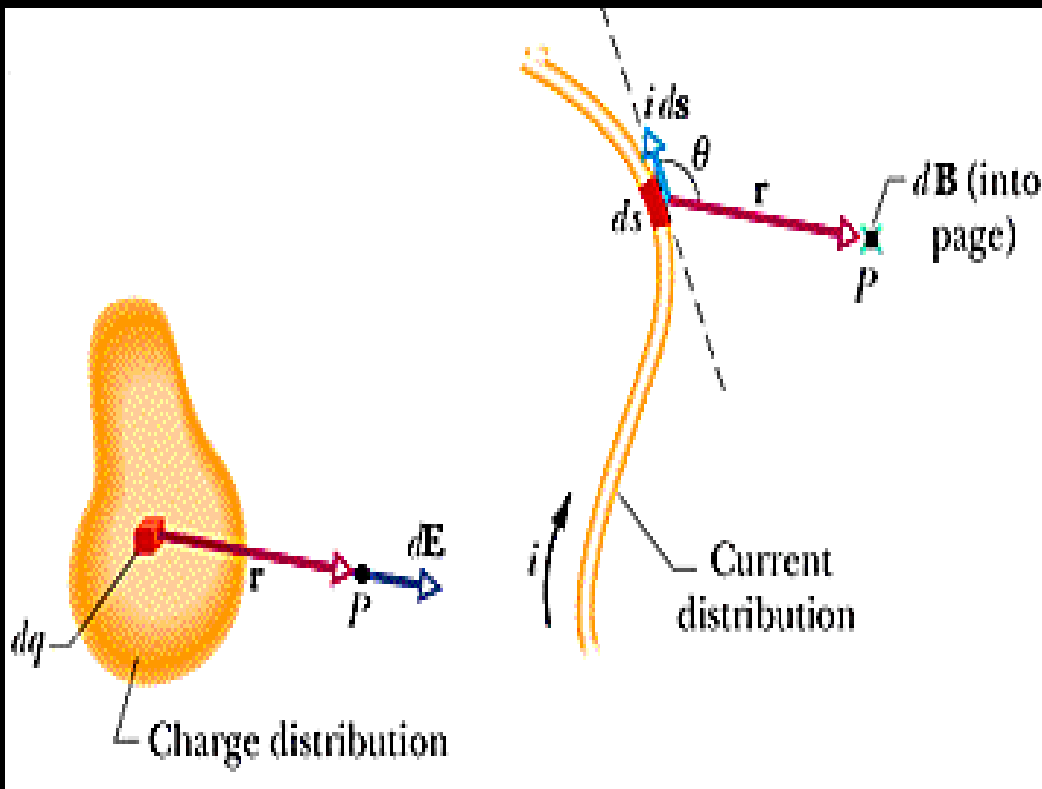
The space shuttle



Conventional rocket

EM Rail Gun

8-1 Calculating the Magnetic Field due to a current



$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

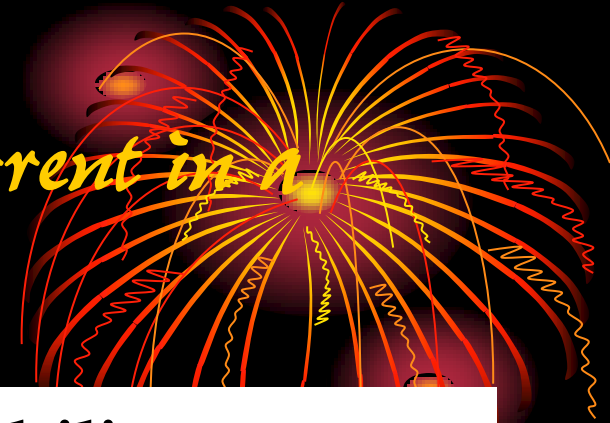
$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^3} \vec{r}$$

$$dB = \frac{\mu_0}{4\pi} \frac{id s \sin \theta}{r^2}$$

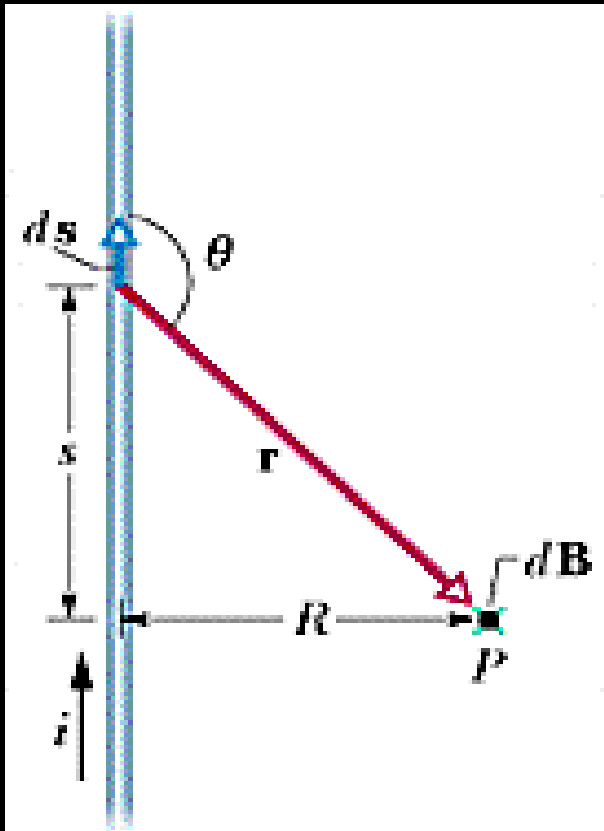
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id \vec{s} \times \vec{r}}{r^3}$$

The law of Biot and Savart (必歐-沙伐定律)

Magnetic Field Due to a Current in a Long Straight Wire



- $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ permeability

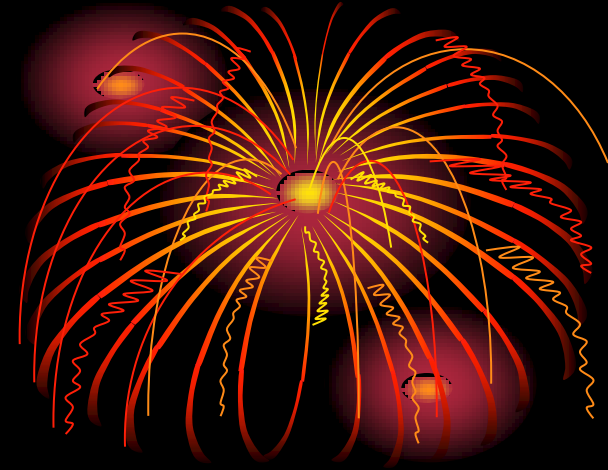


$$dB = \frac{\mu_0}{4\pi} \frac{id s \sin \theta}{r^2}$$

$$B = \int_{-\infty}^{\infty} dB = 2 \int_0^{\infty} dB$$

$$= \frac{\mu_0 i}{2\pi} \int_0^{\infty} \frac{\sin \theta ds}{r^2}$$

Integration

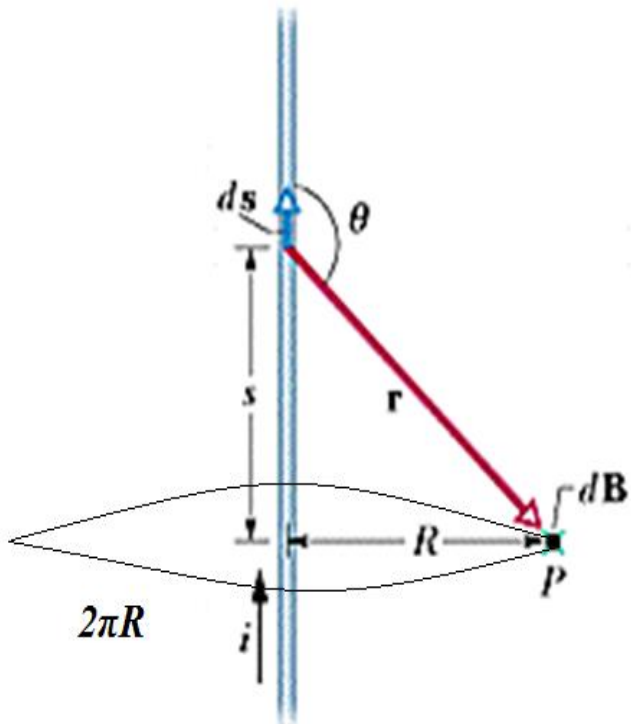
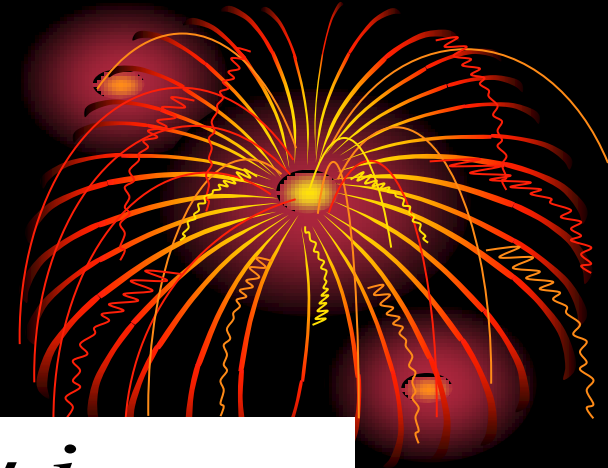


$$r = \sqrt{s^2 + R^2}, \sin \theta = \frac{R}{\sqrt{s^2 + R^2}}$$

$$B = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{R ds}{(s^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi R} \left[\frac{s}{(s^2 + R^2)^{1/2}} \right]_0^\infty = \frac{\mu_0 i}{2\pi R}$$

A hint of Ampere's Law



$$B = \frac{\mu_0 i}{2\pi R}$$

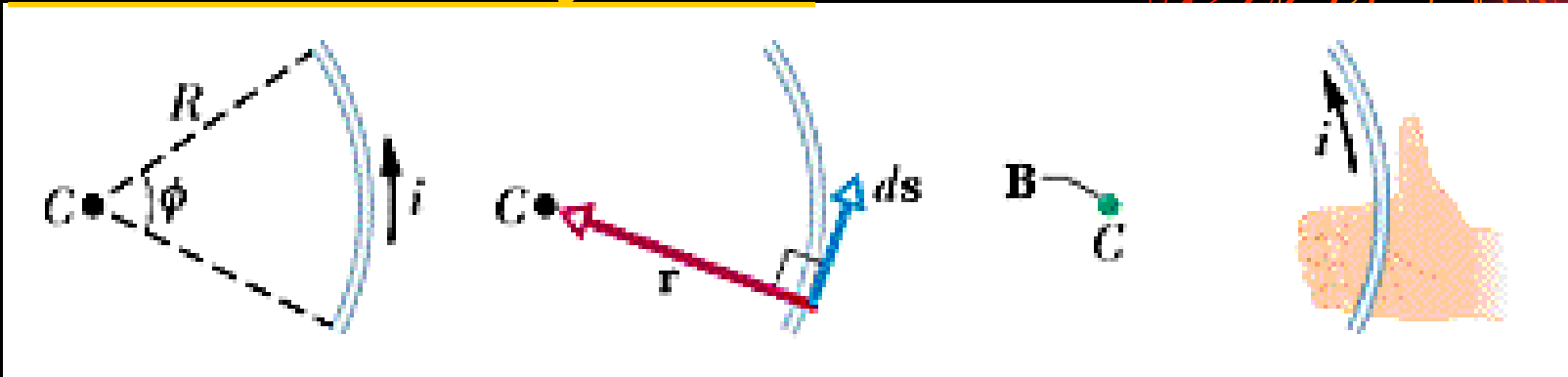
$$B \times 2\pi R = \mu_0 i$$

$$B \times \oint ds = \mu_0 i$$

$$\oint B ds = \mu_0 i$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

Magnetic Field Due to a Current in a Circular Arc of Wire

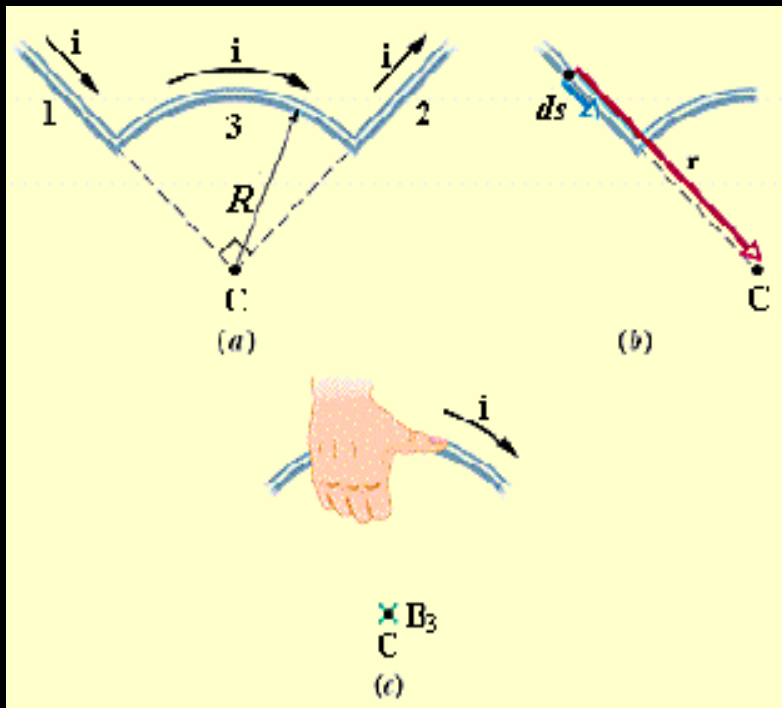
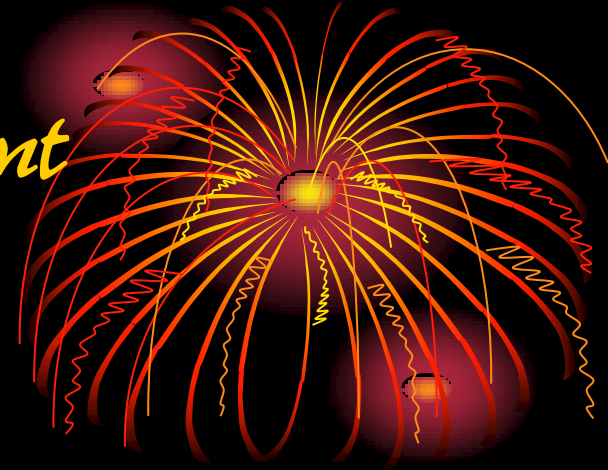


$$dB = \frac{\mu_0}{4\pi} \frac{id s \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} \frac{id s}{R^2}$$

$$B = \int dB = \int_0^\phi \frac{\mu_0}{4\pi} \frac{iR d\phi}{R^2}$$

$$= \frac{\mu_0 i}{4\pi R} \int_0^\phi d\phi = \frac{\mu_0 i \phi}{4\pi R} \xrightarrow{0} \frac{\mu_0 i}{2R}$$

例 1 What \mathbf{B} does the current produce?

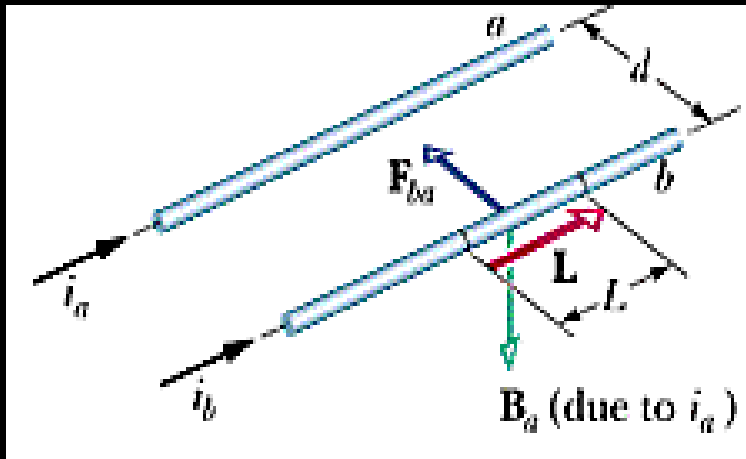
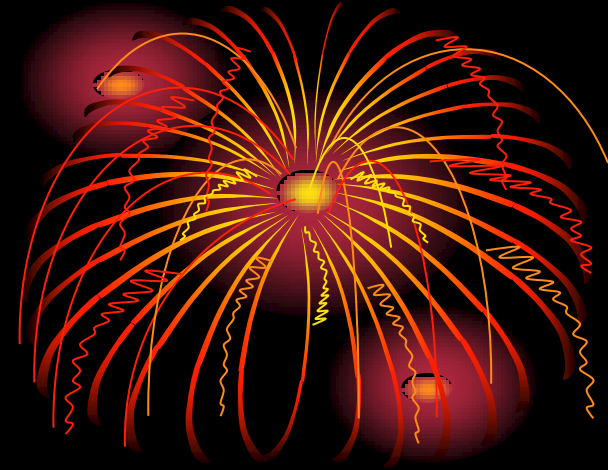


$$d\vec{s} \times \vec{r} = 0$$

$$B_1 = B_2 = 0$$

$$B_3 = \frac{\mu_0 i (\pi / 2)}{4\pi R} = \frac{\mu_0 i}{8R}$$

8-2 Two Parallel Currents



$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

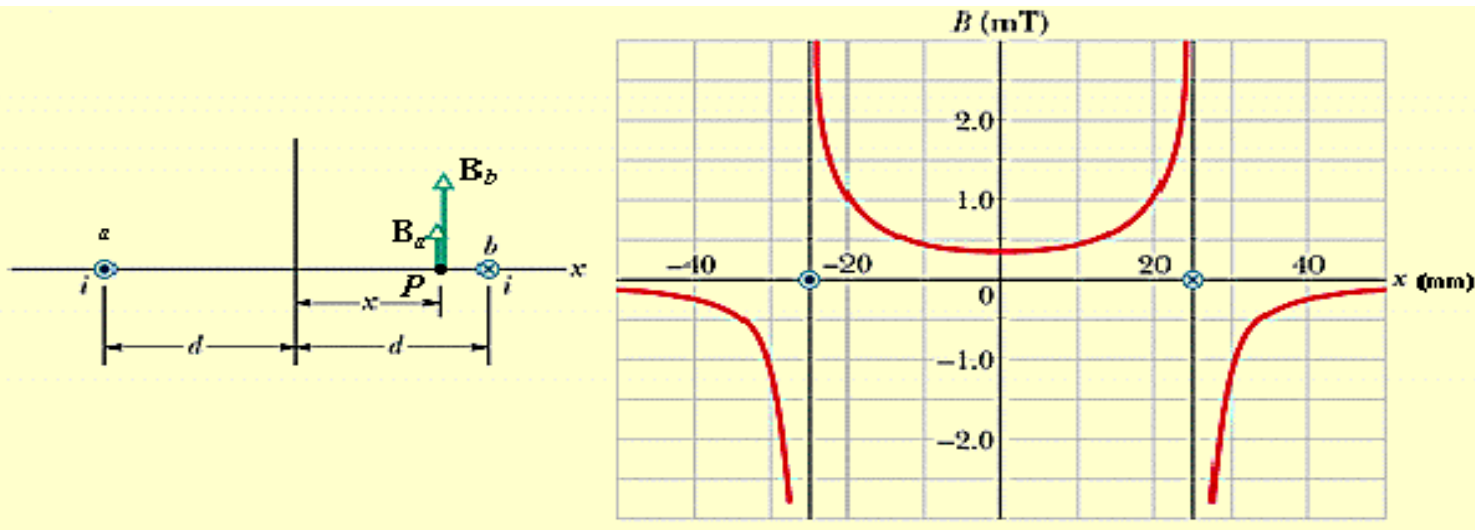
$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}$$

例 2 The Field Between Two Wires

$$B(x) = B_a(x) + B_b(x)$$

$$= \frac{\mu_0 i}{2\pi(d+x)} + \frac{\mu_0 i}{2\pi(d-x)} = \frac{\mu_0 i d}{\pi(d^2 - x^2)}$$

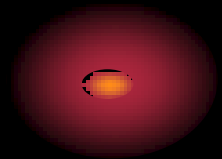
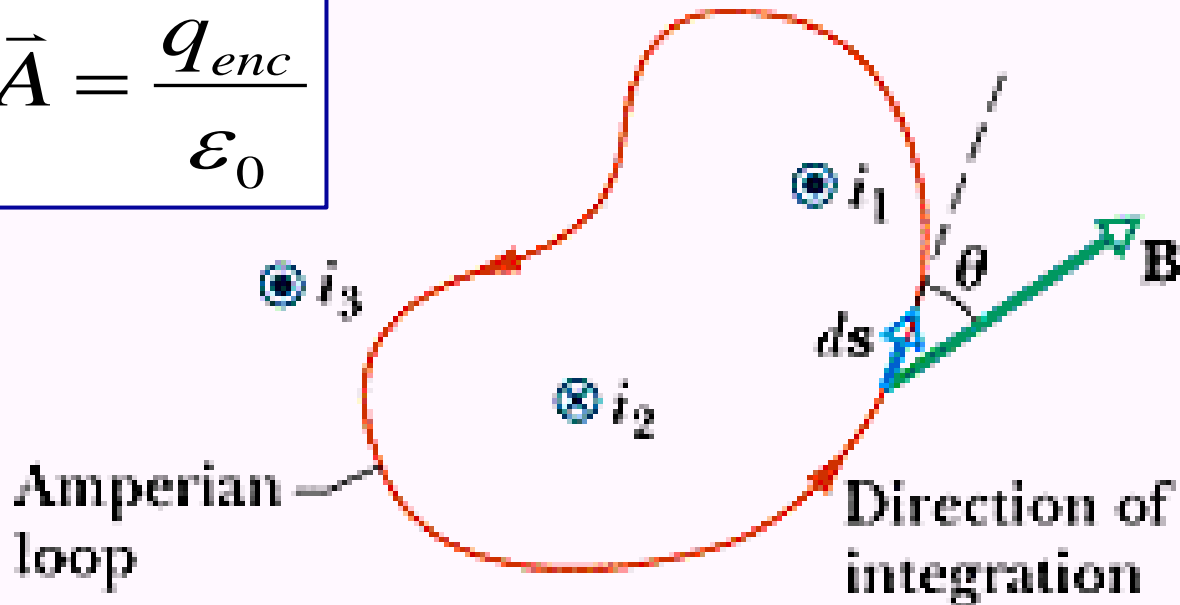


8-3 Ampere's Law

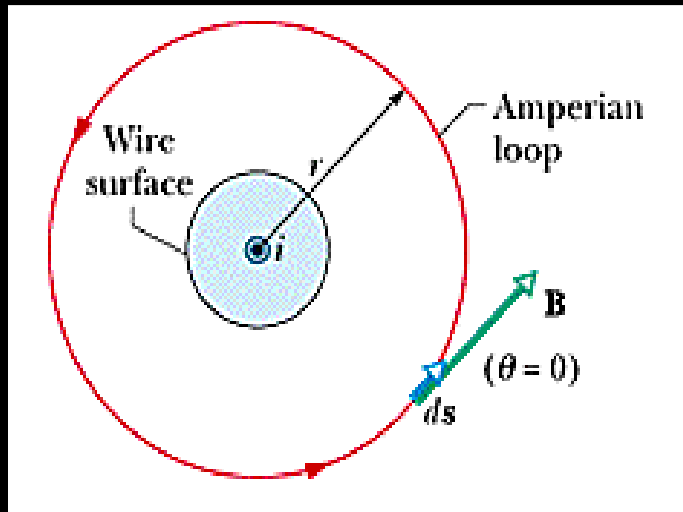
- Comparing Gauss' law and Ampere's law
- Ampere's law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

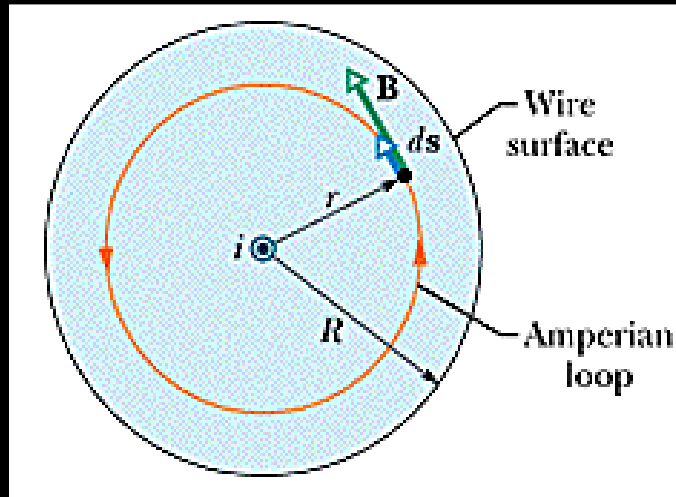


The Magnetic Field Outside a Long Straight Wire with Current



$$\begin{aligned}\oint \vec{B} \cdot d\vec{s} &= \oint B \cos \theta ds \\ &= B \oint ds = B(2\pi r) \\ B(2\pi r) &= \mu_0 i\end{aligned}$$

The Magnetic Field Inside a Long Straight Wire with Current

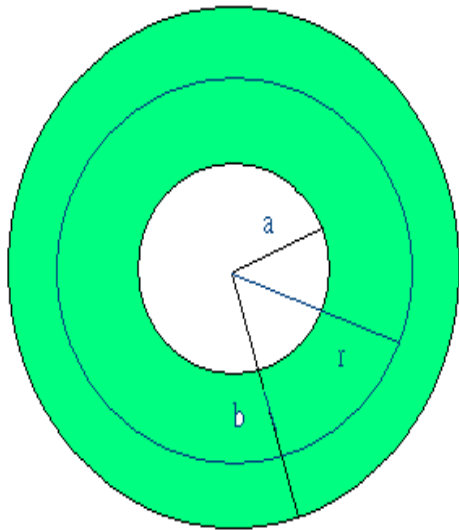
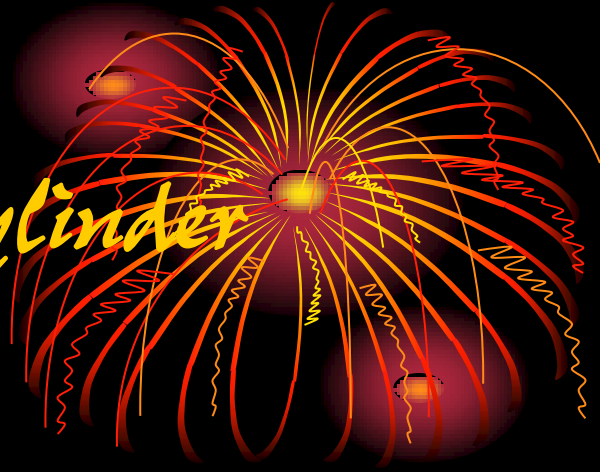


$$\oint \vec{B} \cdot d\vec{s} = B(2\pi r)$$

$$i_{enc} = i \frac{\pi r^2}{\pi R^2}$$

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

例 3 A hollow conducting cylinder

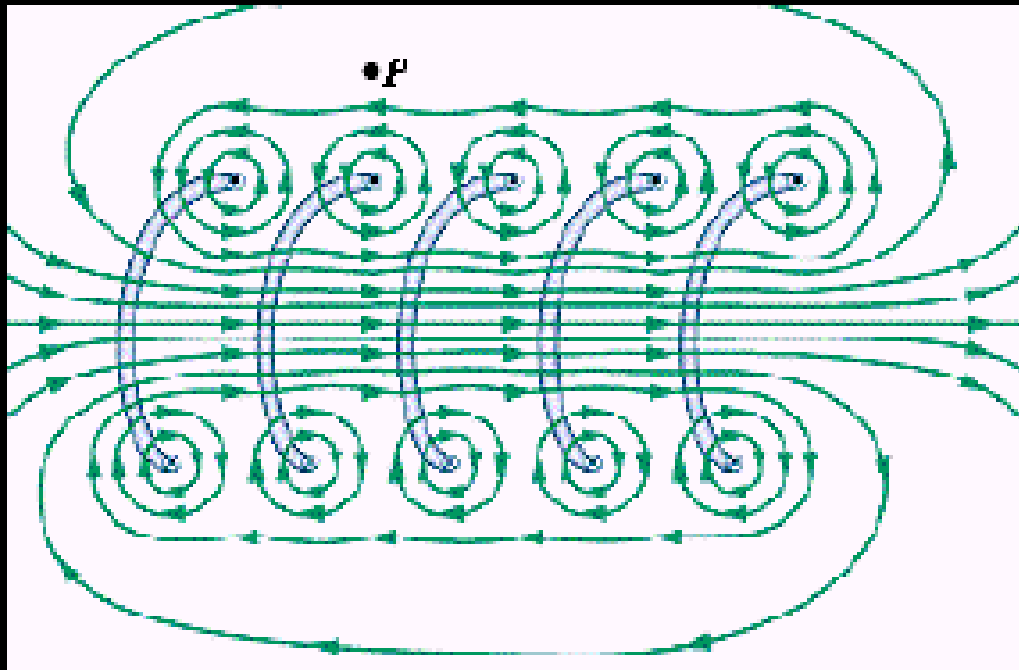


$$i_{enc} = \int J dA = \int_a^r c r^2 (2\pi r dr)$$
$$= \frac{\pi c (r^4 - a^4)}{2}$$

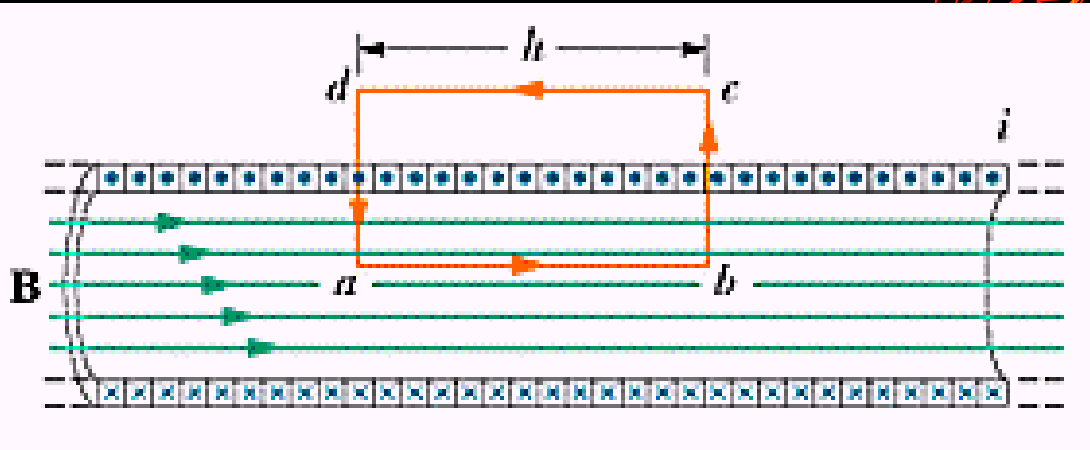
$$B(2\pi r) = -\frac{\mu_0 \pi c (r^4 - a^4)}{2}$$

8-4 Solenoids and Toroids

- Magnetic Field of a Solenoid (螺線管)
- Magnetic Field of a Toroid (螺線環)



Magnetic Field of a Solenoid

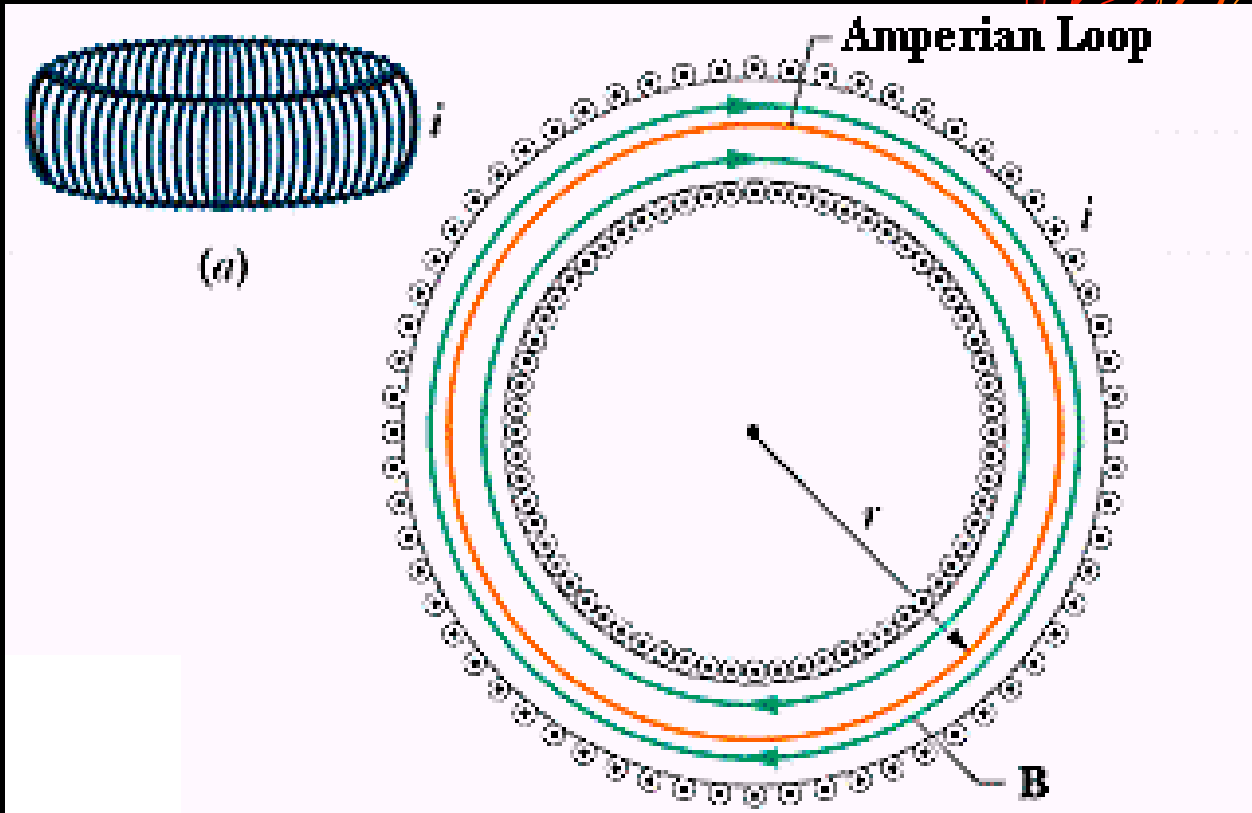


$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s}$$

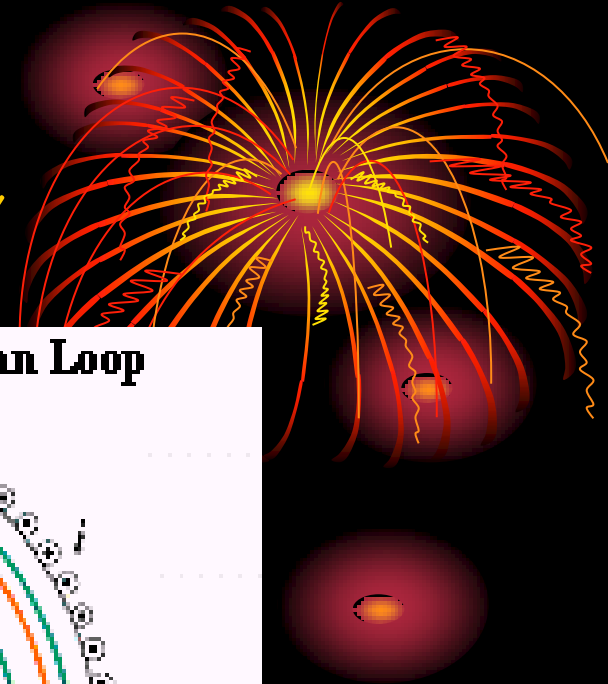
$$+ \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}$$

$$Bh = \mu_0 in h \quad B = \mu_0 in$$

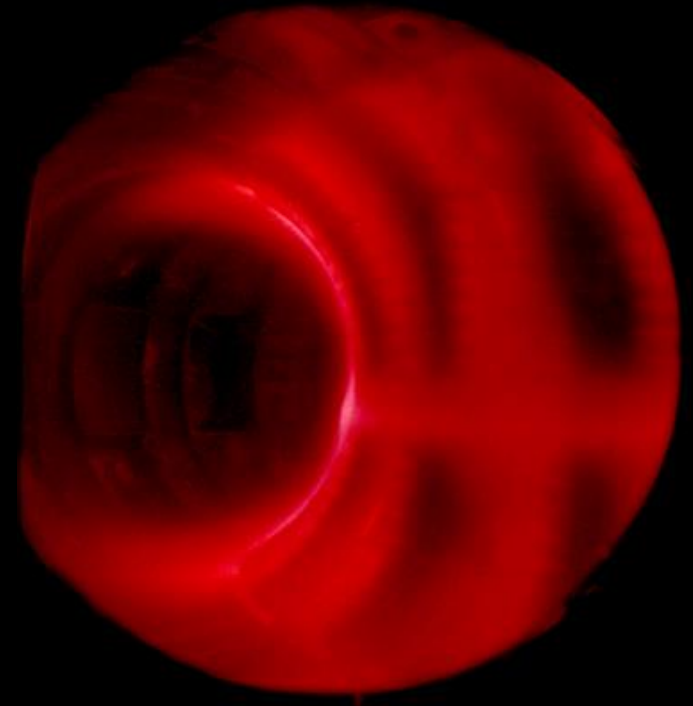
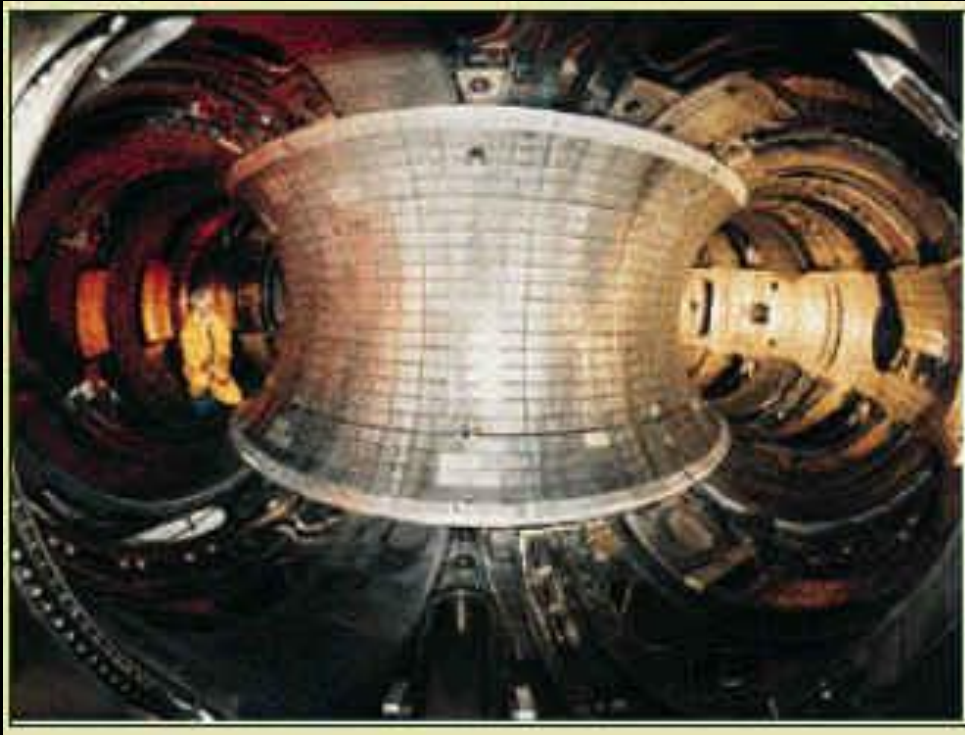
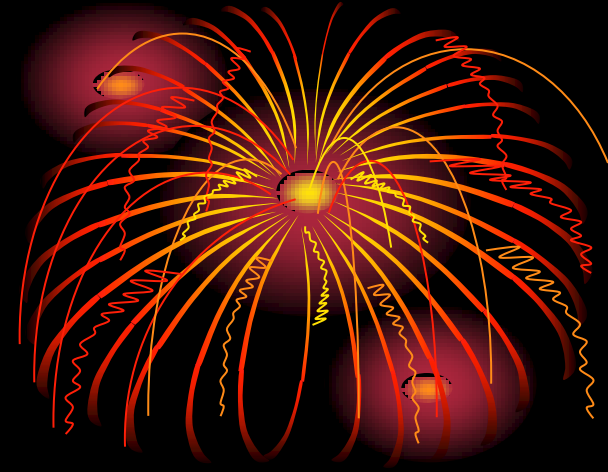
Magnetic Field of a Toroid



$$B(2\pi r) = \mu_0 i N \quad B = \frac{\mu_0 i N}{2\pi r}$$



磁圍阻核融合反應器



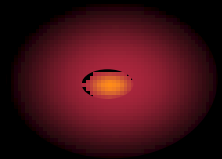
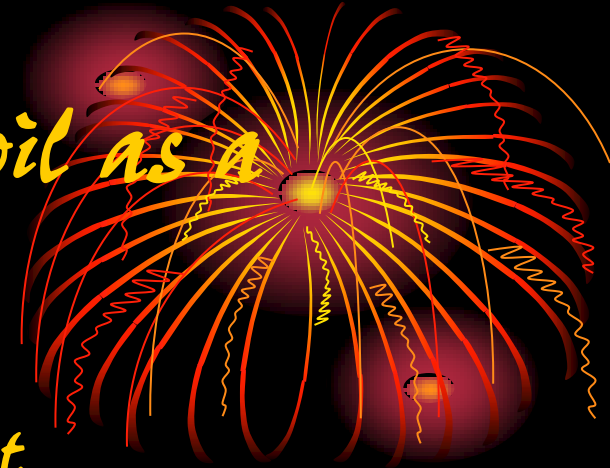
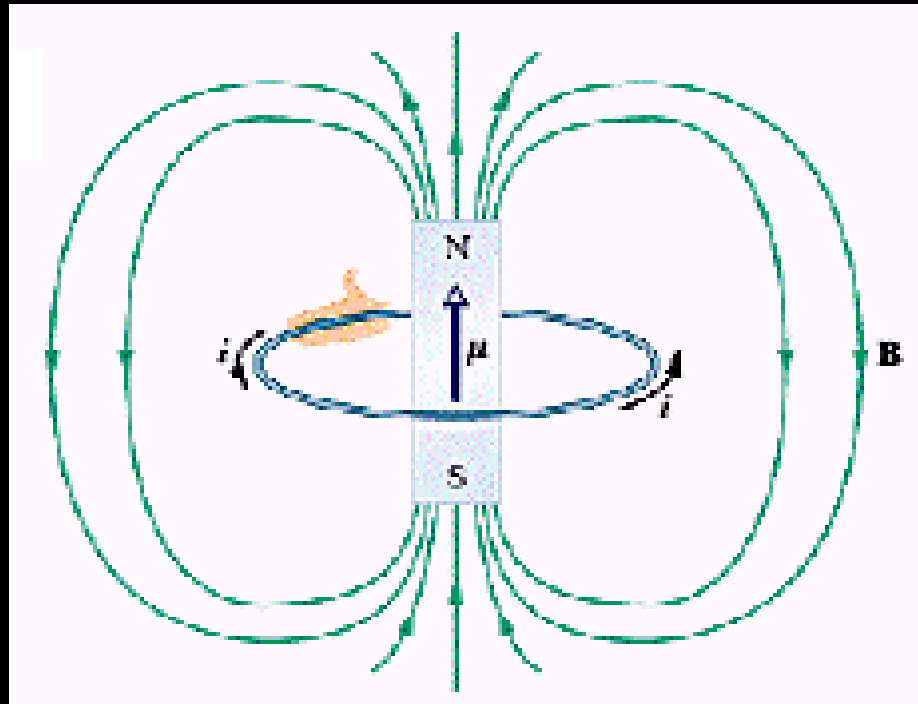
*Tokamak Fusion Test Reactor 1982 -
1997*

JET

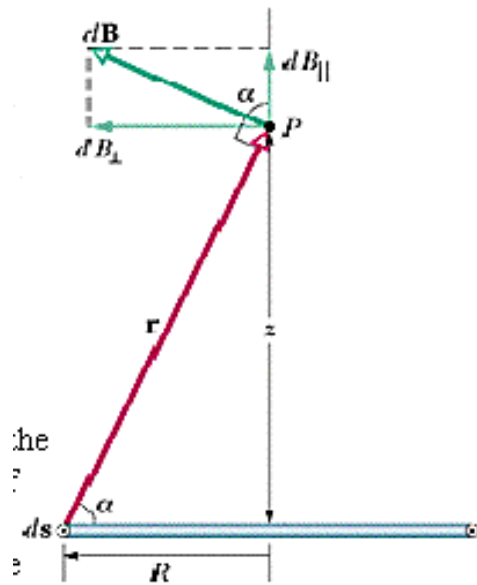
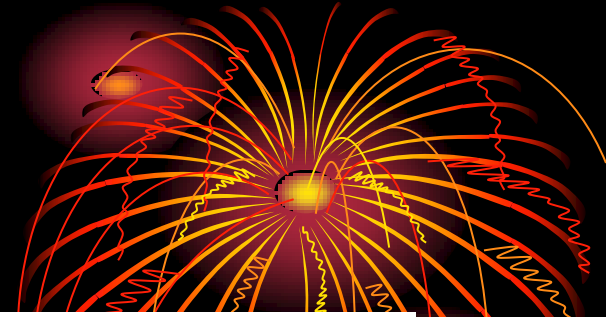
ITER

8-5 A Current Carrying Coil as a Magnetic Dipole

- A current loop and a bar magnet



Magnetic Field of a Coil



$$B = \int dB \cos \alpha$$

$$= \int \frac{\mu_0 i ds}{4\pi r^2} \frac{R}{r}$$

$$= \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} 2\pi R$$

$$= \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}$$

$$\rightarrow \frac{\mu_0 i}{2R} \text{ (at center of coil)}$$

