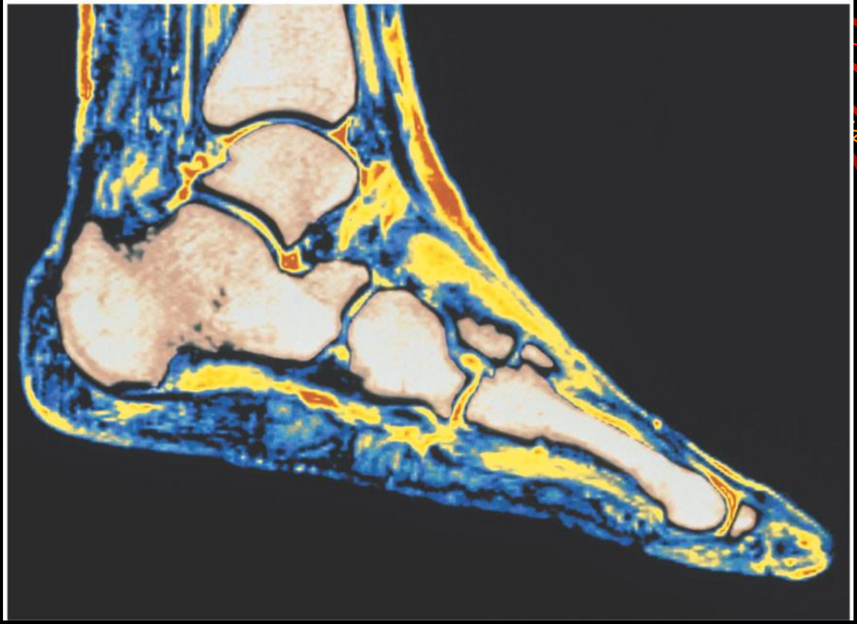
7 磁場

How is an aurora so thin yet so tall and wide?

Introduction

How does magnetic resonance imaging (MRI) allow us to see details in soft nonmagnetic tissue?



How can magnetic forces, which act only on moving charges, explain the behavior of a compass needle?

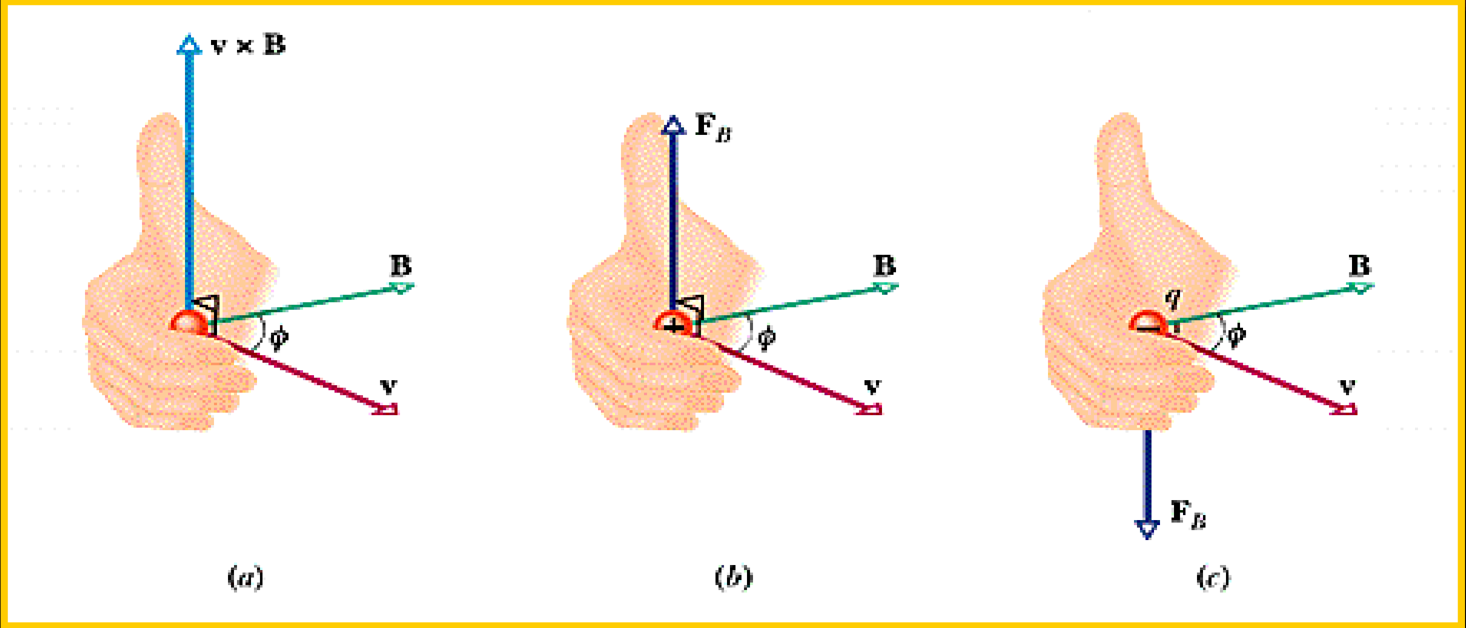
The magnetic field

- The electric field and the magnetic field
- Electromagnets and permanent magnets

7-1 The definition of B

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{E} = \frac{\vec{F}}{q_0} \, (\mathrm{N/C}) \, (\mathrm{V \cdot m})$$



The tracks in a bubble chamber

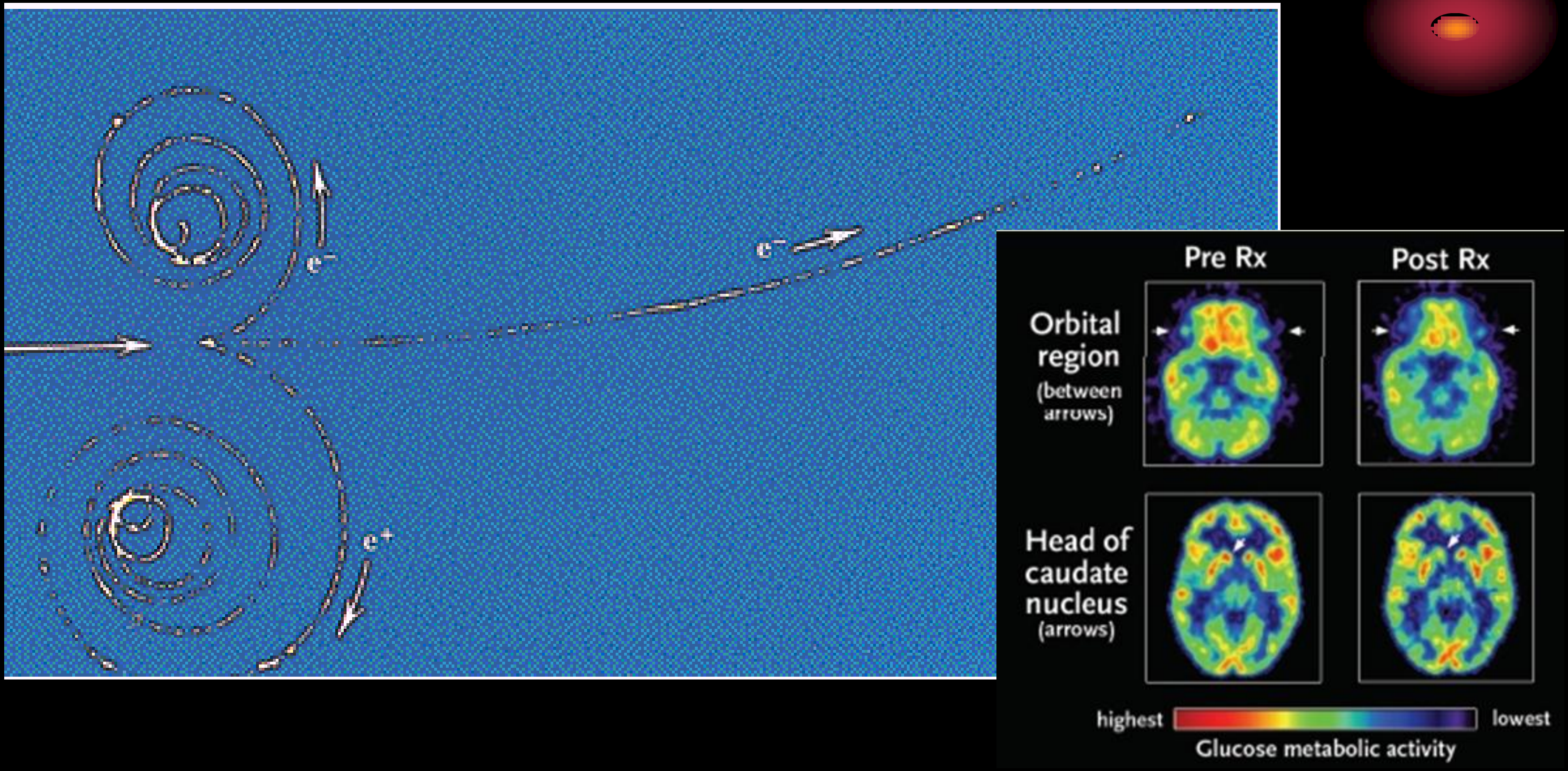
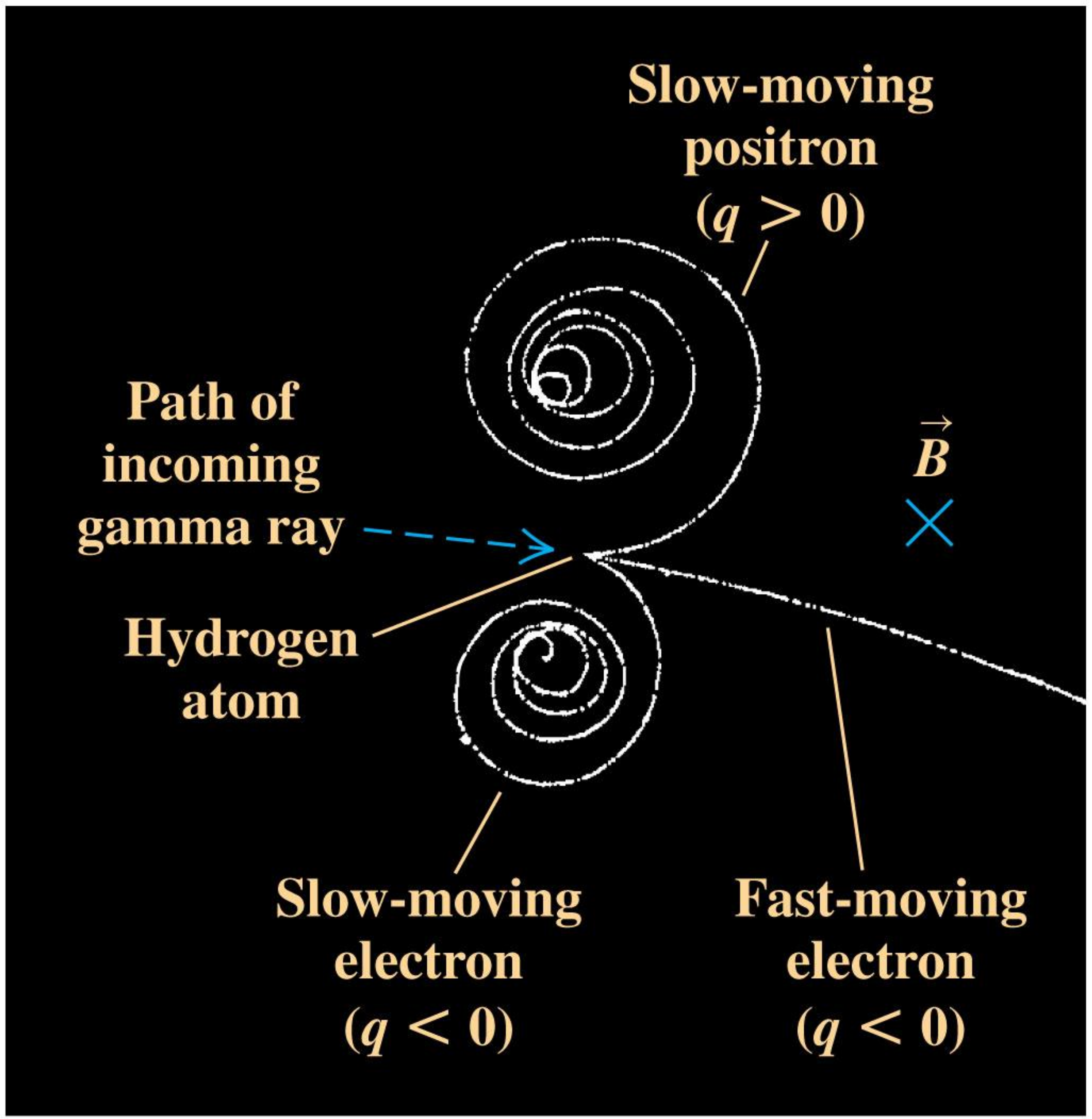


Figure 27.21



Slow-moving positron ($q > 0$)
Path of incoming gamma ray
$\vec{B}$
Hydrogen atom
Slow-moving electron ($q < 0$)
Fast-moving electron ($q < 0$)

The SI unit for B

$$1 \text{ tesla} = 1\text{T} = 1 \text{ N/A} \cdot \text{m} = 10^4 \text{ gauss}$$

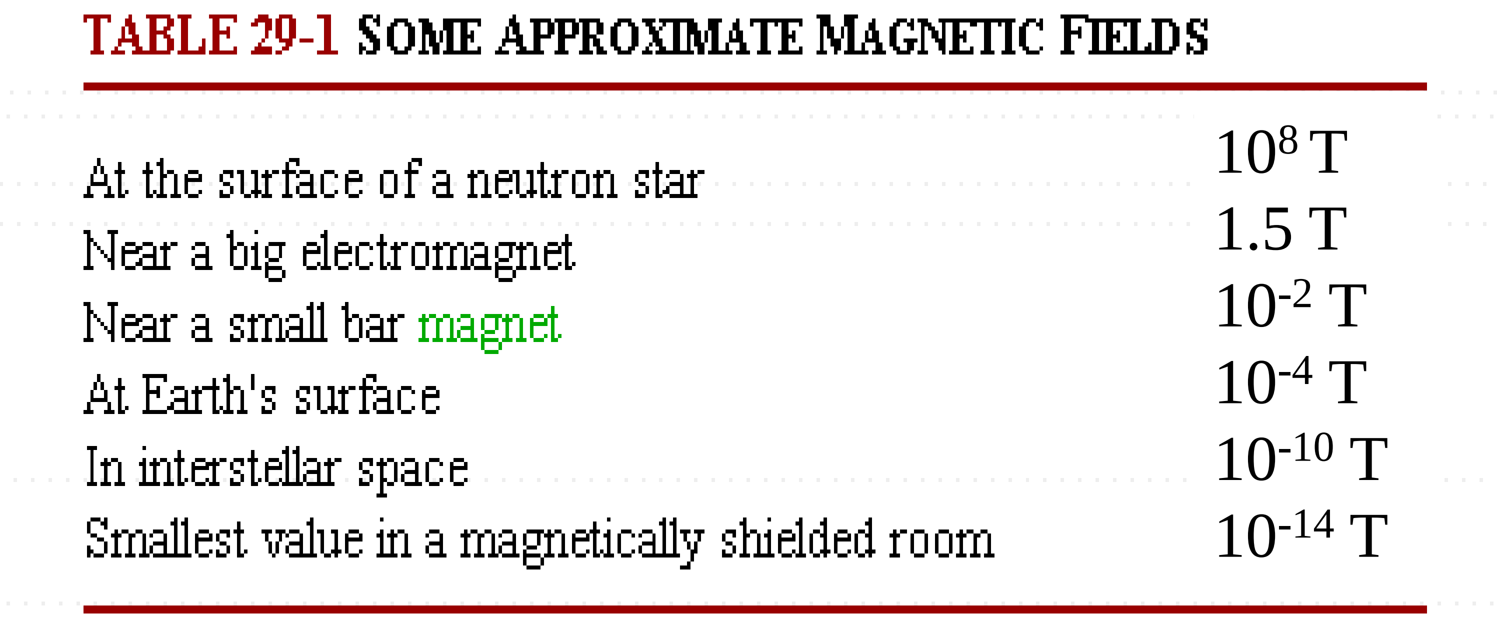
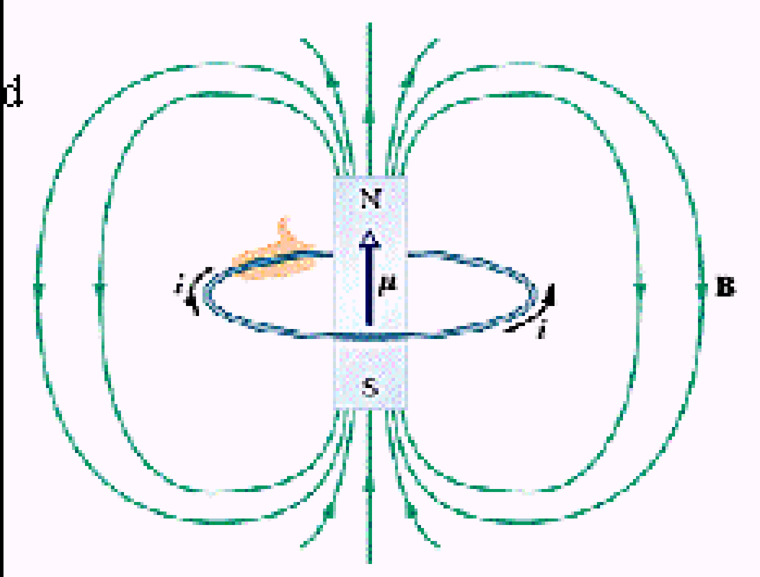


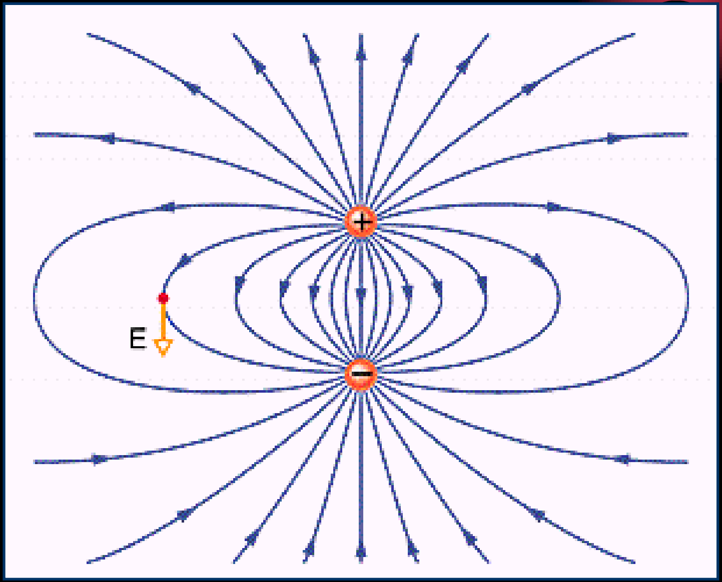
TABLE 29-1 SOME APPROXIMATE MAGNETIC FIELDS

At the surface of a neutron star	10^8 T
Near a big electromagnet	1.5 T
Near a small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in a magnetically shielded room	10^{-14} T

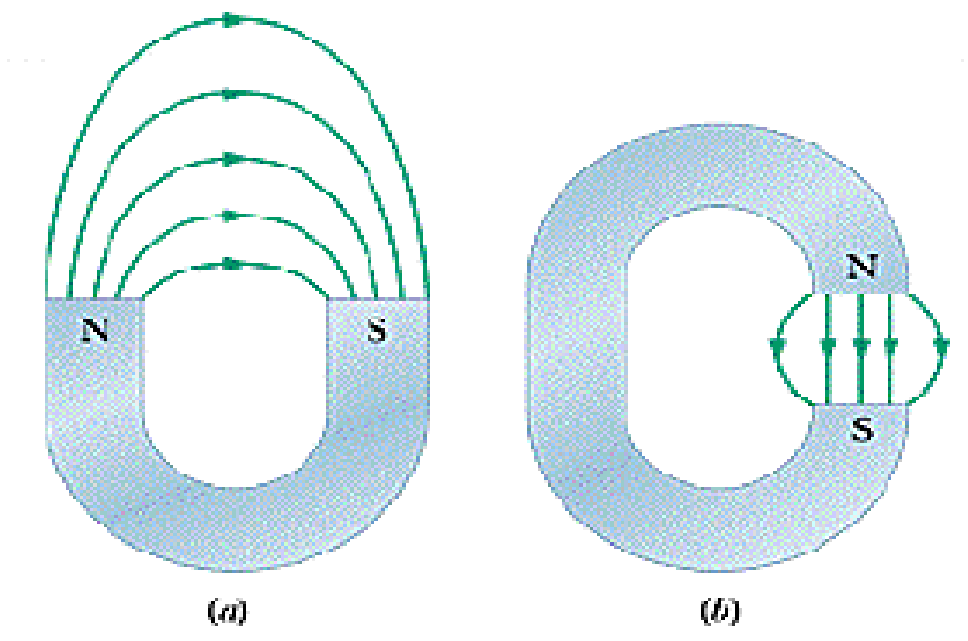
Magnetic Field Lines

- Magnetic vs. electric dipoles





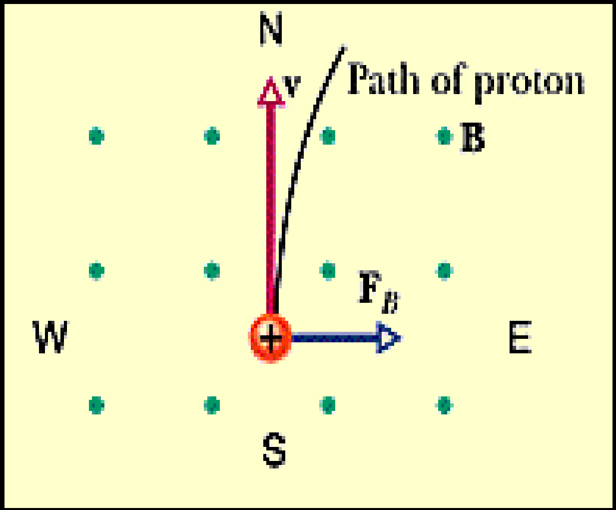
A horseshoe and a C-shaped magnets

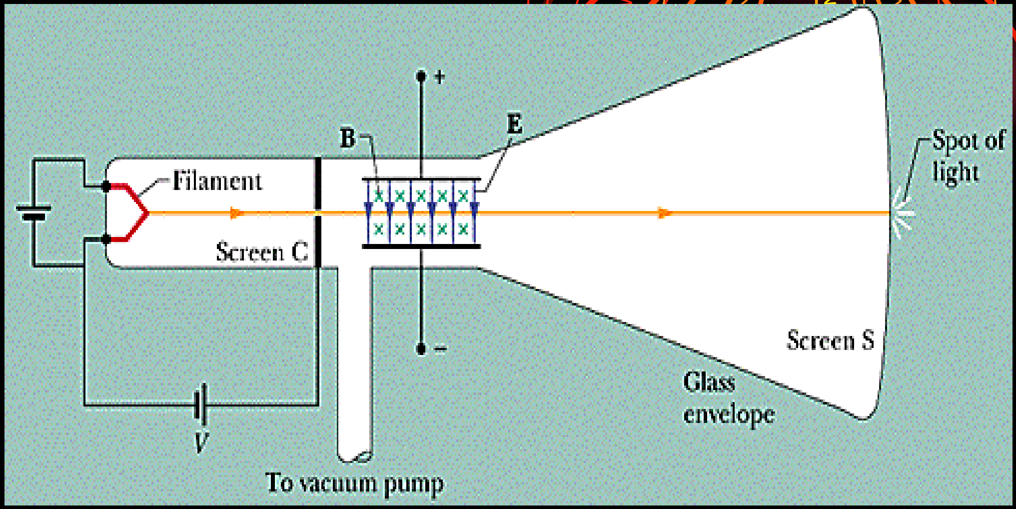


(a)　　(b)

例1 A 5.3 MeV proton

B = 1.2 mT





$$v = \sqrt{2K/m} = 3.2 \times 10^{7}\,\mathrm{m/s}$$

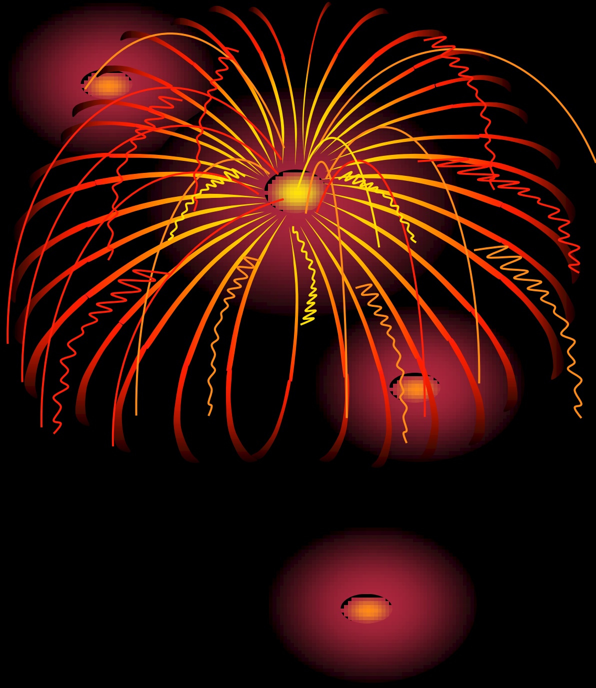
$$F_B = qvB\sin\phi = 6.1 \times 10^{-15}\,\mathrm{N}$$

$$a = F_B/m = 3.7 \times 10^{12}\,\mathrm{m/s^2}$$

7-2 Crossed Fields: Discovery of the Electron

- A cathode ray tube
- Thomson’s procedure:
  - 設定$E=0$、$B=0$，並記錄光點位置
  - 開啟電場、測光點偏移量
  - 開啟磁場，並調整其值，直至光點回復未偏移位置

Calculation



$$y = \frac{qEL^2}{2mv^2}, \quad qE = qvB$$

$$v = E/B, \quad \frac{m}{q} = \frac{B^2L^2}{2yE}$$

$$a_y = \frac{F}{m} = \frac{QE}{m}$$

$$y = \frac{1}{2}a_y t^2, L = v_x t$$

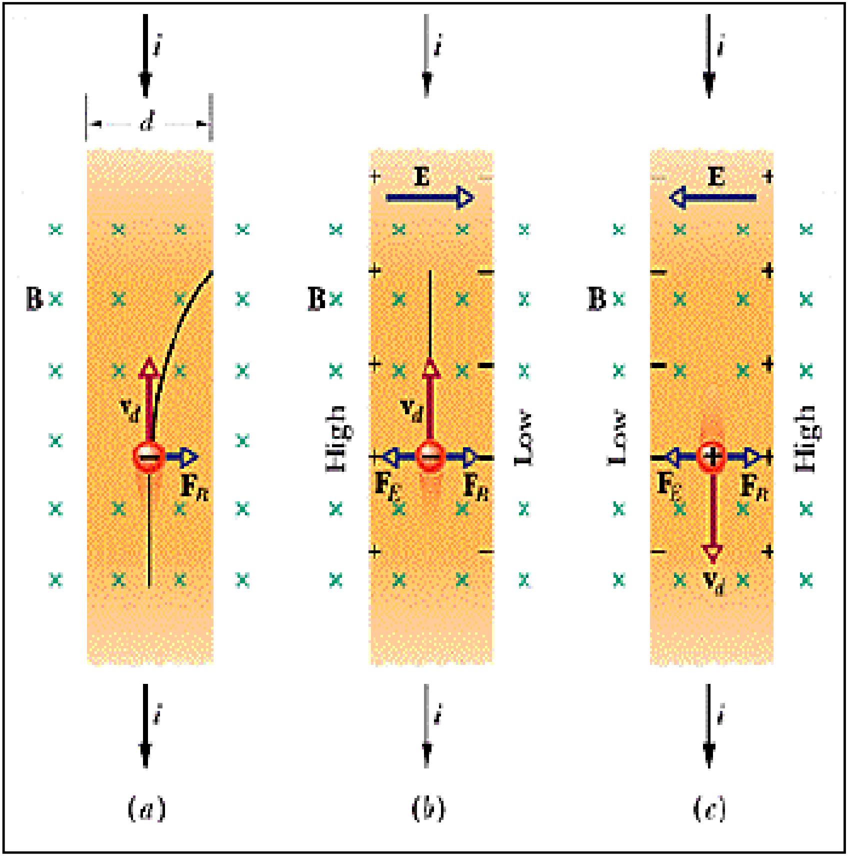
$$y = \frac{QEL^2}{2mv_x^2}$$

the charge-to-mass ratio of the electron ：

$1.7588196 \times 10^{11}\ \mathrm{C \cdot kg^{-1}}$

7-3 Crossed Fields: The Hall Effect

- **By the conduction electrons in copper:**



$$V = Ed$$

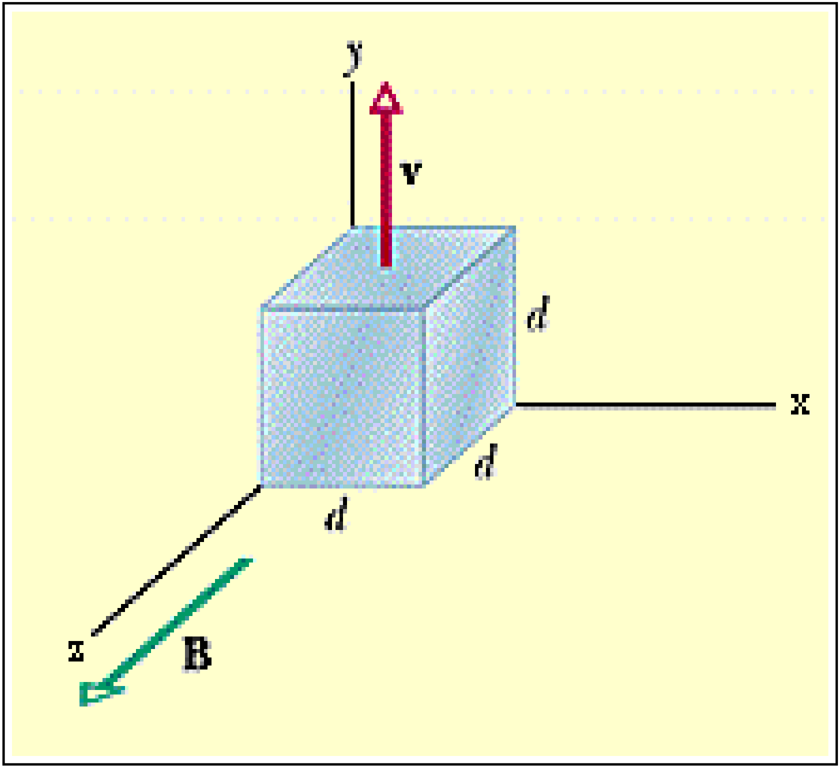
$$eE = ev_d B$$

$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

$$n = \frac{Bi}{Vle} \quad (l = \frac{A}{d})$$

例 2 A cube generator

d = 1.5 cm, v = 4.0m/s, B = 0.05T



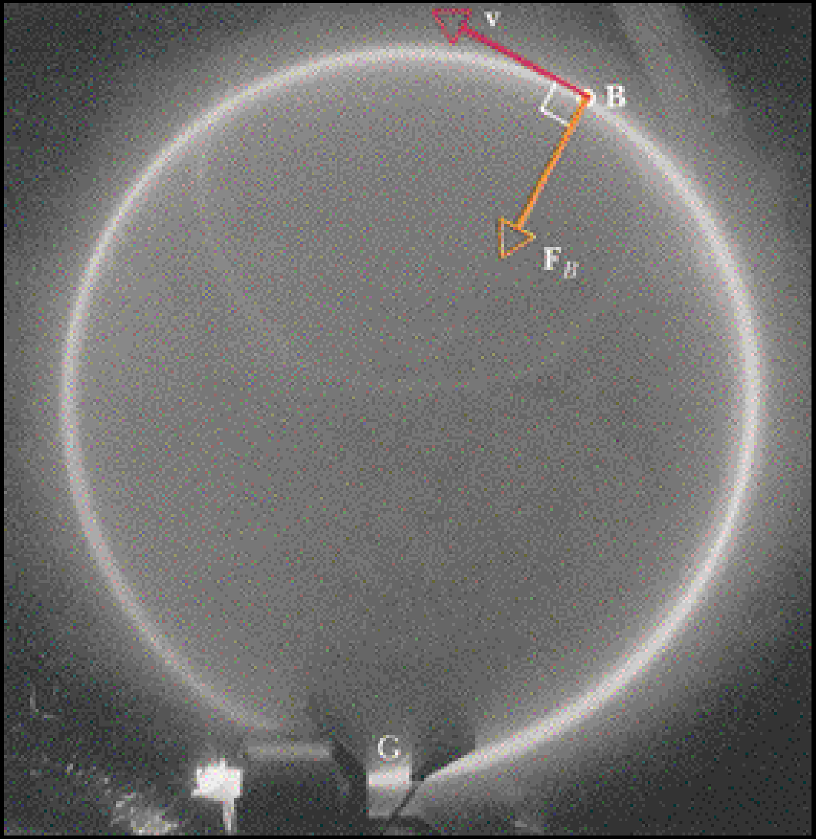
$$eE = evB$$

$$V = Ed$$

$$V = dvB$$

$$V = 3.0\text{mV}$$

7-4 A Circulating Charged Particle



$$F = ma = mv^2 / r$$

$$qvB = mv^2 / r$$

$$r = mv / qB$$

$$T = 2\pi r / v$$

$$= 2\pi m / qB$$

Figure 27.17a

(a) The orbit of a charged particle in a uniform magnetic field

A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.

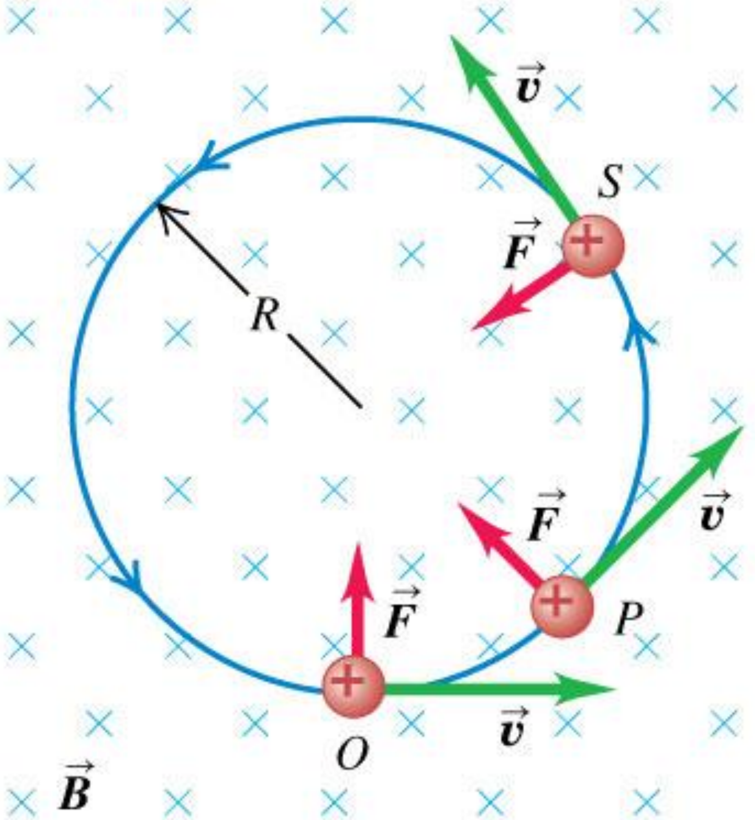
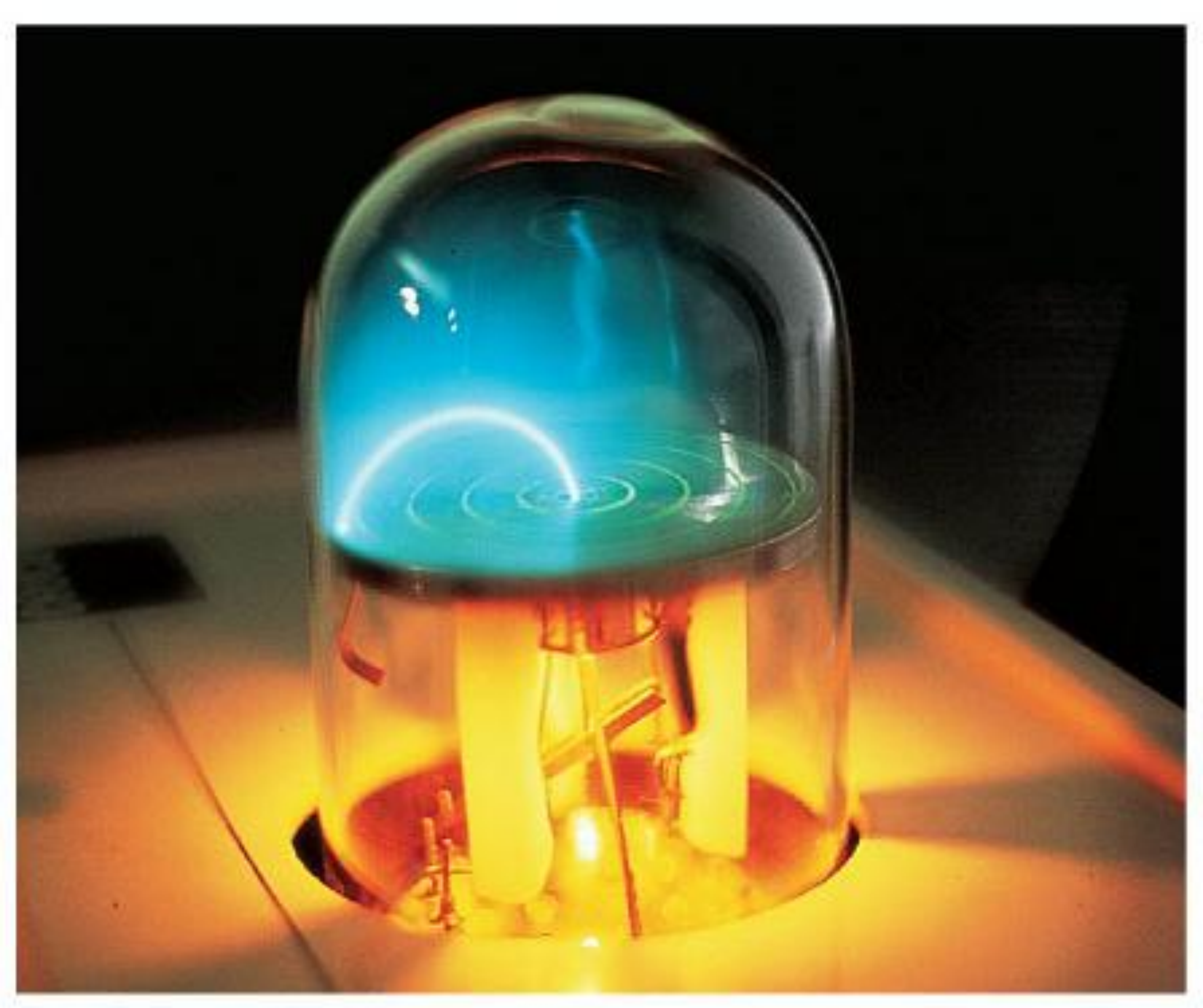
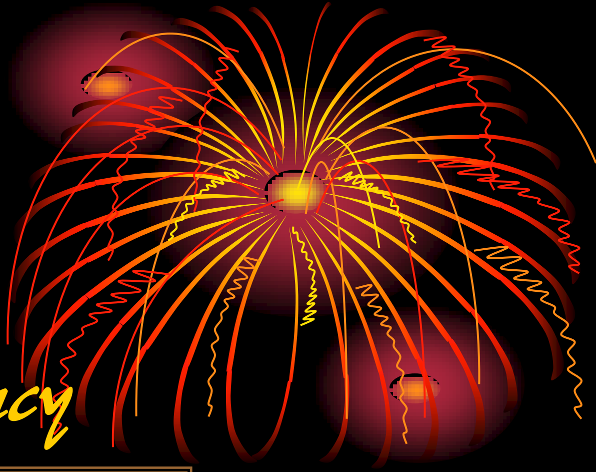


Figure 27.17b



頻率與軌跡

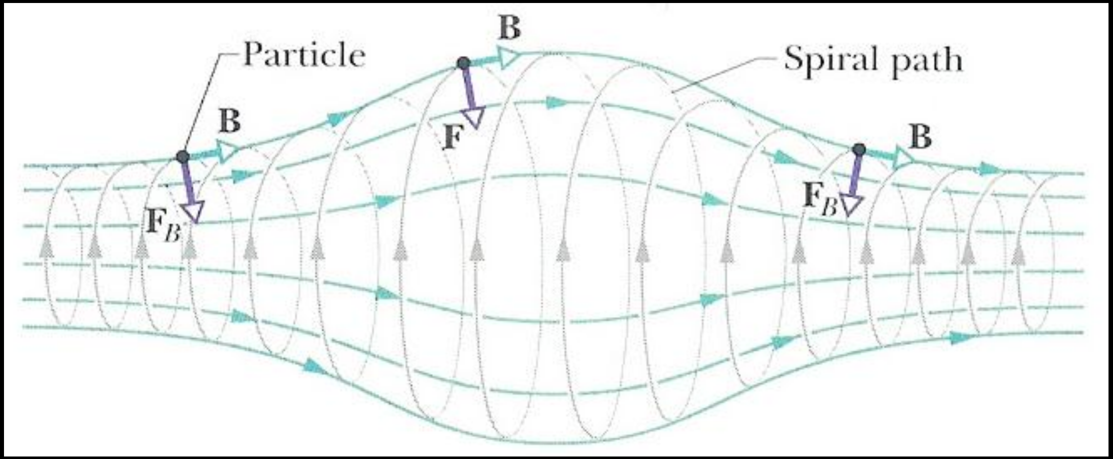


The frequency and angular frequency

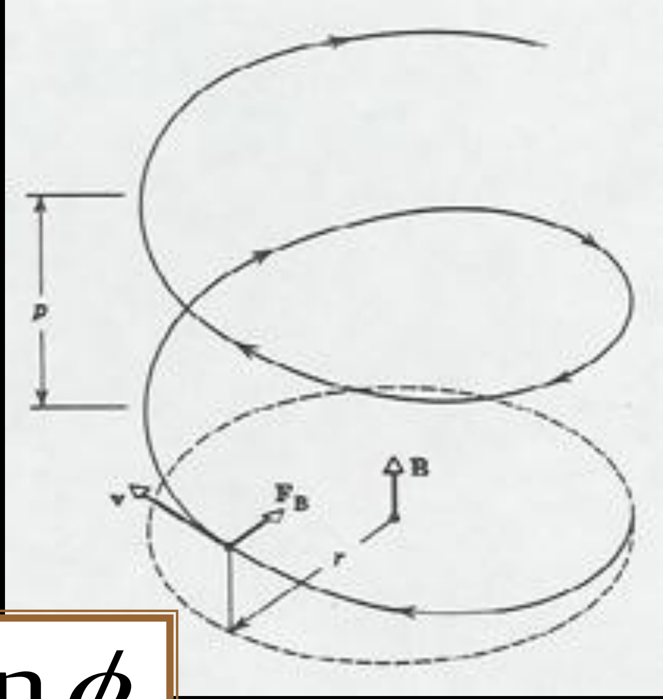
$$f=\frac{1}{T}=\frac{qB}{2\pi m} \qquad \omega=2\pi f=\frac{qB}{m}$$

$$f=\frac{qB}{2\pi m_0}\sqrt{1-v^2/c^2}$$

The magnetic bottle machine



Helical Paths



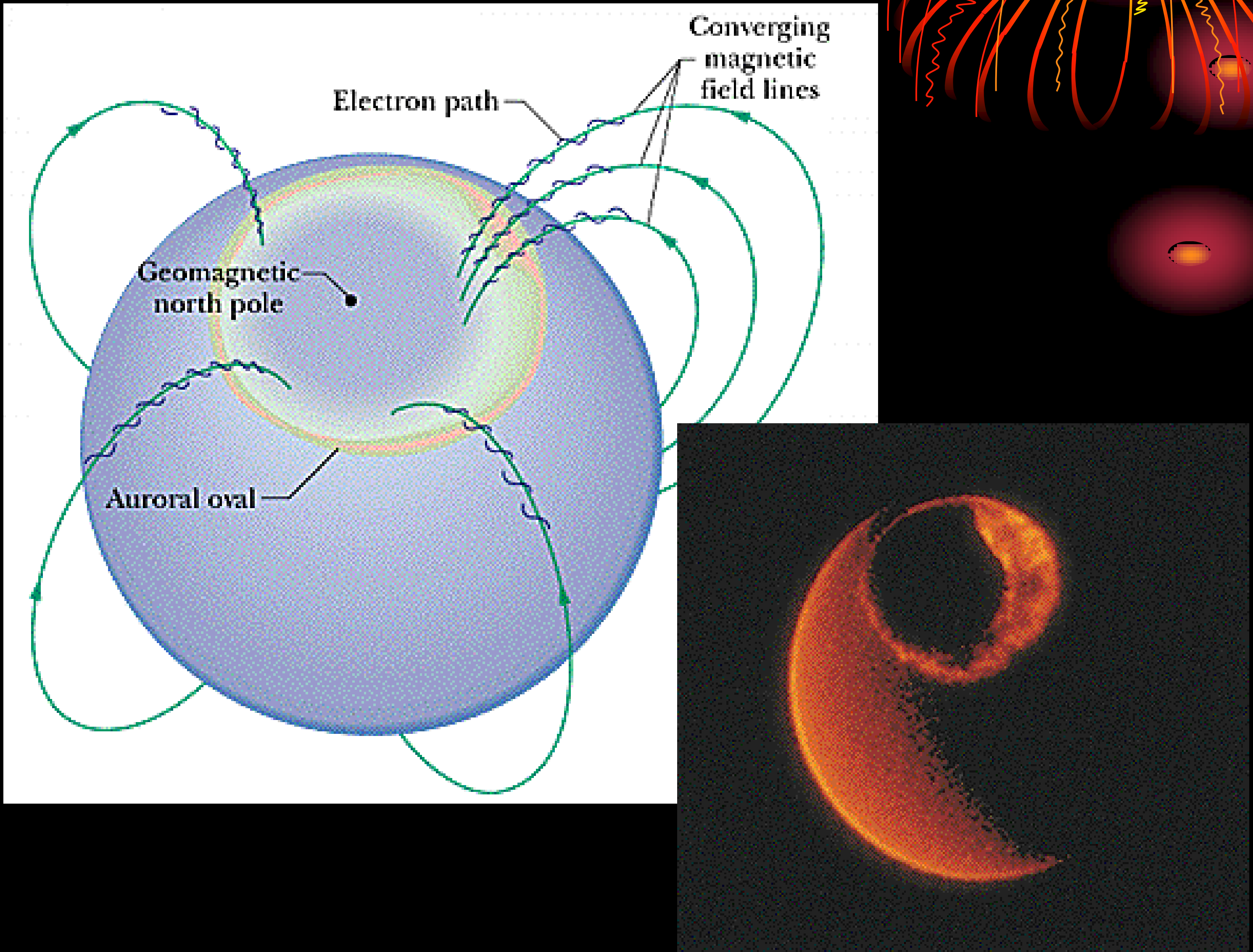
$v_{\parallel}$ and $v_{\perp}$

$$v_{\parallel} = v\cos\phi \quad v_{\perp} = v\sin\phi$$

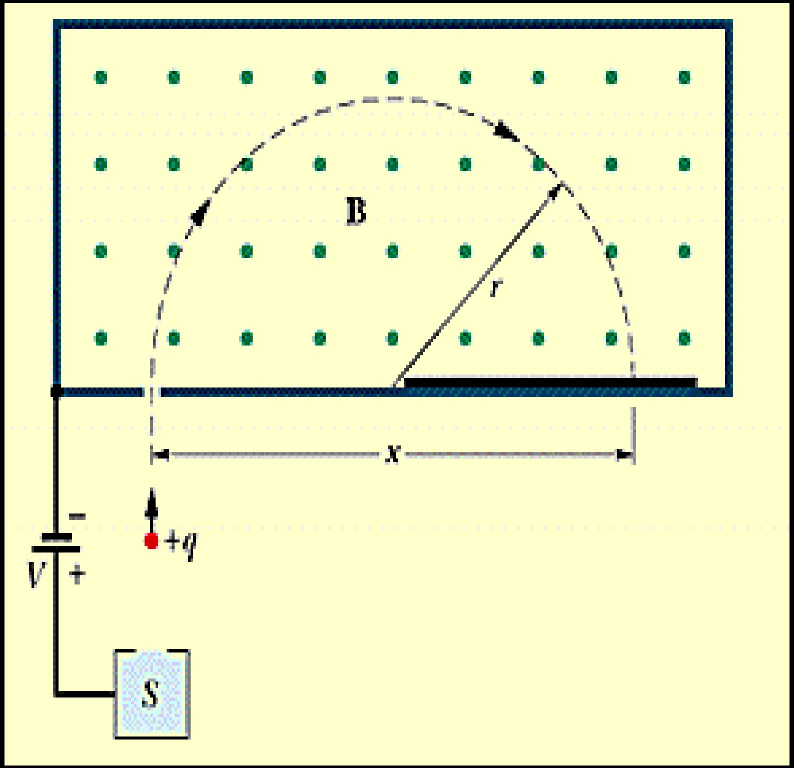
The pitch (螺距) of the helical path

$$p = v_{\parallel}T = v\cos\phi\,\frac{2\pi m}{qB}$$

極光橢圓圈


Converging magnetic field lines
Electron path
Geomagnetic north pole
Auroral oval

例 3 The Mass Spectrometer (質譜儀)



$$\frac{1}{2}mv^2 = qV$$

$$v = \sqrt{2qV/m}$$

$$r = \frac{mv}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$x = 2r$$

質譜儀

x = 1.6254m, V = 1000.0V, B = 80.000mT

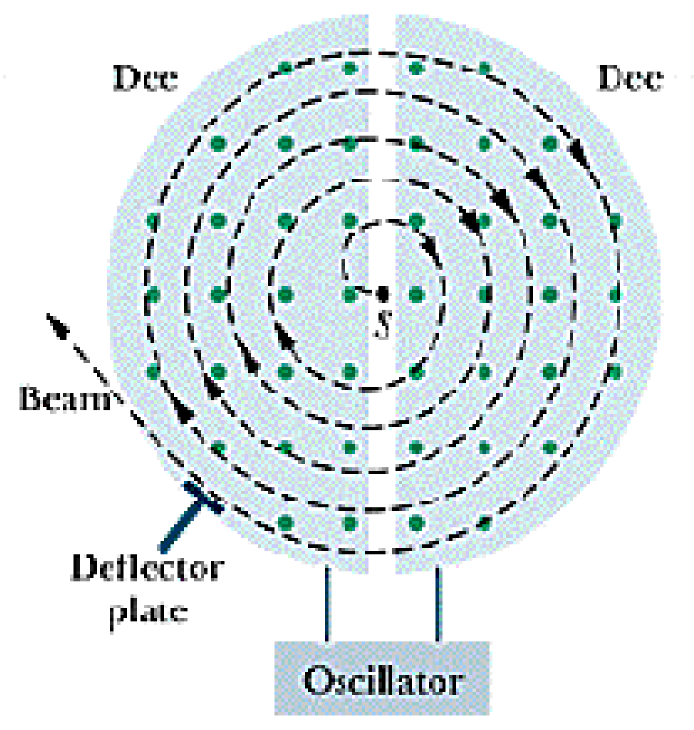
$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}$$

$$\to m = \frac{B^2 q x^2}{8V} = 203.93\text{u}$$

Isotope Separation

Centrifuge and diffusion chamber

7-5 Cyclotrons and Synchrotrons (迴旋加速器與同步加速器)



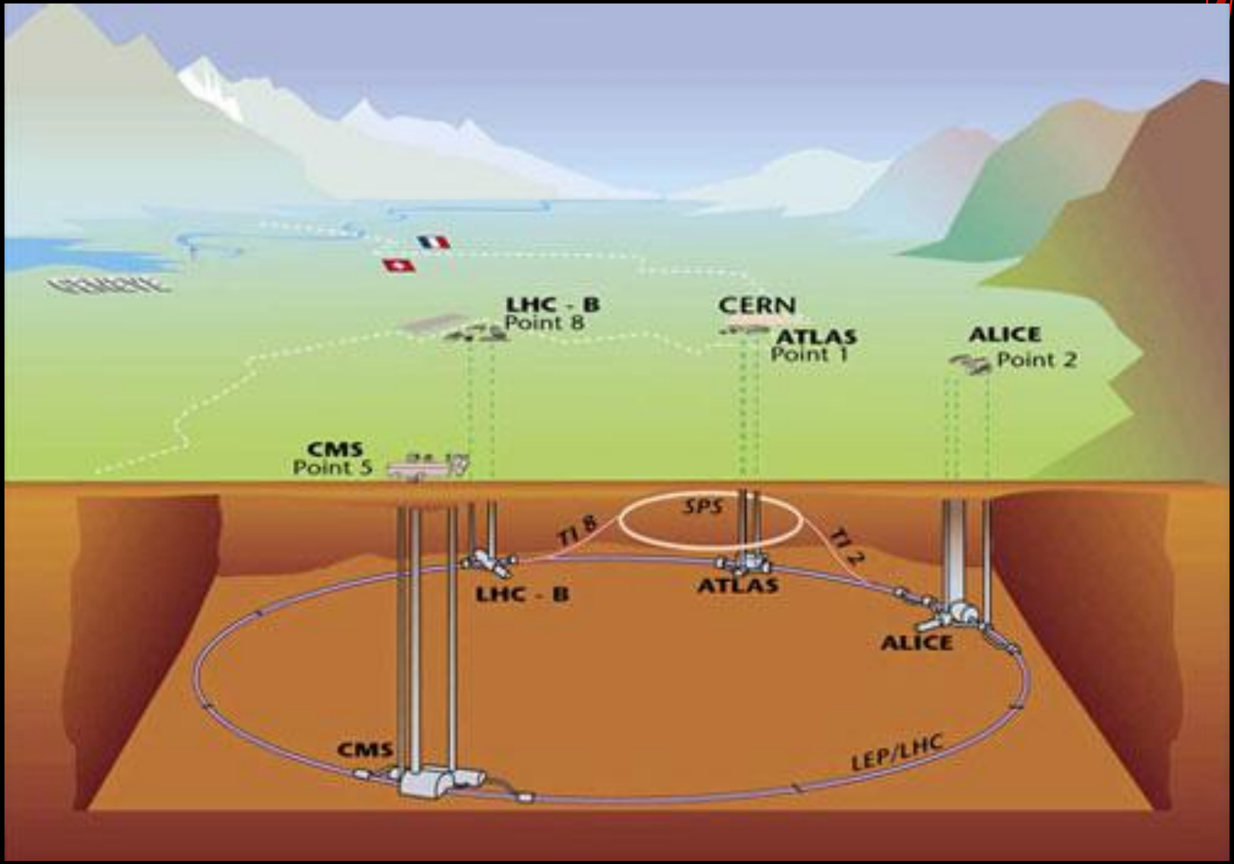
Fermilab: 6.3km ring

Synchrotrons

$$f = f_{osc}$$

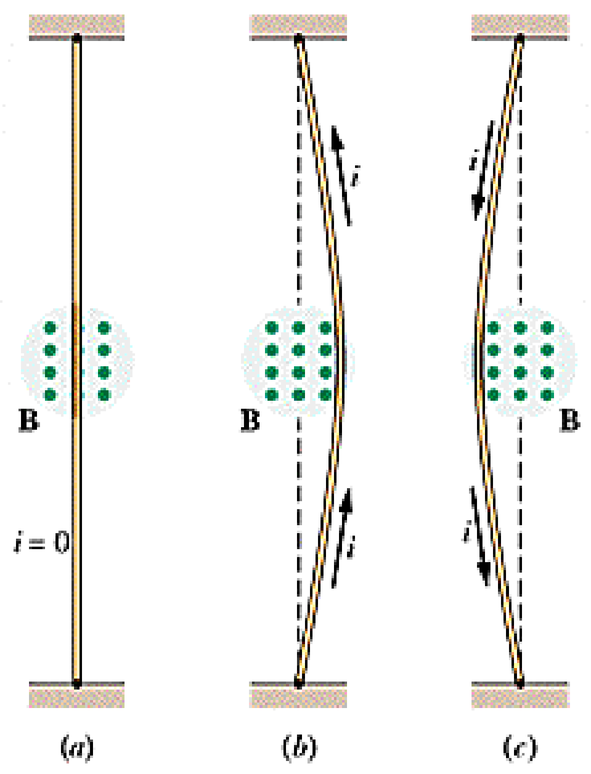
- The resonance condition:
- When proton energy > 50Mev:
  - Out of resonance (relativistic effect)
  - A huge magnet (4×10^6 m^2) is needed for high energy (500Gev) protons
- The proton sychrotron at Fermilab can produces 1Tev proton

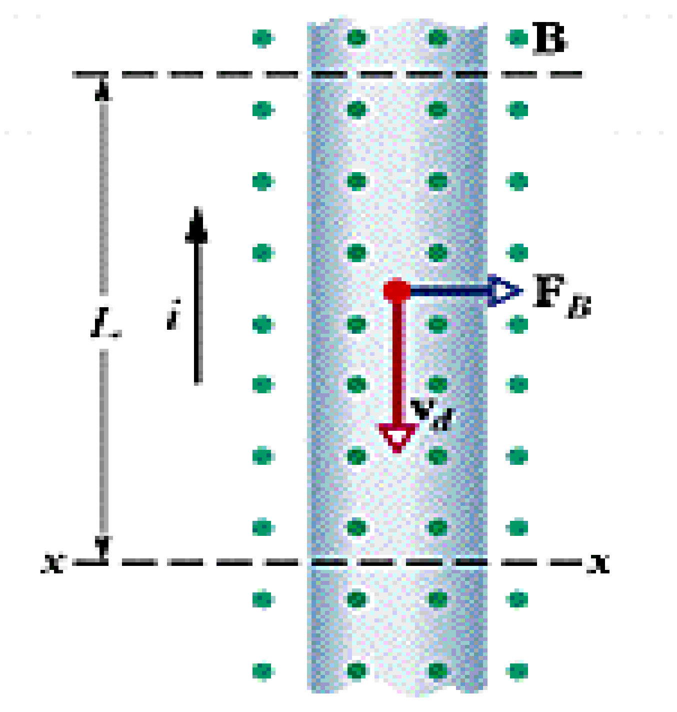
CERN LHC



The LHC is 27km long and sits 100m below the surface.

7-6 Magnetic Force on a Current-Carrying Wire





Magnetic Force

$$q = it = iL / v_d$$

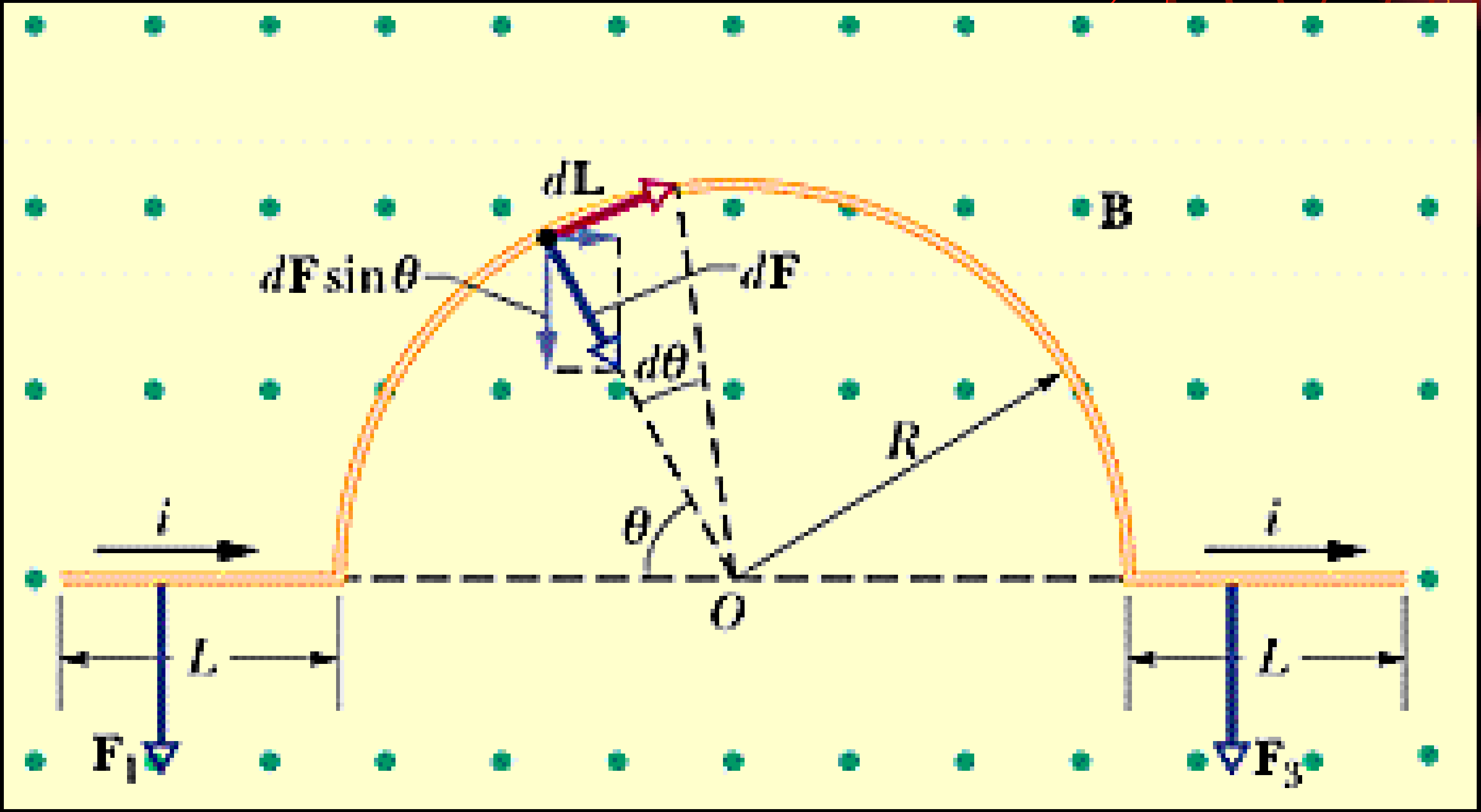
$$F_B = qv_d B \sin\phi = iLB$$

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

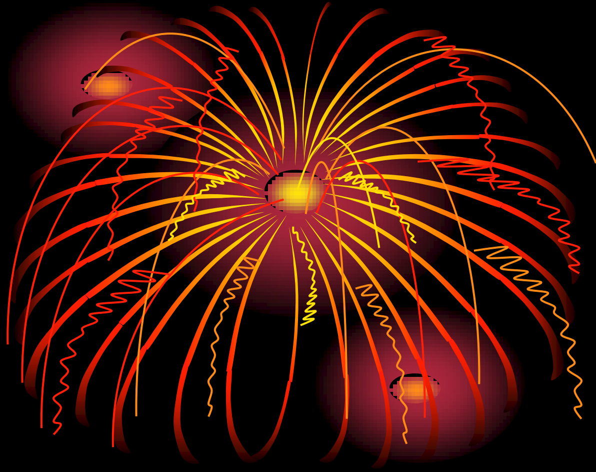
For a wire segment:

$$d\vec{F}_B = id\vec{L} \times \vec{B}$$

例4 A length of wire with a semicircular arc



Calculation



$$F_1 = F_3 = iLB$$

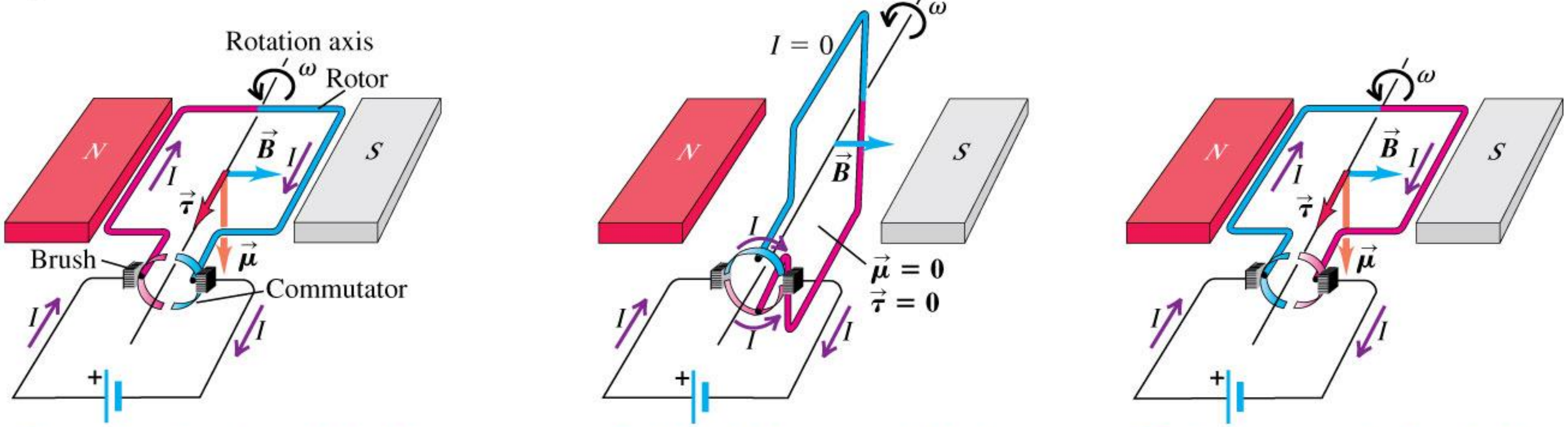
$$dF = iBdL = iB(Rd\theta)$$

$$F_2 = \int_0^{\pi} dF \sin\theta = iBR\int_0^{\pi} \sin\theta d\theta$$

$$= -iBR\cos\theta\Big|_0^{\pi} = 2iBR$$

$$F = F_1 + F_2 + F_3 = 2iB(L+R)$$

The direct-current motor



(a) Brushes are aligned with commutator segments.

- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

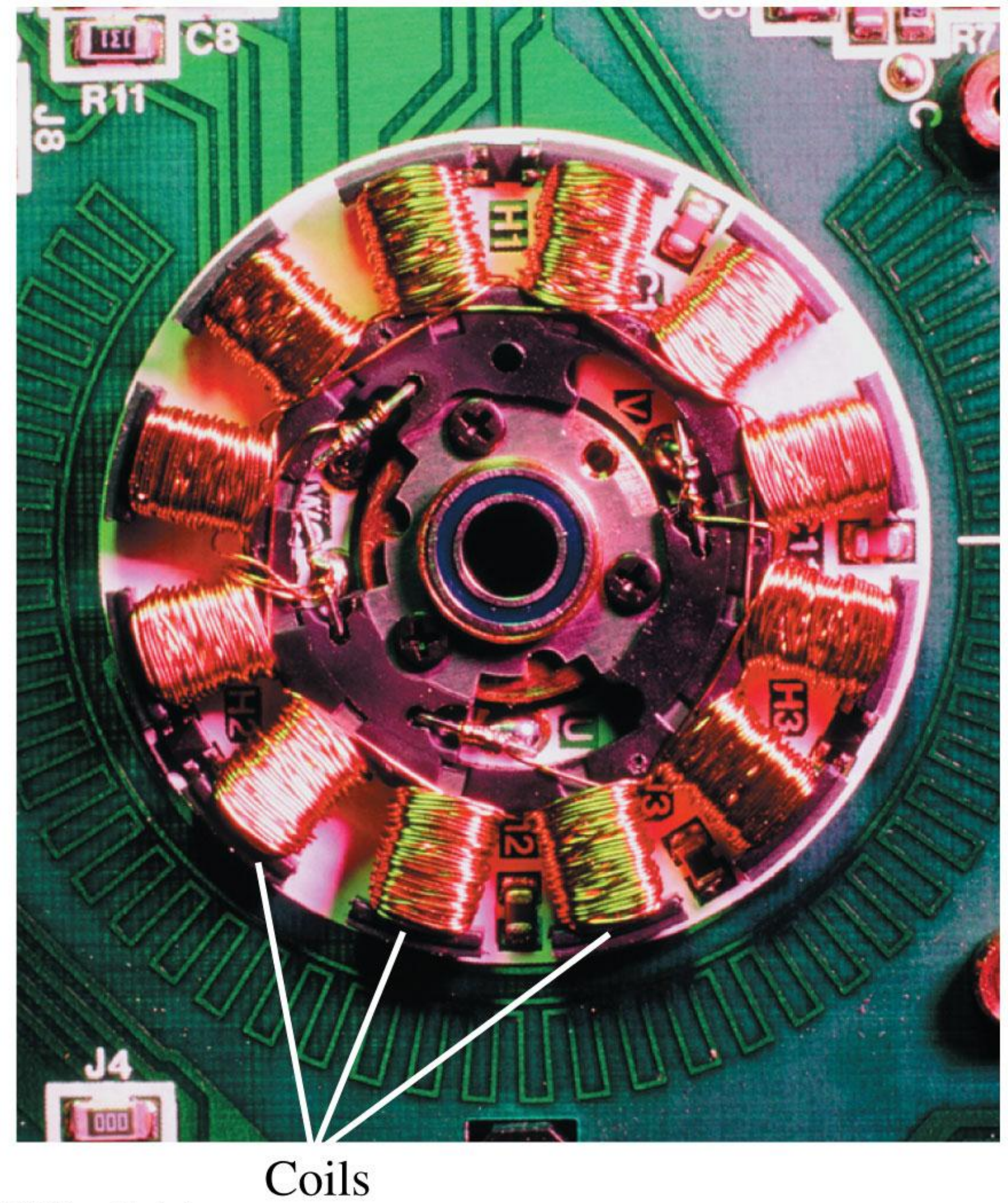
(b) Rotor has turned 90°.

- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

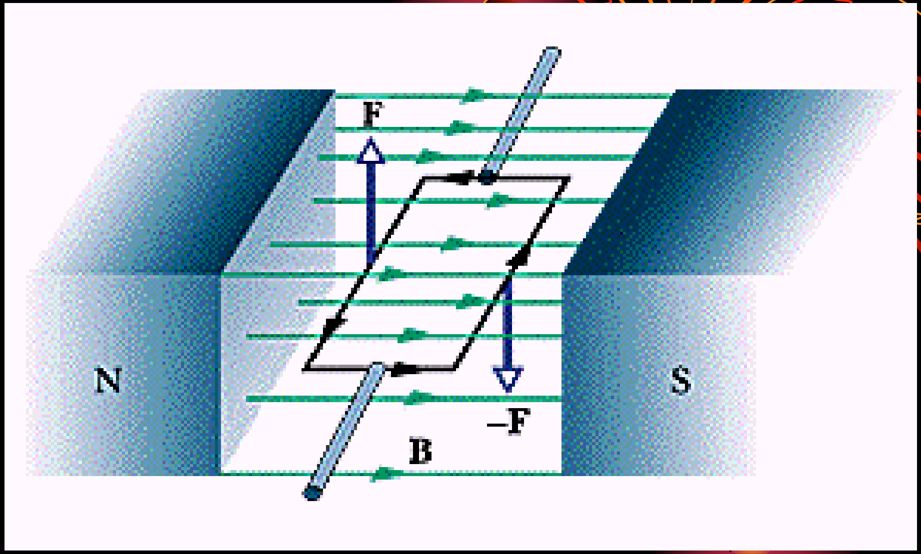
(c) Rotor has turned 180°.

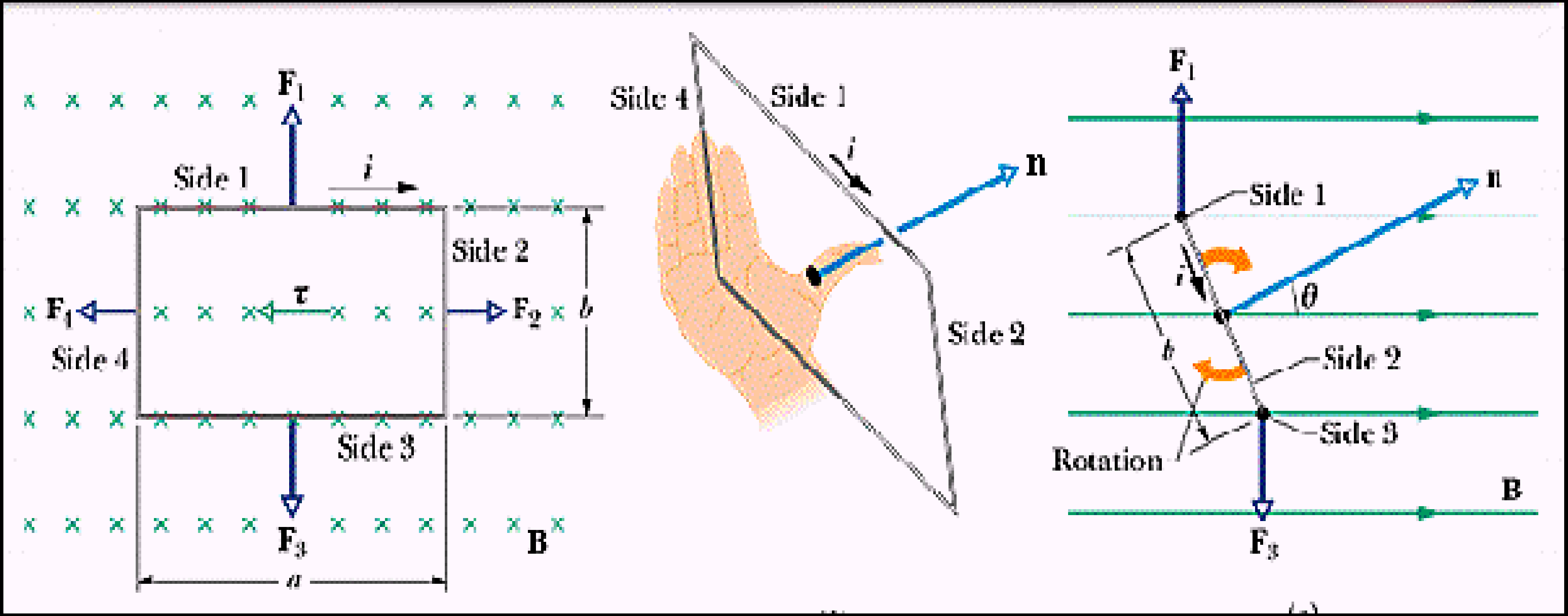
- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

Figure 27.40



線圈


F
N
S
−F
B


F1
Side 1
i
Side 2
τ
F1
F2
b
Side 4
Side 3
F3
B
a
Side 4
Side 1
i
n
Side 2
F1
Side 1
n
i
θ
b
Side 2
Side 3
Rotation
B
F3

7-7 Torque on A Current Loop

- F_2 and F_4 cancel
- F_1 and F_3 form a force couple

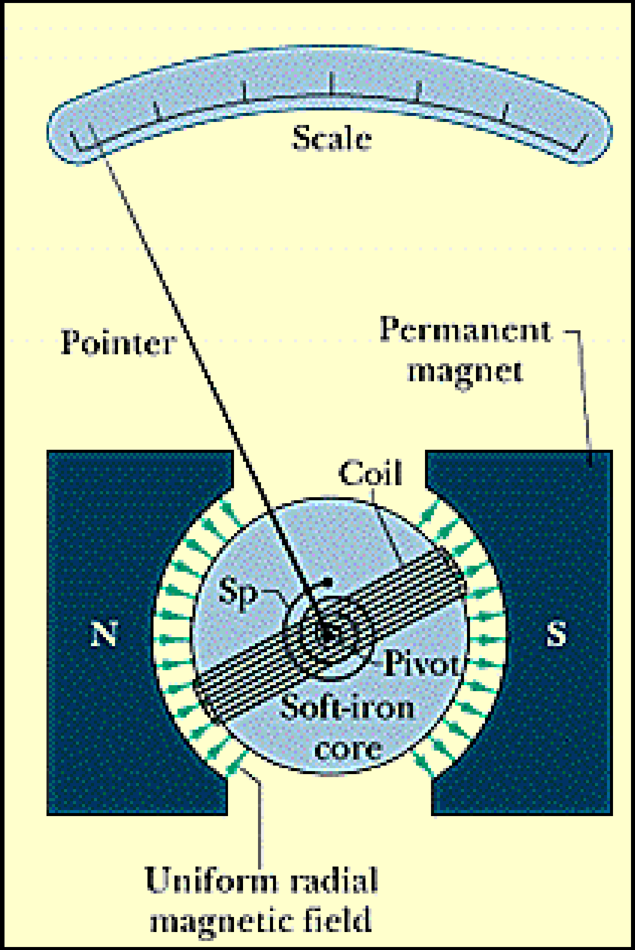
$$F_2 = ibB\sin(90^\circ - \theta) = ibB\cos\theta$$

$$\tau' = (iaB\frac{b}{2}\sin\theta) + (iaB\frac{b}{2}\sin\theta)$$

$$= iabB\sin\theta$$

$$\tau = N\tau' = (NiA)B\sin\theta$$

例 5 A galvanometer for analog meters



$$\tau = NiAB\sin\theta = \kappa\phi$$

$$\kappa = \frac{NiAB\sin\theta}{\phi}$$

$$= [(250)(100\times10^{-6}\,\mathrm{A})(2.52\times10^{-4}\,\mathrm{m}^2)$$

$$\times(0.23\mathrm{T})(\sin 90^\circ)]/28^\circ$$

$$= 5.2\times10^{-8}\,\mathrm{M}\cdot\mathrm{m/degree}$$

7-8 The Magnetic Dipole

- The magnetic dipole moments

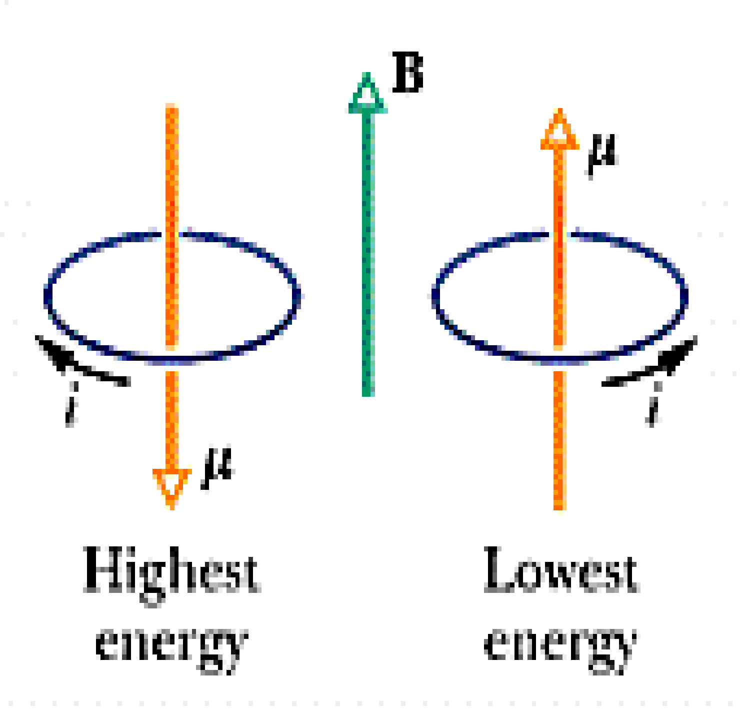
$$\mu = NiA \rightarrow \tau = \mu B \sin\theta$$

$$\rightarrow \vec{\tau} = \vec{\mu} \times \vec{B} \quad (\mathrm{cf}: \vec{\tau} = \vec{p} \times \vec{E})$$

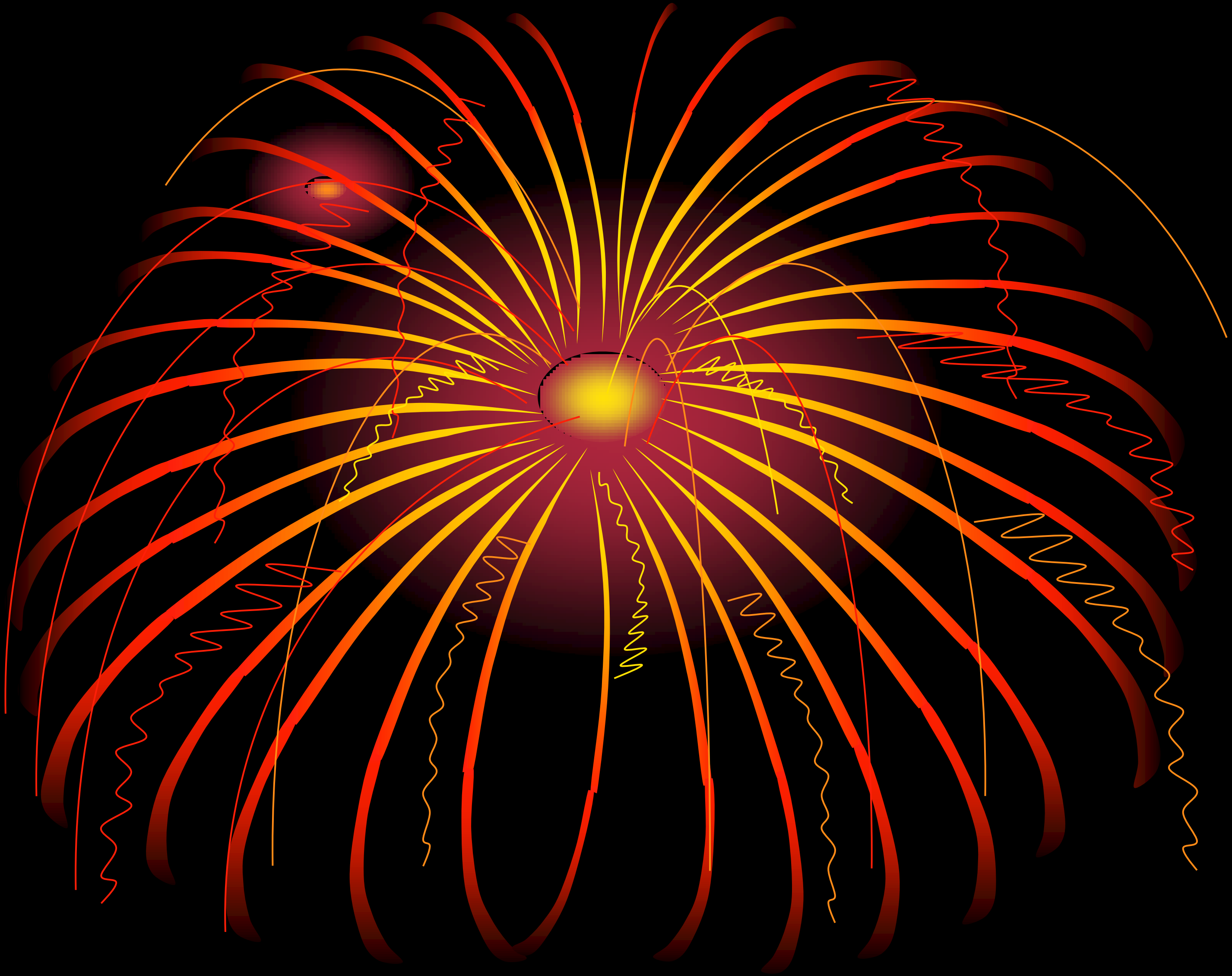
- The magnetic potential energy

$$U(\theta) = -\vec{p} \cdot \vec{E} \leftrightarrow U(\theta) = -\vec{\mu} \cdot \vec{B}$$

磁能



$$\boxed{\begin{aligned}\Delta U &= (+\mu B) - (-\mu B)\\ &= 2\mu B\end{aligned}}$$

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