

# 6 電路

# Outline

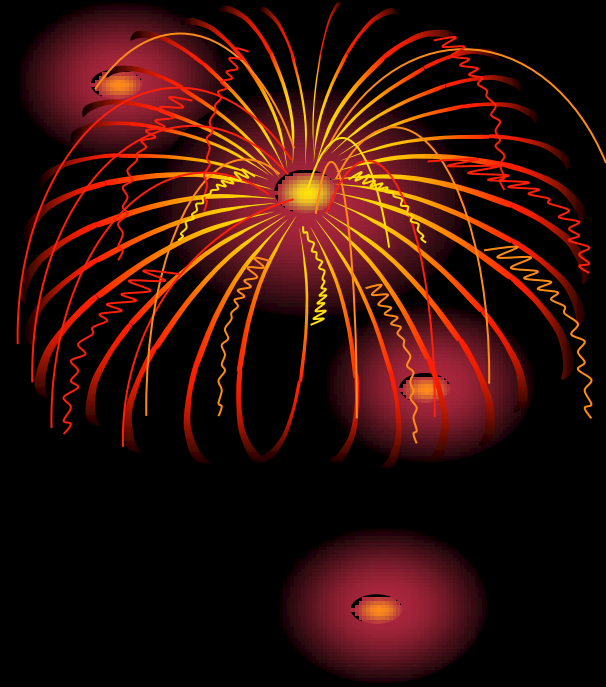
6.1 直流電路

6.2 串聯電阻

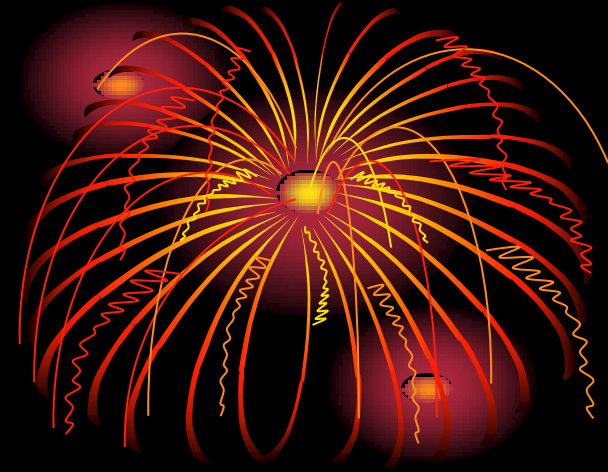
6.3 並聯電阻

6.4 交流電路

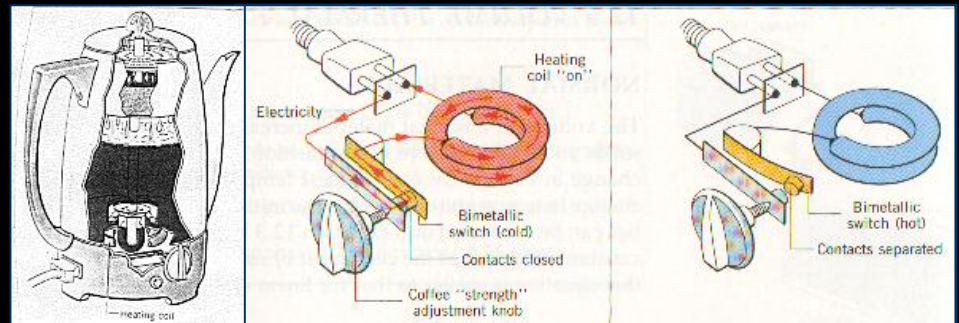
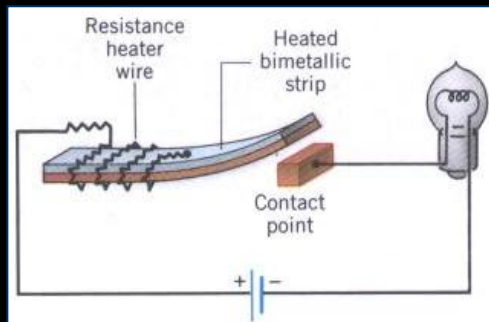
6.5 虛擬電路實驗



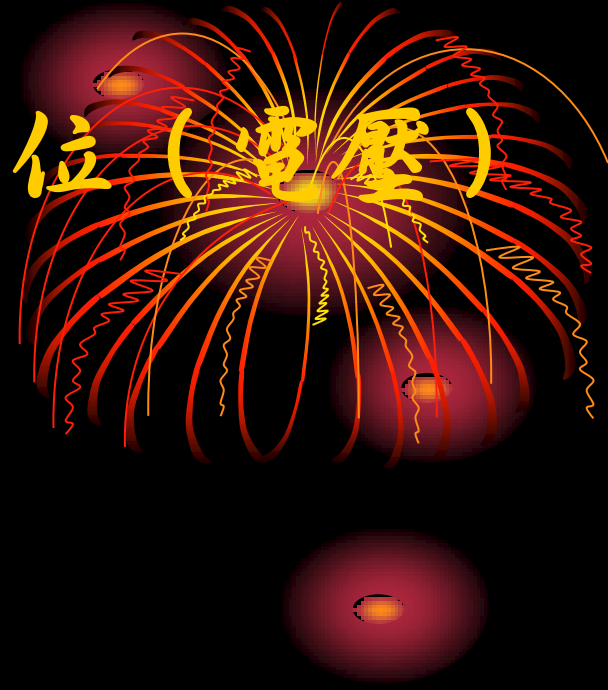
# 6.1 直流電路



- *Direct Current Circuit*
- *A bimetallic strip flasher*
- 以電阻電熱絲加熱 bimetallic strip，約一秒啟閉一次，可用作警示裝置。

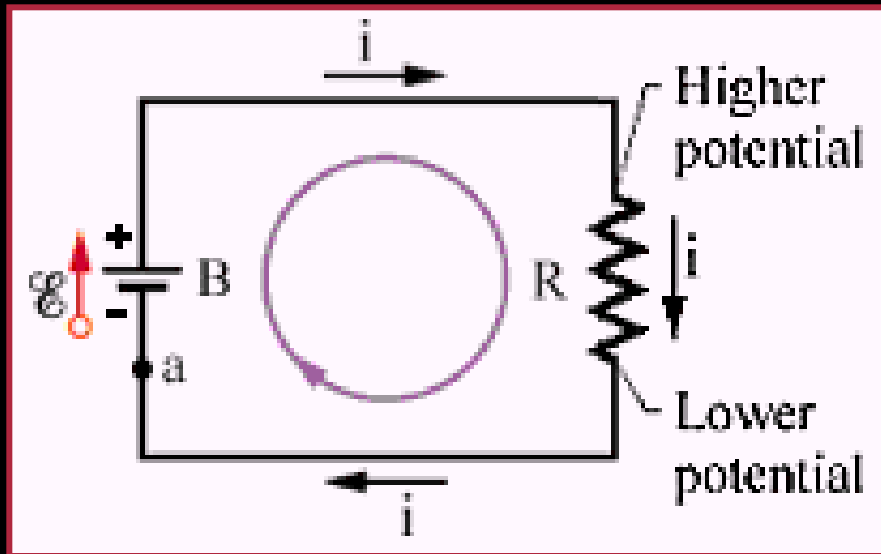


# Ex.1 簡單直流電路內之電位 (電壓) 升降



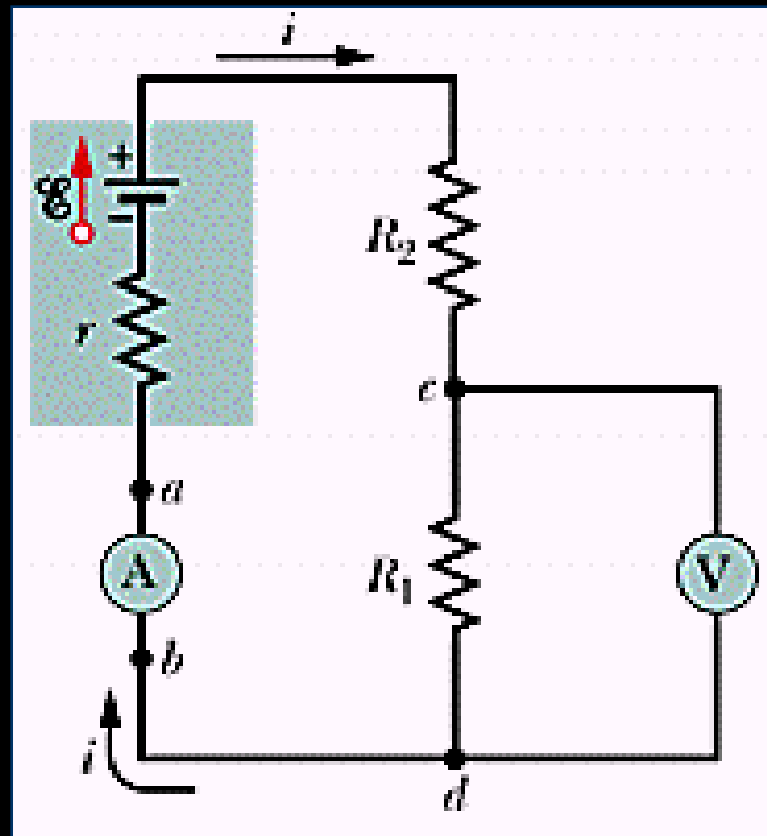
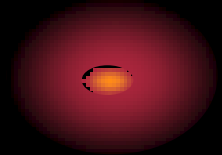
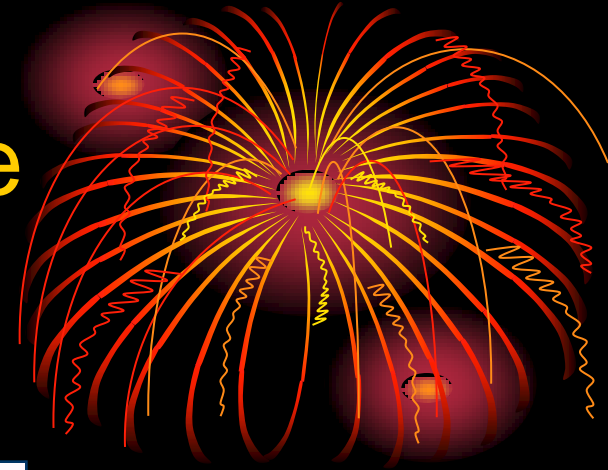
$$V_a + E - iR = V_a$$

$$E = iR$$



克希荷夫  
Kirchhoff 迴  
路定理

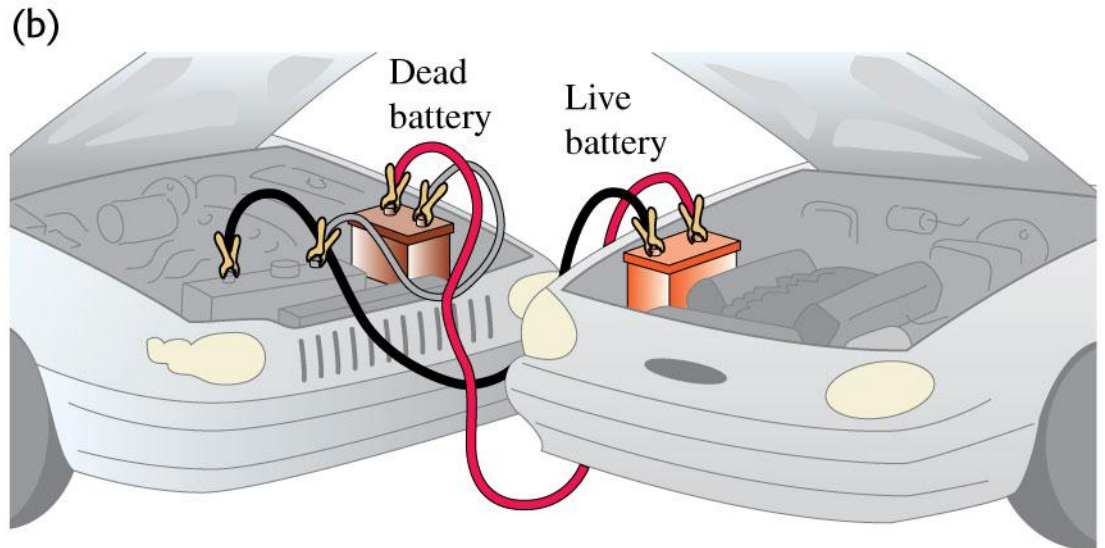
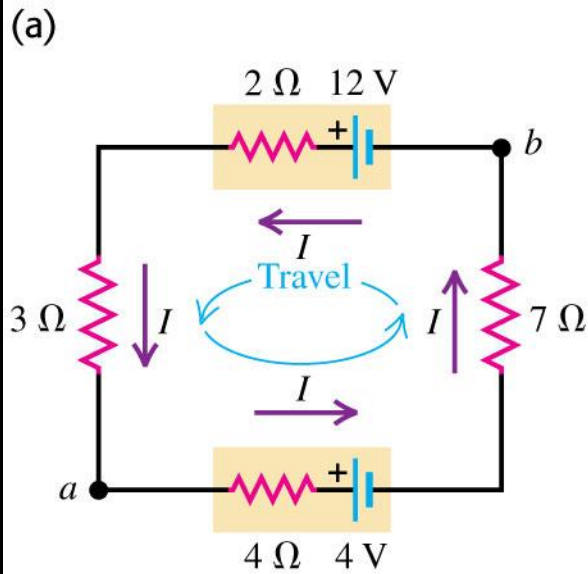
# Ex.2 The Ammeter and the Voltmeter



# A single-loop circuit

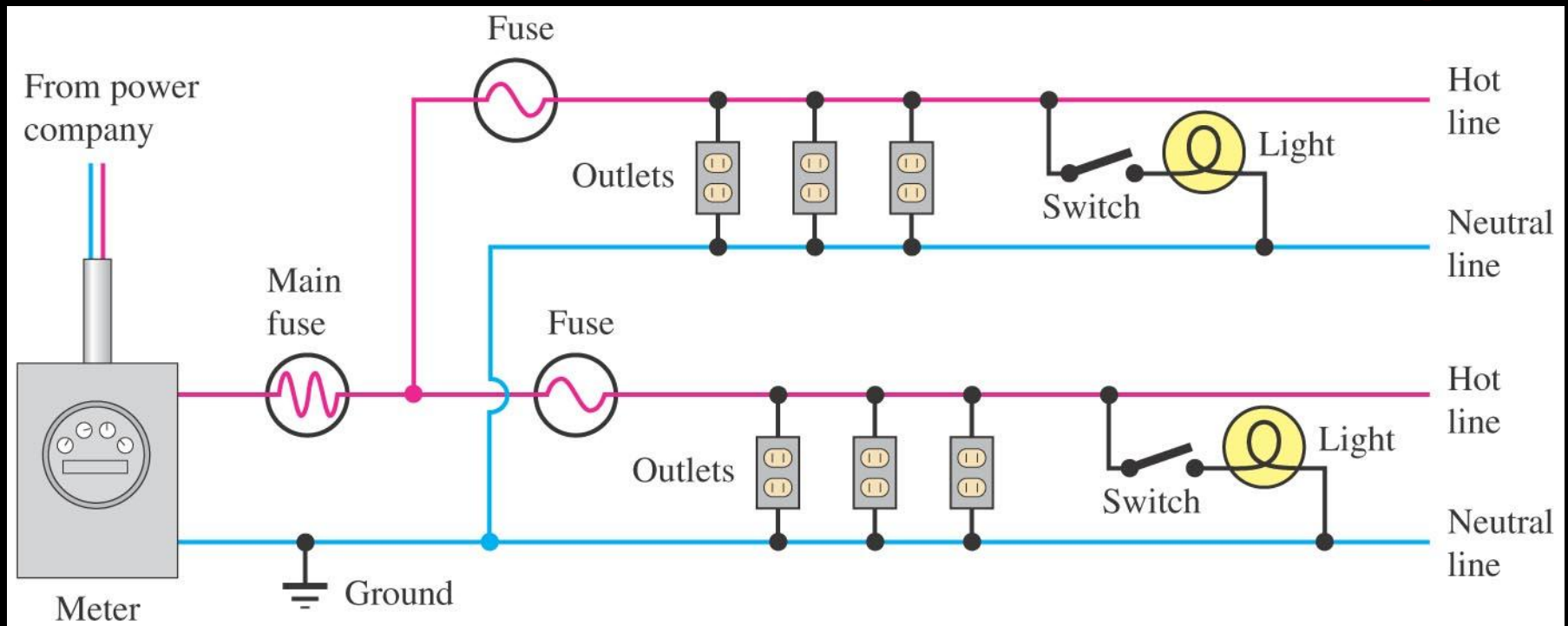


- Follow Example 26.3, using Figure 26.10 below.



# Power distribution systems

- Follow the text discussion using Figure 26.24 below.

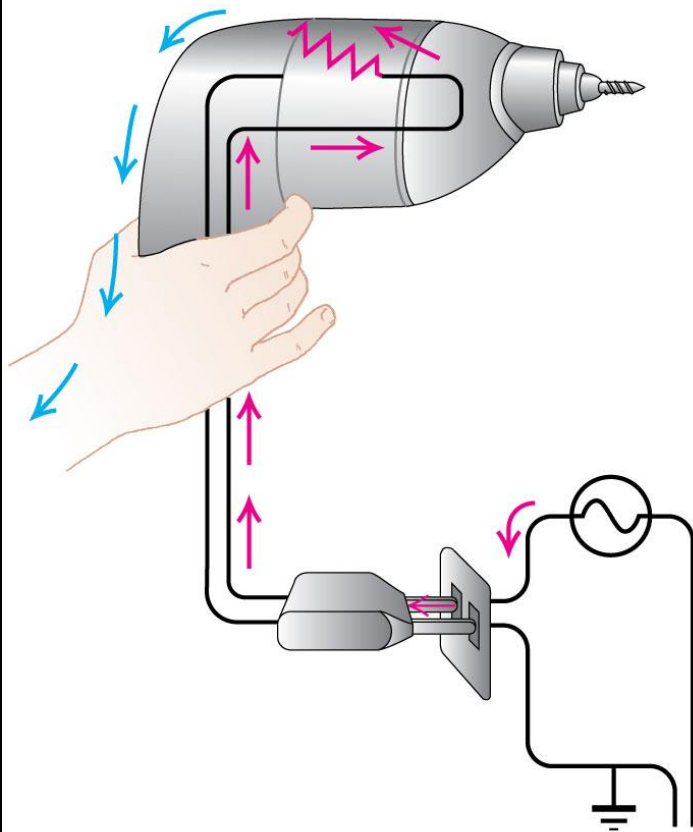


# Household wiring

- **Figure 26.26 at the right shows why it is safer to use a three-prong plug for electrical appliances.**



(a) Two-prong plug



(b) Three-prong plug

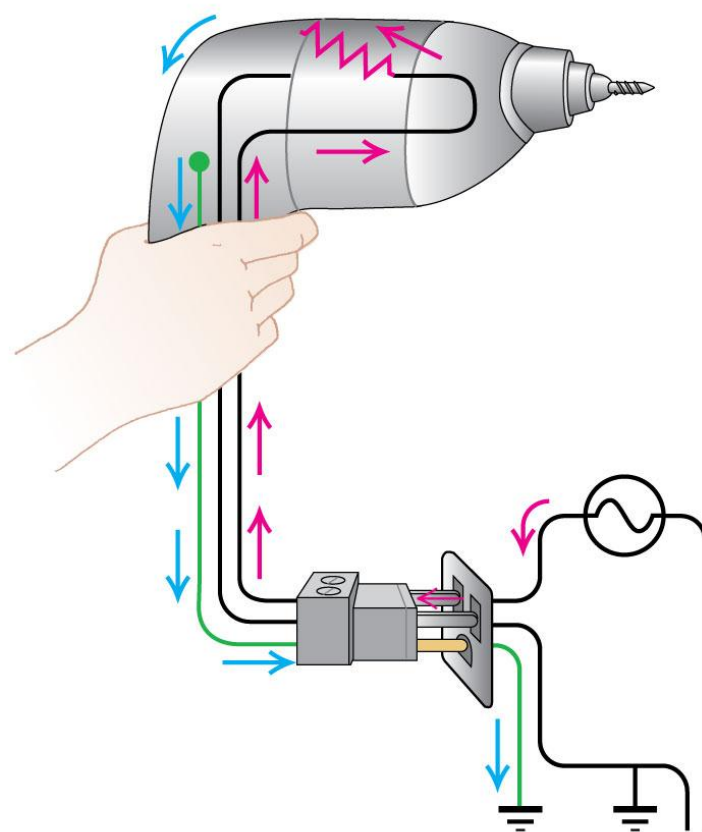
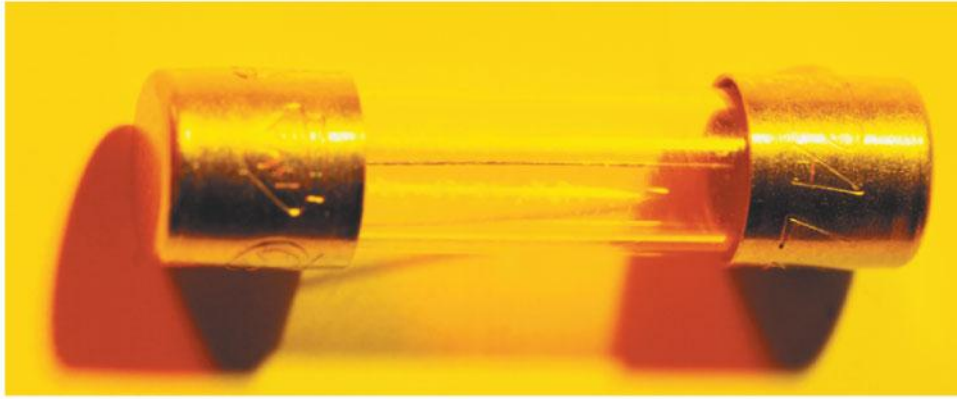


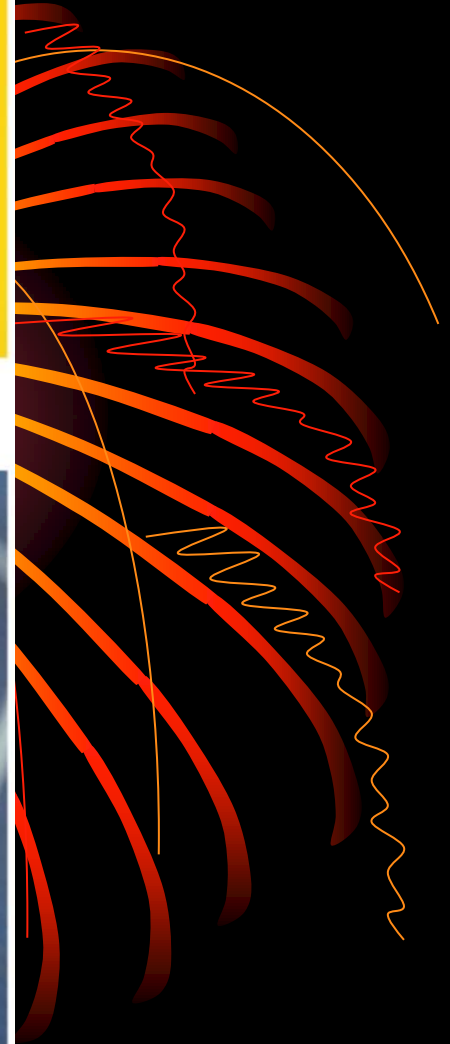
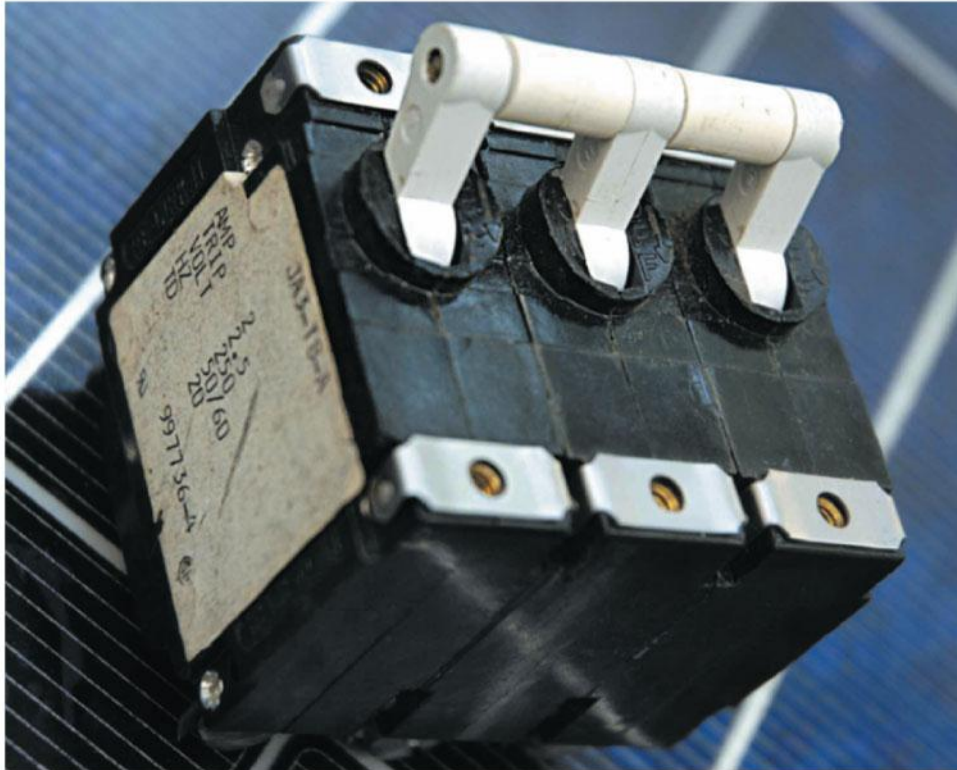


Figure 26.25

(a)

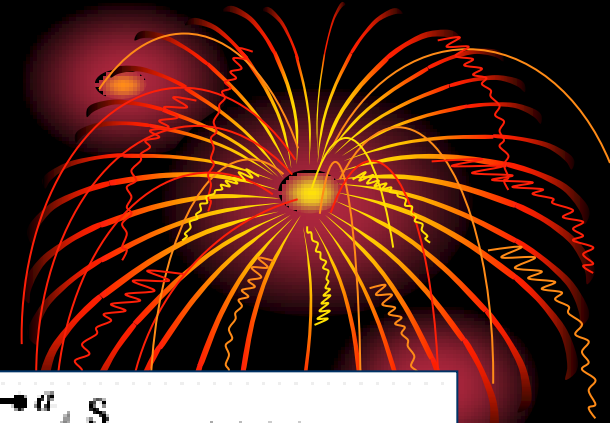


(b)

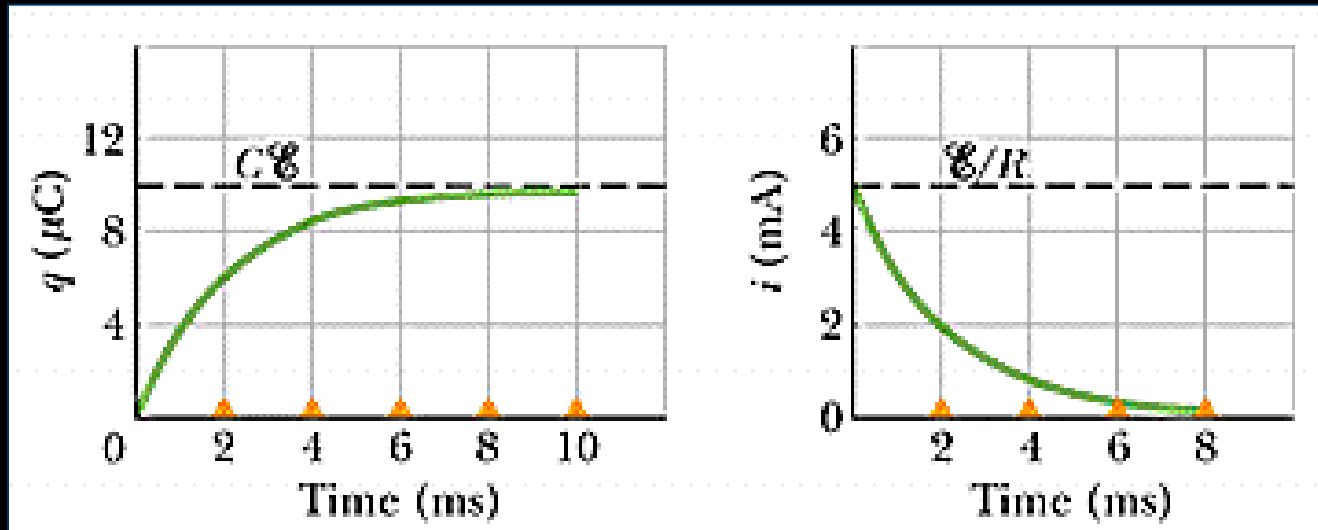
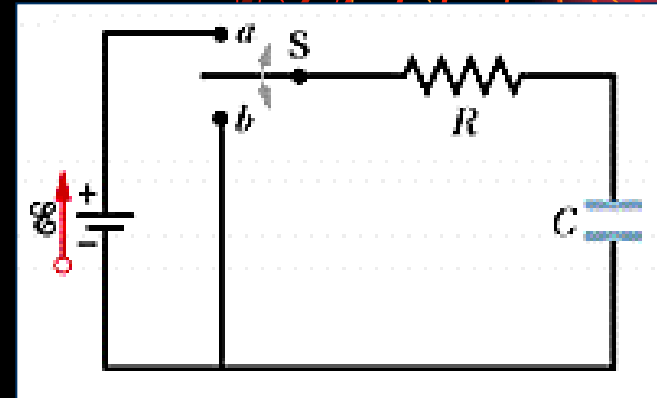




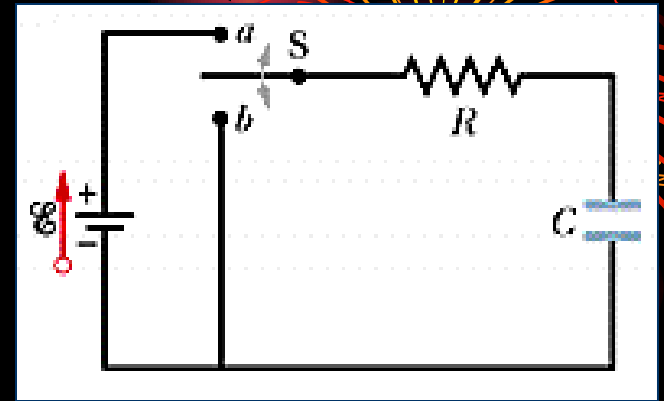
# Ex.3 RC 電路



- 電阻—電容電路



## Ex.3 (cont) Charging a Capacitor



$$\mathcal{E} - iR - \frac{q}{C} = 0, \quad i = \frac{dq}{dt}$$

$$R \frac{dq}{dt} + \frac{q}{C} = \mathcal{E} \rightarrow \frac{dq}{dt} + \frac{q}{RC} = \frac{\mathcal{E}}{R}$$

$$\rightarrow \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC} (q - C\mathcal{E})$$

$$\rightarrow \frac{dq}{(q - C\mathcal{E})} = -\frac{dt}{RC} \rightarrow \frac{d(q - C\mathcal{E})}{(q - C\mathcal{E})} = -\frac{dt}{RC}$$

## Ex.3 (cont)

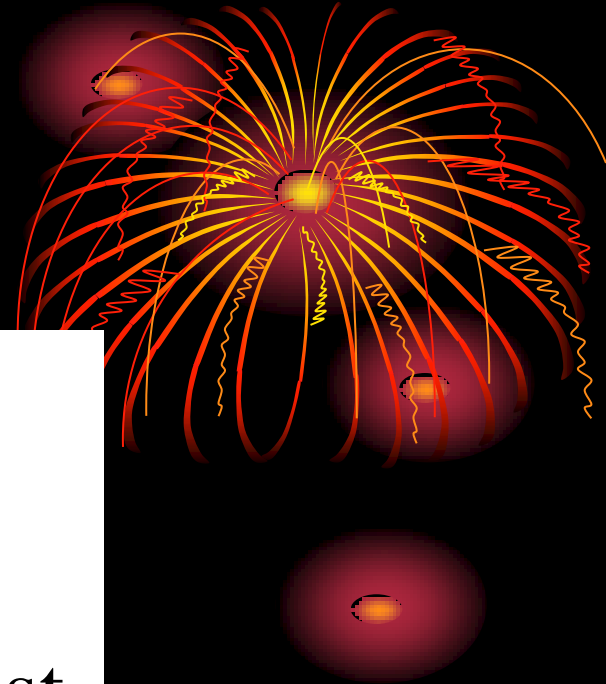
$$\rightarrow \int \frac{d(q - C\mathcal{E})}{(q - C\mathcal{E})} = -\int \frac{dt}{RC}$$

$$\rightarrow \ln(q - C\mathcal{E}) = -\frac{t}{RC} + \text{Const.}$$

$$\rightarrow q - C\mathcal{E} = e^{-\frac{t}{RC} + \text{const}}$$

$$\rightarrow q - C\mathcal{E} = e^{-\frac{t}{RC}} e^{\text{const}} = C' e^{-\frac{t}{RC}}$$

What is  $C'$ ?



## Ex.3 (Initial Condition)

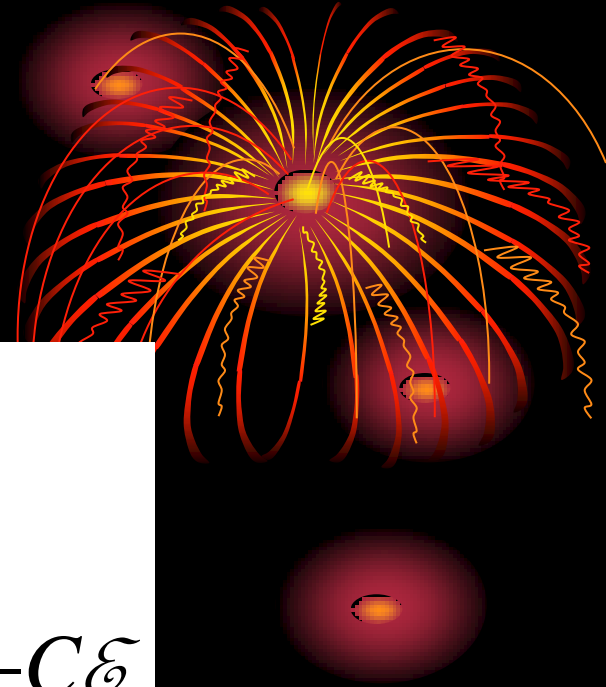
$$\rightarrow q(t) - C\mathcal{E} = C' e^{-\frac{t}{RC}}$$

$$\xrightarrow{t=0} 0 - C\mathcal{E} = C' e^{-\frac{0}{RC}} \rightarrow C' = -C\mathcal{E}$$

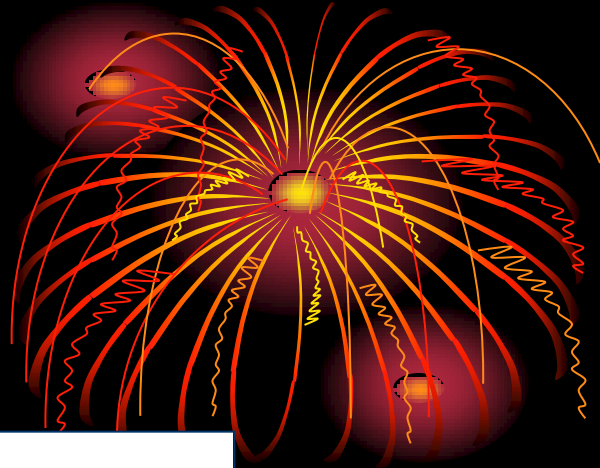
$$\rightarrow q(t) - C\mathcal{E} = -C\mathcal{E} e^{-\frac{t}{RC}}$$

$$\rightarrow q(t) = C\mathcal{E}(1 - e^{-t/RC})$$

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R}\right) e^{-t/RC}$$



## Ex.3 (cont)

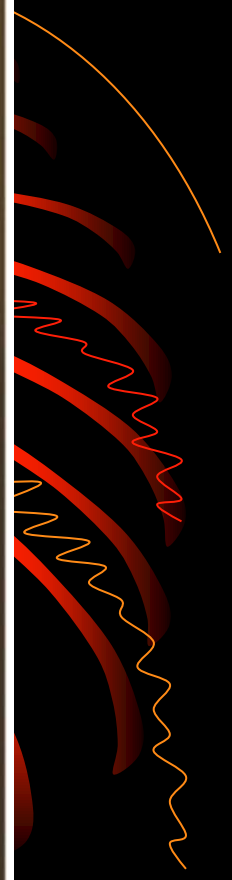
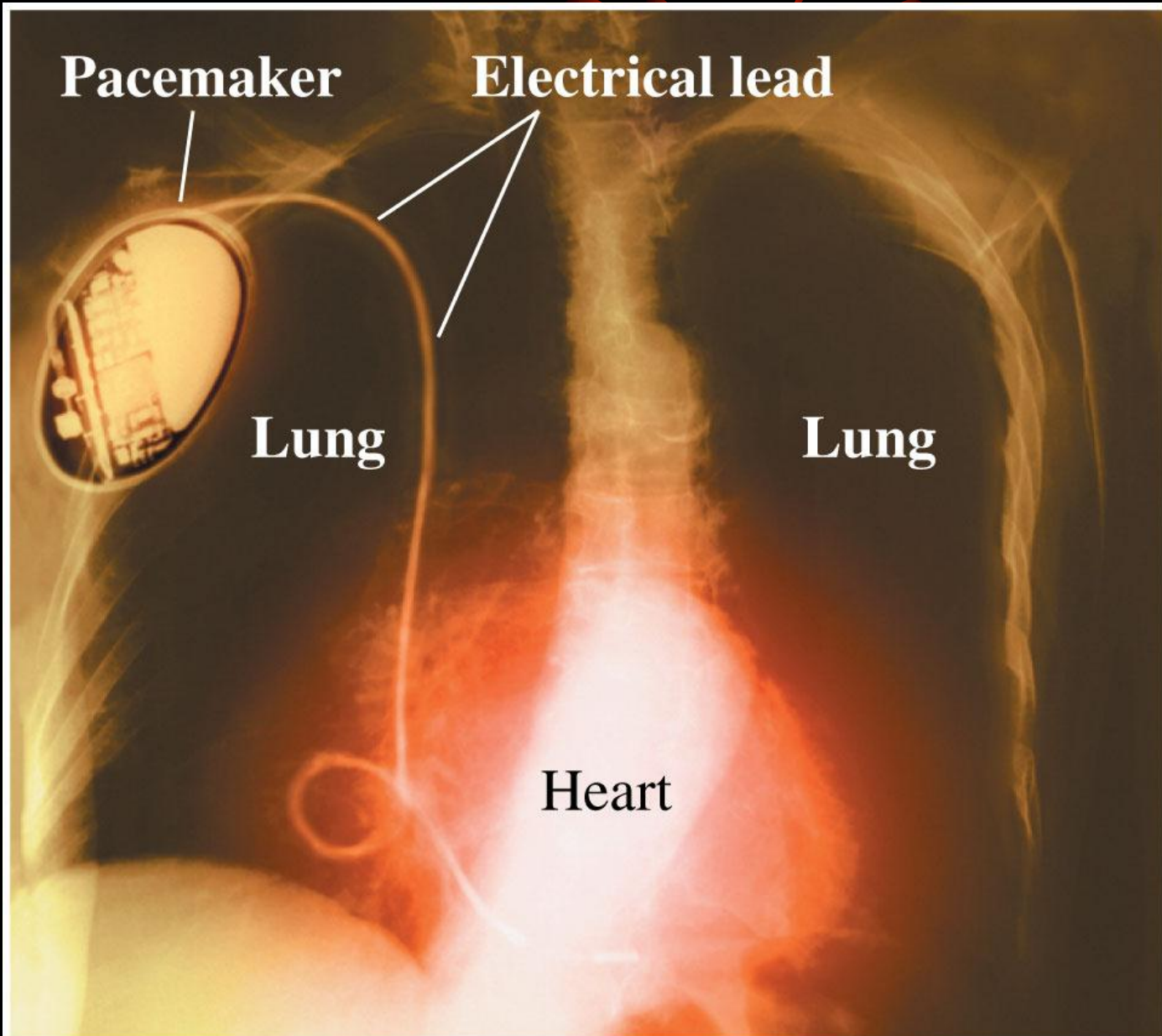


$$q(t) = C\varepsilon(1 - e^{-t/RC}) = 0.63C\varepsilon$$

- 如果  $R = 10\text{M}\Omega$      $C = 1\mu\text{F}$

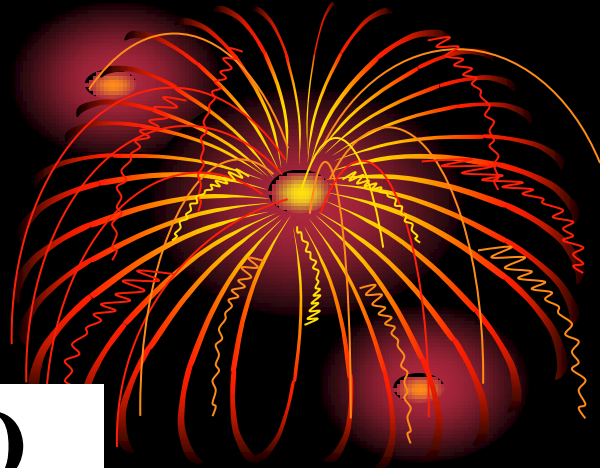
$$RC = (10 \times 10^6 \Omega)(10^{-6} \text{F}) = 10\text{s}$$

- 經過  $46\text{s}$  後， $e = 0.01$ ，充電達  $99\%$
- 可用作間歇式雨刷等電路控制。





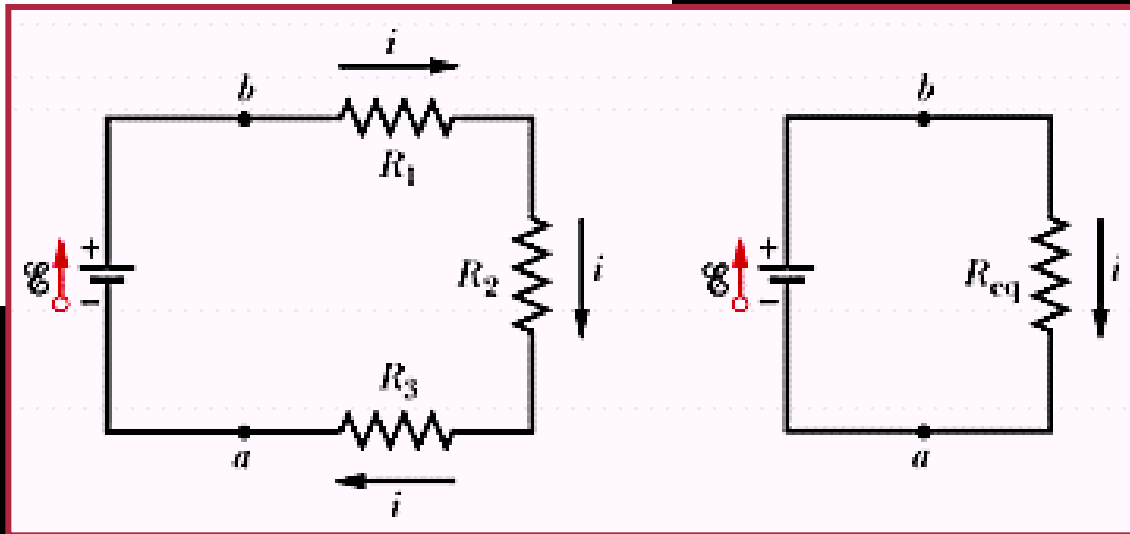
## 6.2 Resistance in series



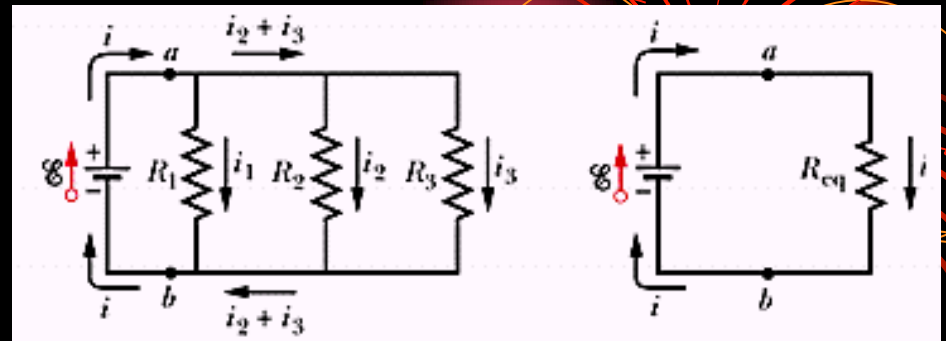
$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0$$

$$i = \frac{\mathcal{E}}{R_1 + R_2 + R_3} = \frac{\mathcal{E}}{R_{eq}}$$

$$R_{eq} = \sum_{j=1}^n R_j$$



## 6.3 Resistors in parallel

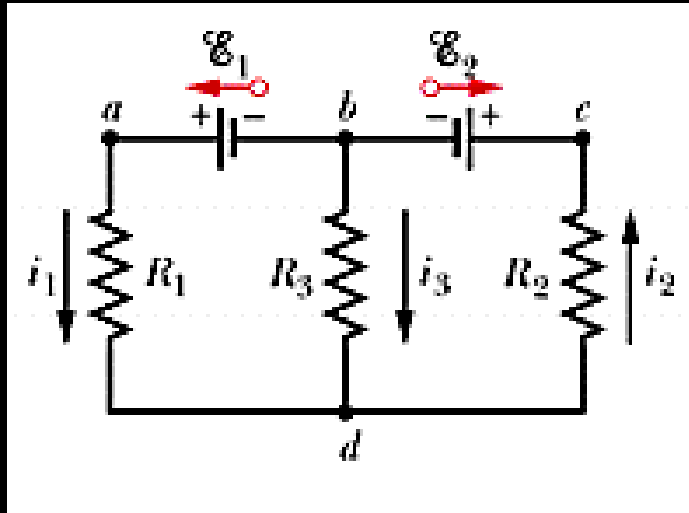


$$i_1 = \frac{\mathcal{E}}{R_1}, \quad i_2 = \frac{\mathcal{E}}{R_2}, \quad i_3 = \frac{\mathcal{E}}{R_3}$$

$$i = i_1 + i_2 + i_3 = \mathcal{E} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

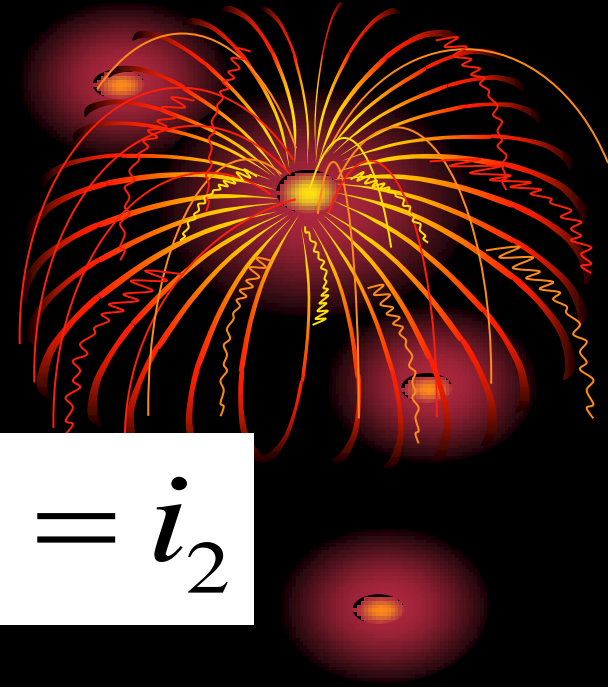
$$i = \frac{\mathcal{E}}{R_{eq}} \rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

# Multiloop Circuits

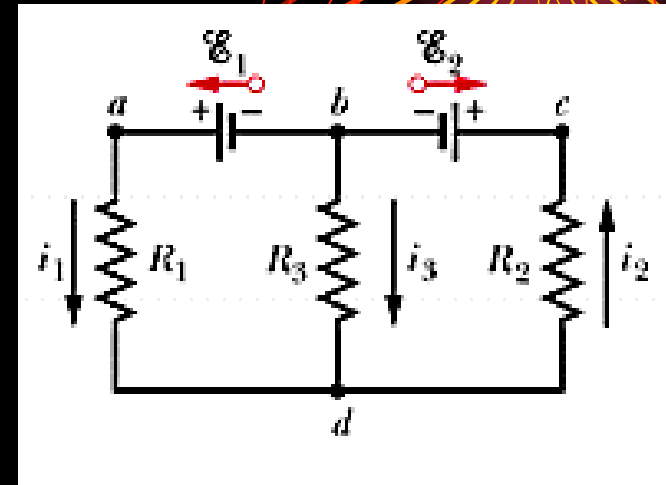


$$i_1 + i_3 = i_2$$

- The Junction rule: 流入接點之電流必等於流出者  
(Kirchhoff's junction/current rule)



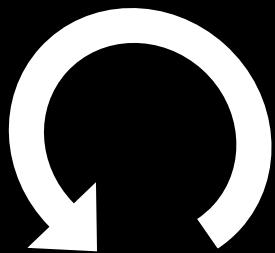
# 解 System of equations (聯立方程組)



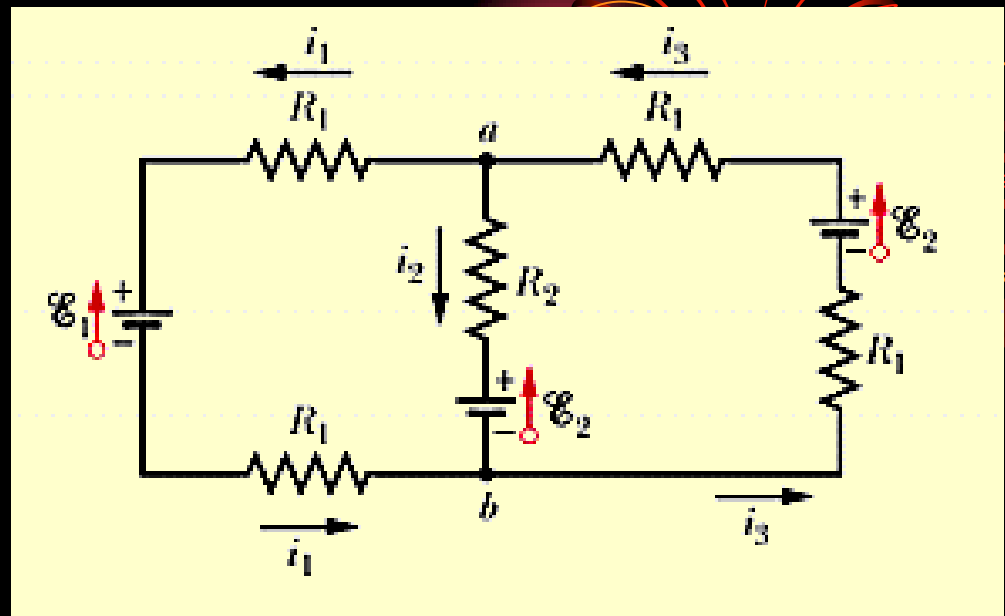
$$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0 \text{ left loop}$$

$$-i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0 \text{ right loop}$$

$$\mathcal{E}_1 - i_1 R_1 - i_2 R_2 - \mathcal{E}_2 = 0 \text{ big loop}$$



Ex.4 What are the currents?



$$i_3 = i_1 + i_2$$

$$-i_1 R_1 - \mathcal{E}_1 - i_1 R_1 + \mathcal{E}_2 + i_2 R_2 = 0$$

$$+ i_3 R_1 - \mathcal{E}_2 + i_3 R_1 + \mathcal{E}_2 + i_2 R_2 = 0$$

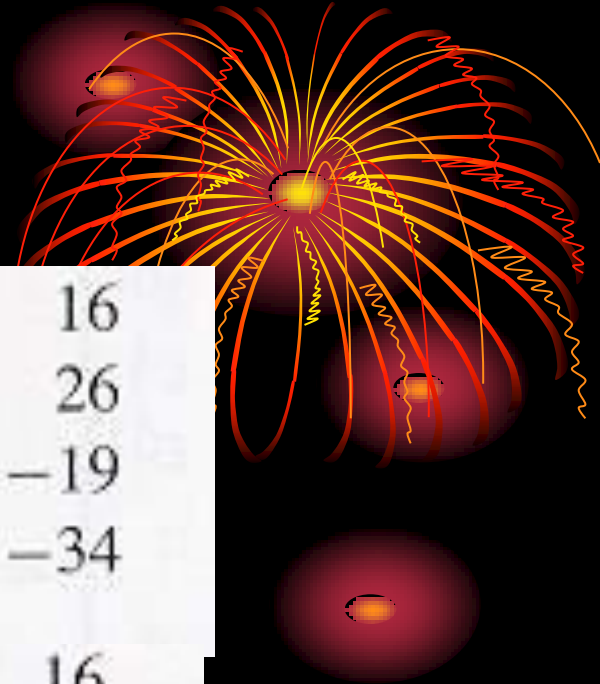
# 解聯立方程組

*pivot  
equation*

$$\begin{cases} 6x_1 - 2x_2 + 2x_3 + 4x_4 = 16 \\ 12x_1 - 8x_2 + 6x_3 + 10x_4 = 26 \\ 3x_1 - 13x_2 + 9x_3 + 3x_4 = -19 \\ -6x_1 + 4x_2 + x_3 - 18x_4 = -34 \end{cases}$$

$$\begin{cases} 6x_1 - 2x_2 + 2x_3 + 4x_4 = 16 \\ -4x_2 + 2x_3 + 2x_4 = -6 \\ -12x_2 + 8x_3 + x_4 = -27 \\ 2x_2 + 3x_3 - 14x_4 = -18 \end{cases}$$

$$\begin{cases} 6x_1 - 2x_2 + 2x_3 + 4x_4 = 16 \\ -4x_2 + 2x_3 + 2x_4 = -6 \\ 2x_3 - 5x_4 = -9 \\ 4x_3 - 13x_4 = -21 \end{cases}$$



# 高斯消去法

$$\begin{cases} 6x_1 - 2x_2 + 2x_3 + 4x_4 = 16 \\ -4x_2 + 2x_3 + 2x_4 = -6 \\ \phantom{-4x_2 + 2x_3 + 2x_4 = -6} 2x_3 - 5x_4 = -9 \\ \phantom{-4x_2 + 2x_3 + 2x_4 = -6} \phantom{2x_3 - 5x_4 = -9} -3x_4 = -3 \end{cases}$$

$$x_4 = \frac{-3}{-3} = 1 \quad 2x_3 - 5 = -9$$

$$x_3 = \frac{-4}{2} = -2$$

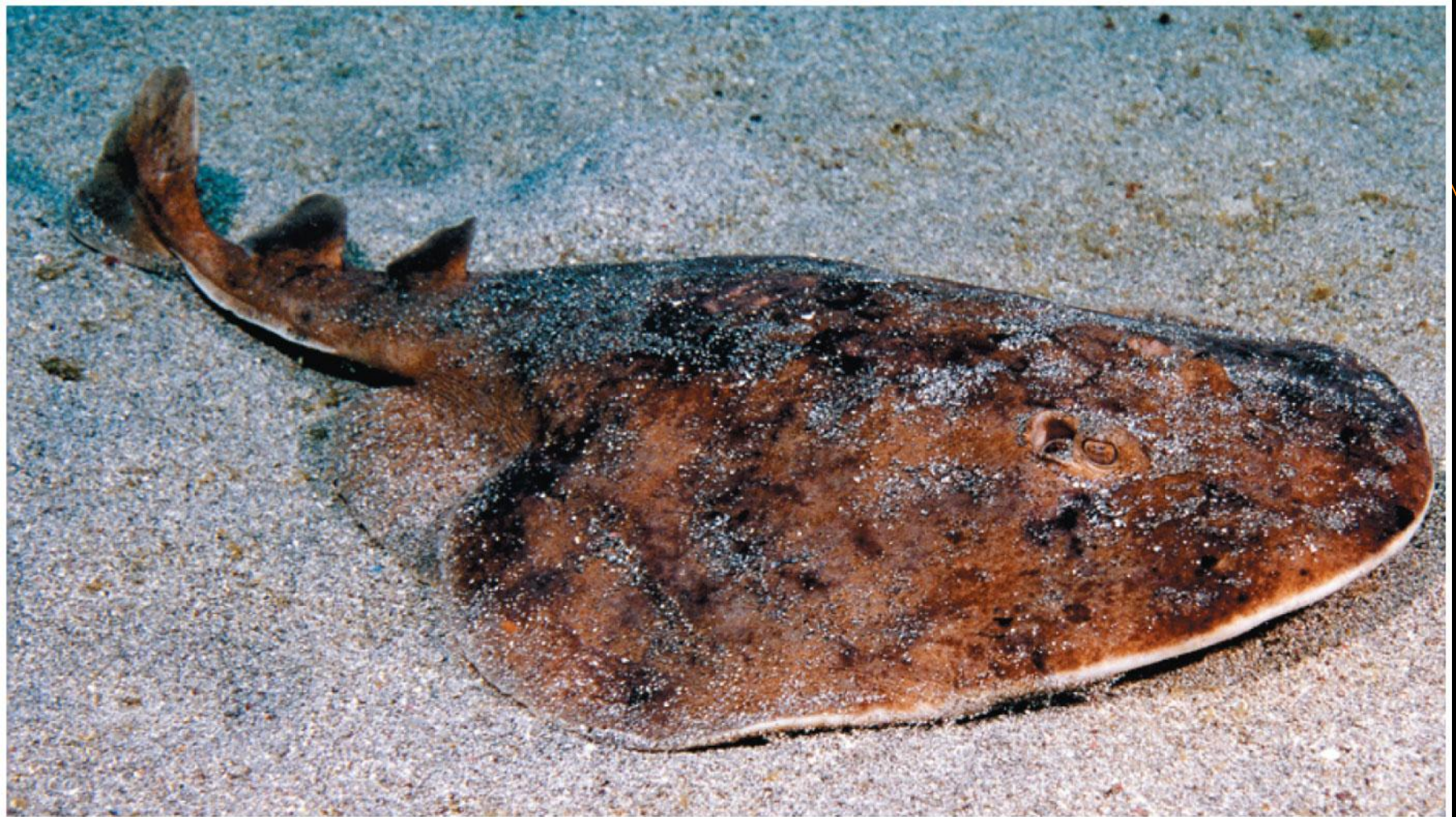
$$x_1 = 3 \quad x_2 = 1 \quad x_3 = -2 \quad x_4 = 1$$



pivot equation

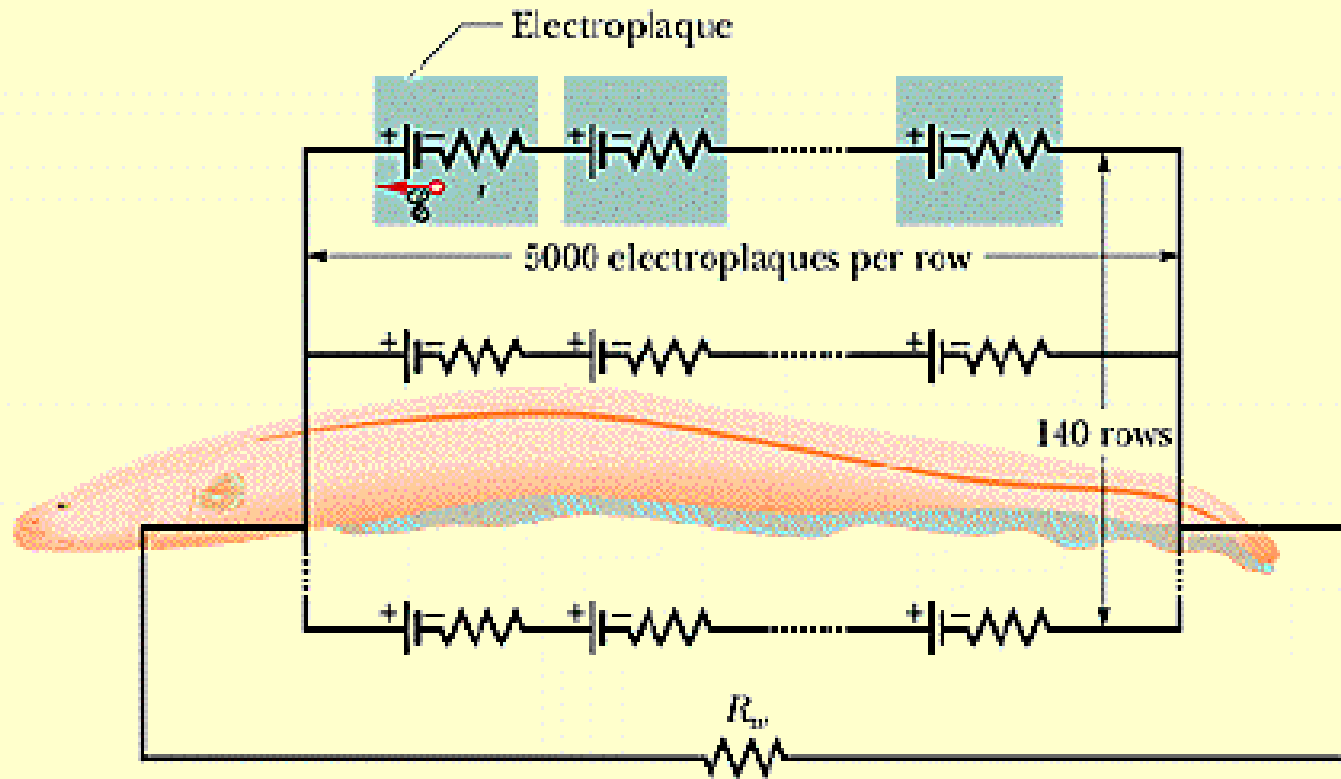
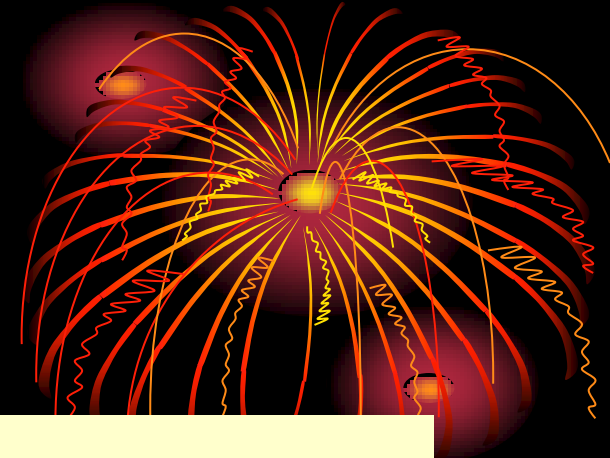
upper  
triangle

**Forward Elimination and Back substitution**



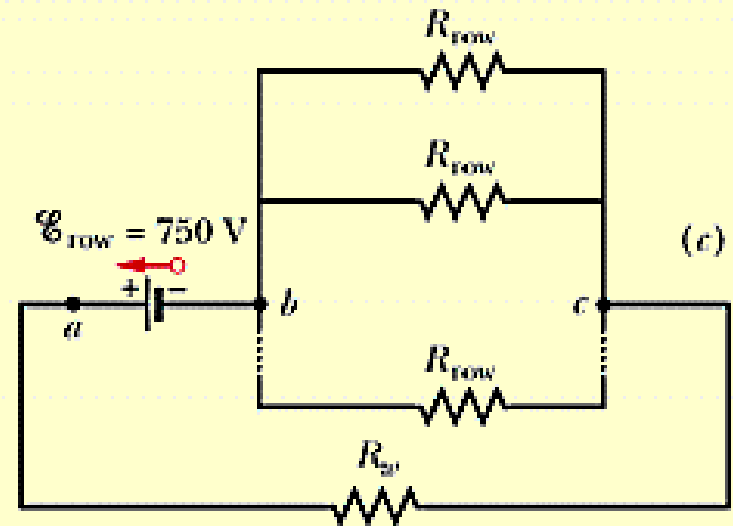
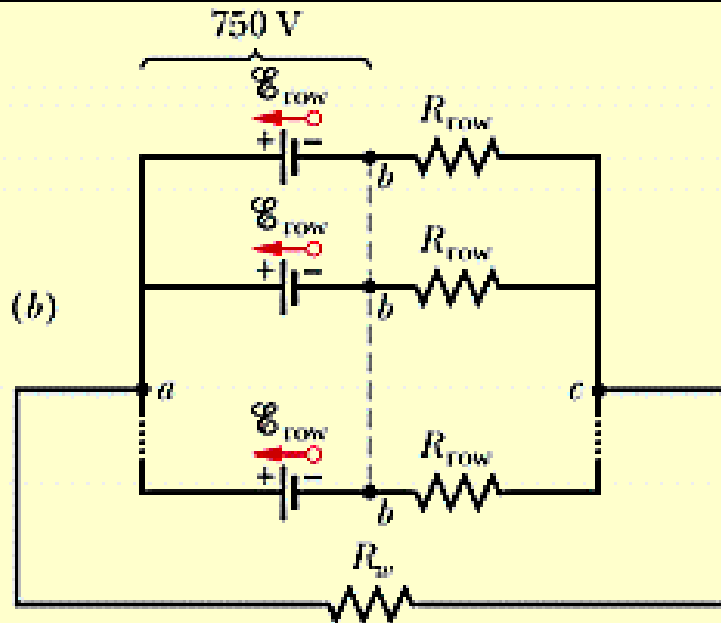


# Ex.5 The electric eel

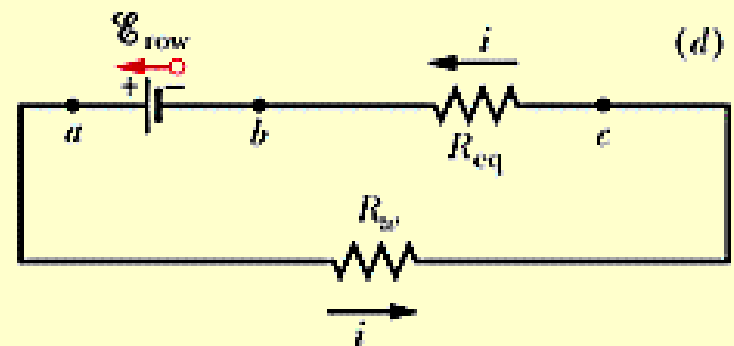


(a)

# The Equivalent circuit



The surrounding water has resistance  $R_w$ . (b) The emf  $\mathcal{E}_{row}$  and resistance  $R_{row}$  of each row. (c) The emf between points  $a$  and  $b$  is  $\mathcal{E}_{row}$ . Between points  $b$  and  $c$  are 140 parallel resistances  $R_{row}$ . (d) The simplified circuit, with  $R_{eq}$  replacing the parallel combination.



# The current

$$0.15V$$

$$\mathcal{E}_{row} = 5000\mathcal{E}_{ep} = 750V$$

$$0.25V$$

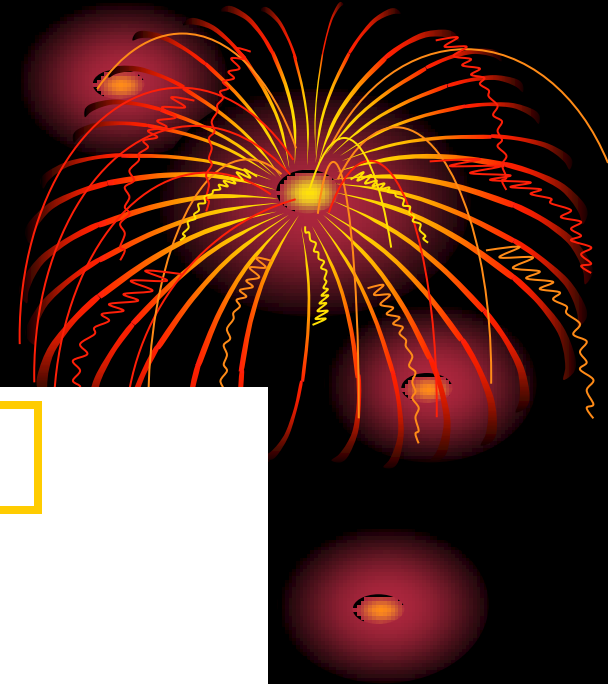
$$R_{row} = 5000r_{ep} = 1250\Omega$$

$$\frac{1}{R_{eq}} = \sum_{j=1}^{140} \frac{1}{R_j} = 140 \frac{1}{R_{row}} \rightarrow R_{eq} = 8.93\Omega$$

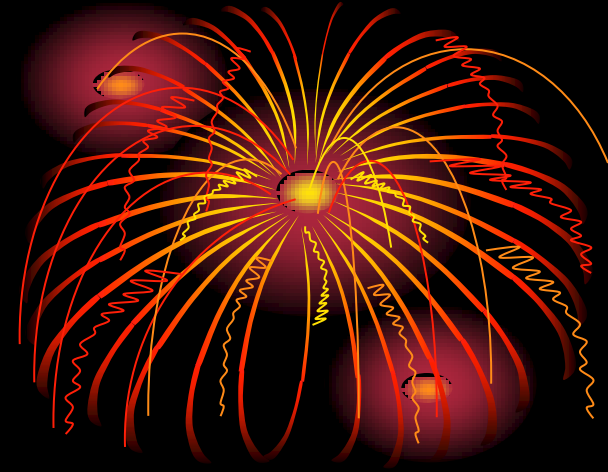
$$R_w = 800\Omega$$

$$\mathcal{E}_{row} - iR_w - iR_{eq} = 0 \rightarrow i = 0.93A$$

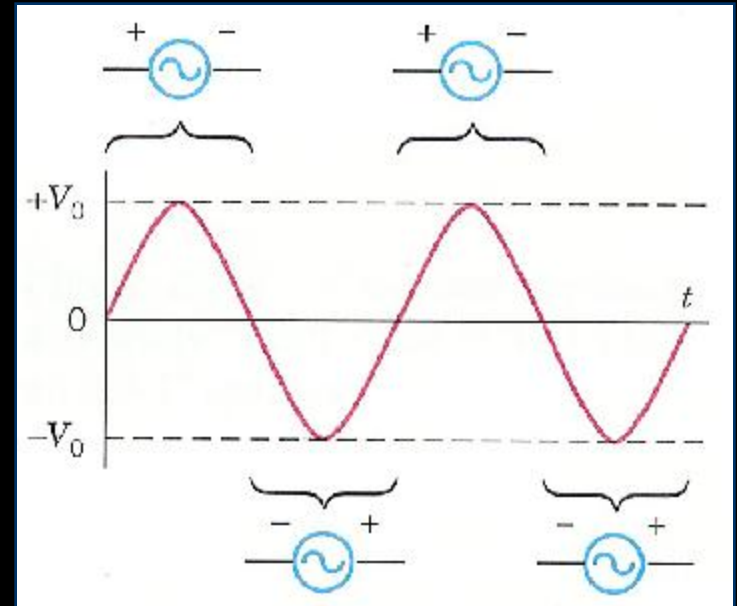
$$i_{row} = i / 140 = 6.6 \times 10^{-3} A$$



# 6.4 交流電路



- *Alternating Current Circuit*
- $V(t) = V_0 \sin 2\pi f t$
- $f$  頻率 = 60Hz (赫)
- $V_0$  尖峰值，  
一般家庭約170V



# 功率



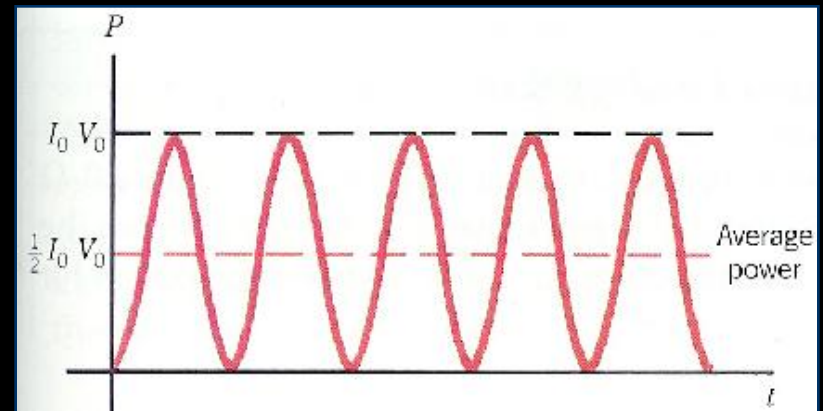
- $P(t) = I_0 V_0 \sin^2 2\pi f t$ ，取時間平均值

$$\bar{P} = \frac{1}{2} I_0 V_0 = \frac{I_0}{\sqrt{2}} \frac{V_0}{\sqrt{2}} = I_{rms} V_{rms}$$

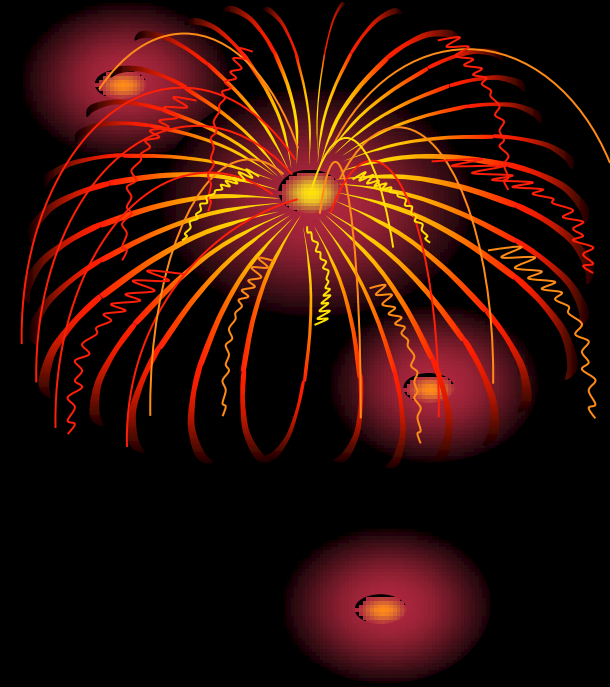
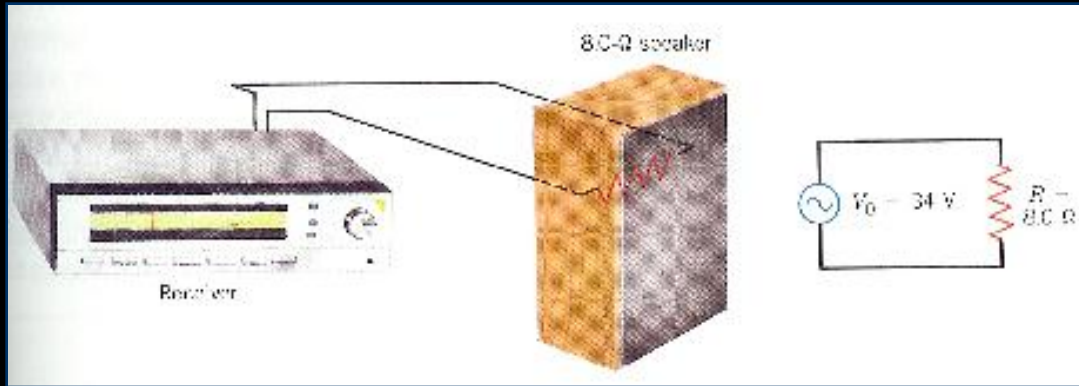
- 對交流電而言，歐姆定律變成

$$V_{rms} = I_{rms} R$$

$$\bar{P} = I_{rms} V_{rms} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$



# Ex.5 72W 音箱



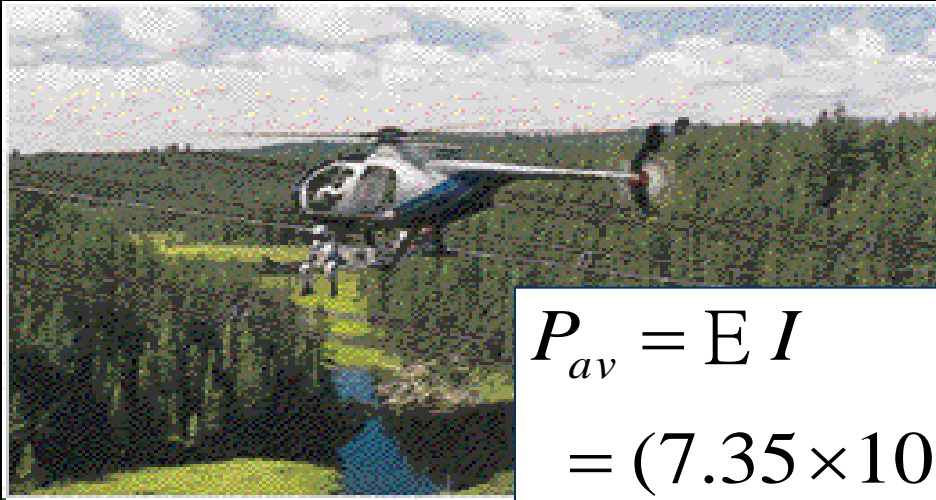
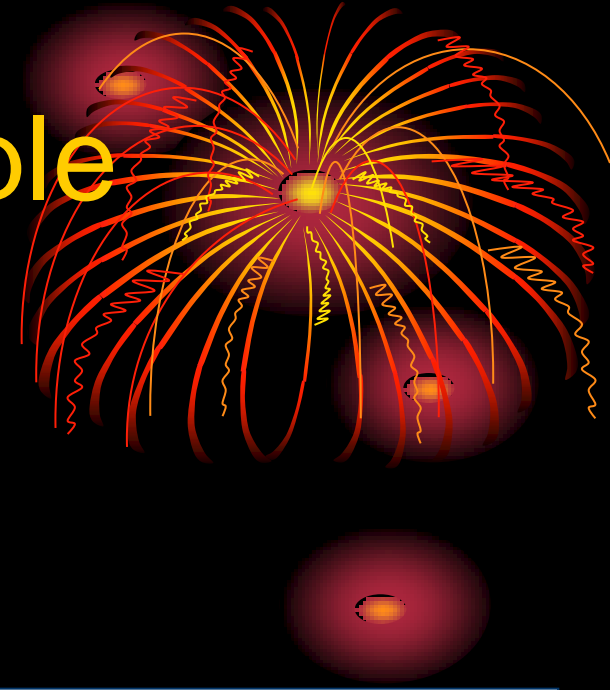
$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{34\text{V}}{\sqrt{2}} = 24\text{V}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{24\text{V}}{8.0\ \Omega} = 3.0\text{A}$$

$$\overline{P} = I_{rms} V_{rms} = 72\text{W}$$

# Ex.6 A practical example

## Quebec → Montreal



$$P_{av} = E I$$

$$= (7.35 \times 10^5 \text{ V})(500 \text{ A}) = 368 \text{ M W}$$

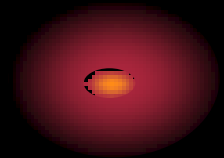
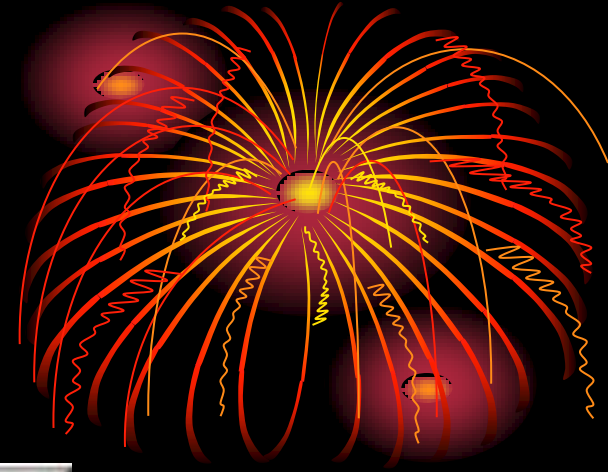
$$P_{av,1} = I^2 R = (1000 \text{ A})^2$$

$$\times (0.22 \Omega / m \times 1000 \text{ km}) = 220 \text{ M W}$$

$$P_{av,2} = I^2 R = (500 \text{ A})^2$$

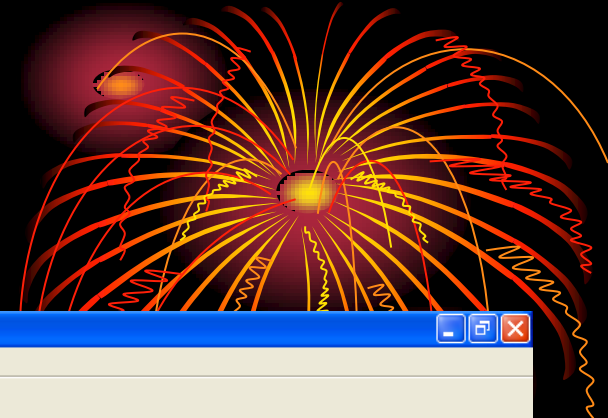
$$\times (0.22 \Omega / m \times 1000 \text{ km}) = 55.0 \text{ M W}$$

# 6.5 虛擬電路實驗

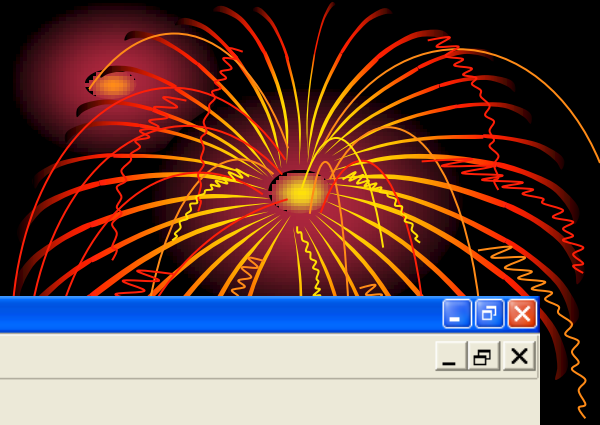




# 6.6 Orcad-PSpice



The screenshot displays the Capture CIS - Demo Edition software interface. The main window shows a schematic diagram of an inverting amplifier circuit. The circuit consists of an operational amplifier (OPAMP, U3) with its non-inverting input (+) connected to ground (0). The inverting input (-) is connected to a 1k resistor (R1) and a 9k resistor (R2). The output (OUT) is connected to a 1k resistor (R3) and ground (0). A sine wave source (V1) is connected to the inverting input. The source parameters are: VOFF = 0, VAMPL = 1.2V, and FREQ = 1k. The software interface includes a menu bar (File, Edit, View, Place, Macro, PSpice, Accessories, Options, Window, Help), a toolbar, and a file explorer on the left showing the project structure. The status bar at the bottom indicates "Run PSpice simulation for active profile.", "0 items selected", and "Scale=200% X=1.50 Y=1.80".



# 6.7 Simulation

