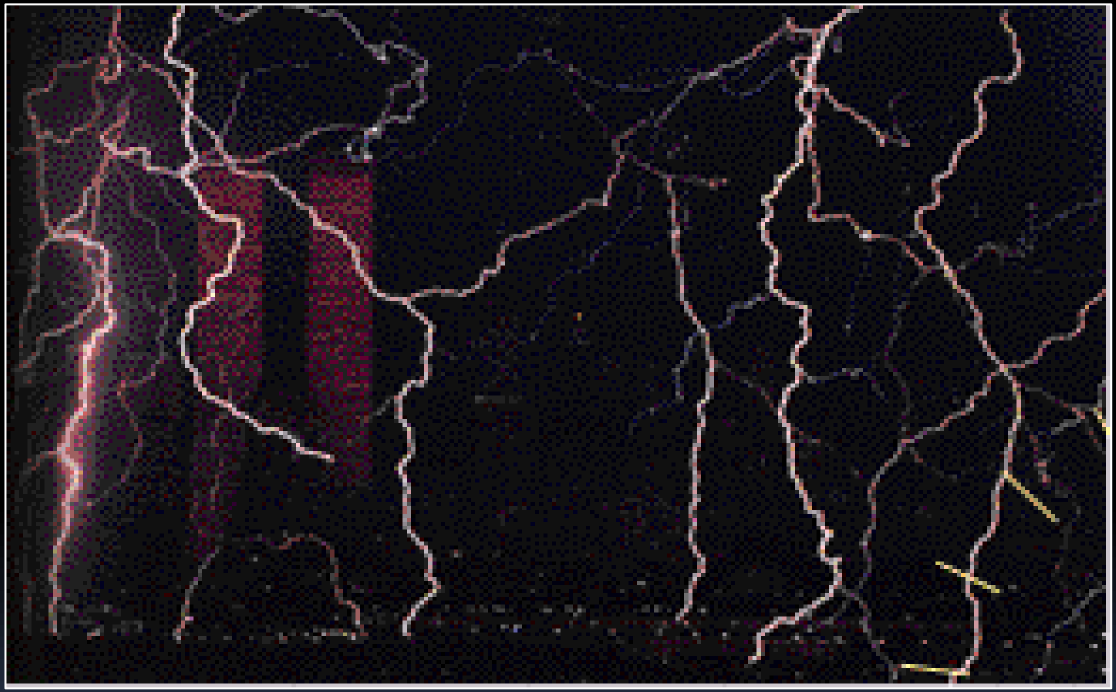
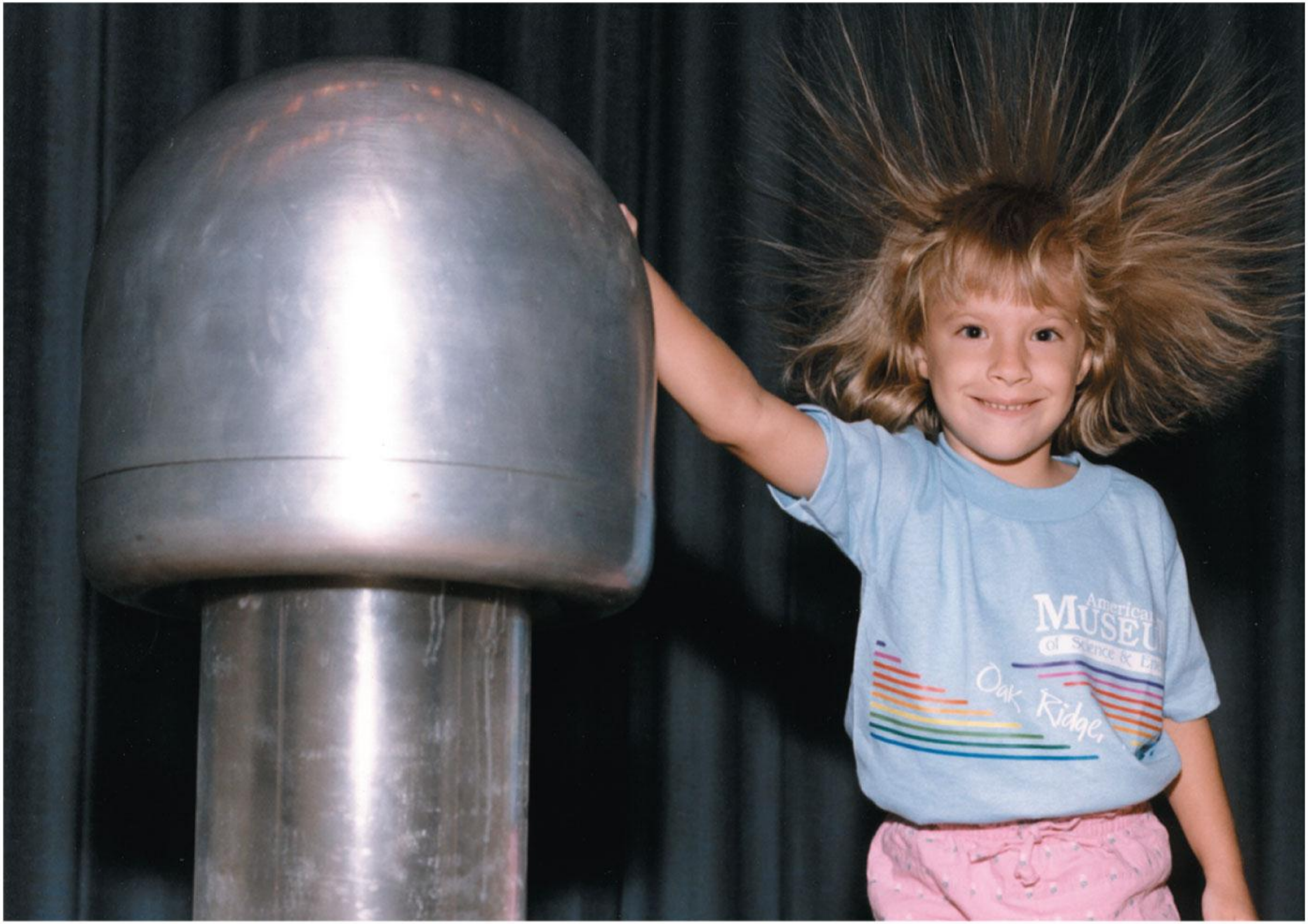
高二物理

1 高斯定律

2 Gauss' Law



How wide is a lightning strike ?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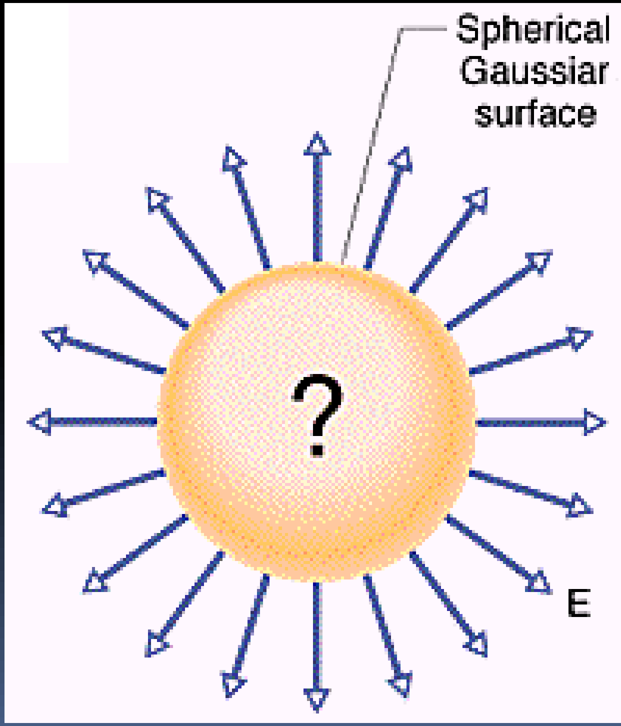
Oak Ridge,

3-1 A New Look at Coulomb's Law

- Using Gauss's law to take advantage of special symmetry situations
- Gaussian surfaces
- 高斯面上各點電場與面內總電荷相關

$$\varepsilon_0 \Phi = q_{enc}$$

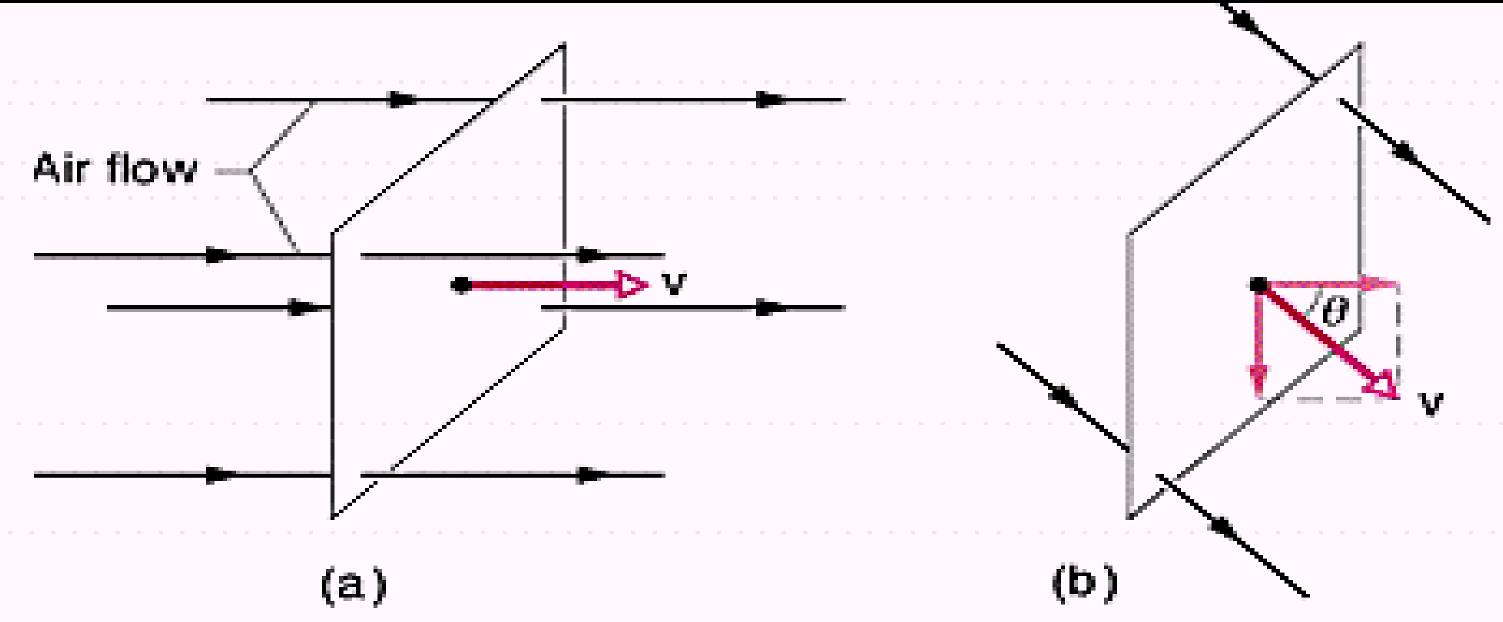


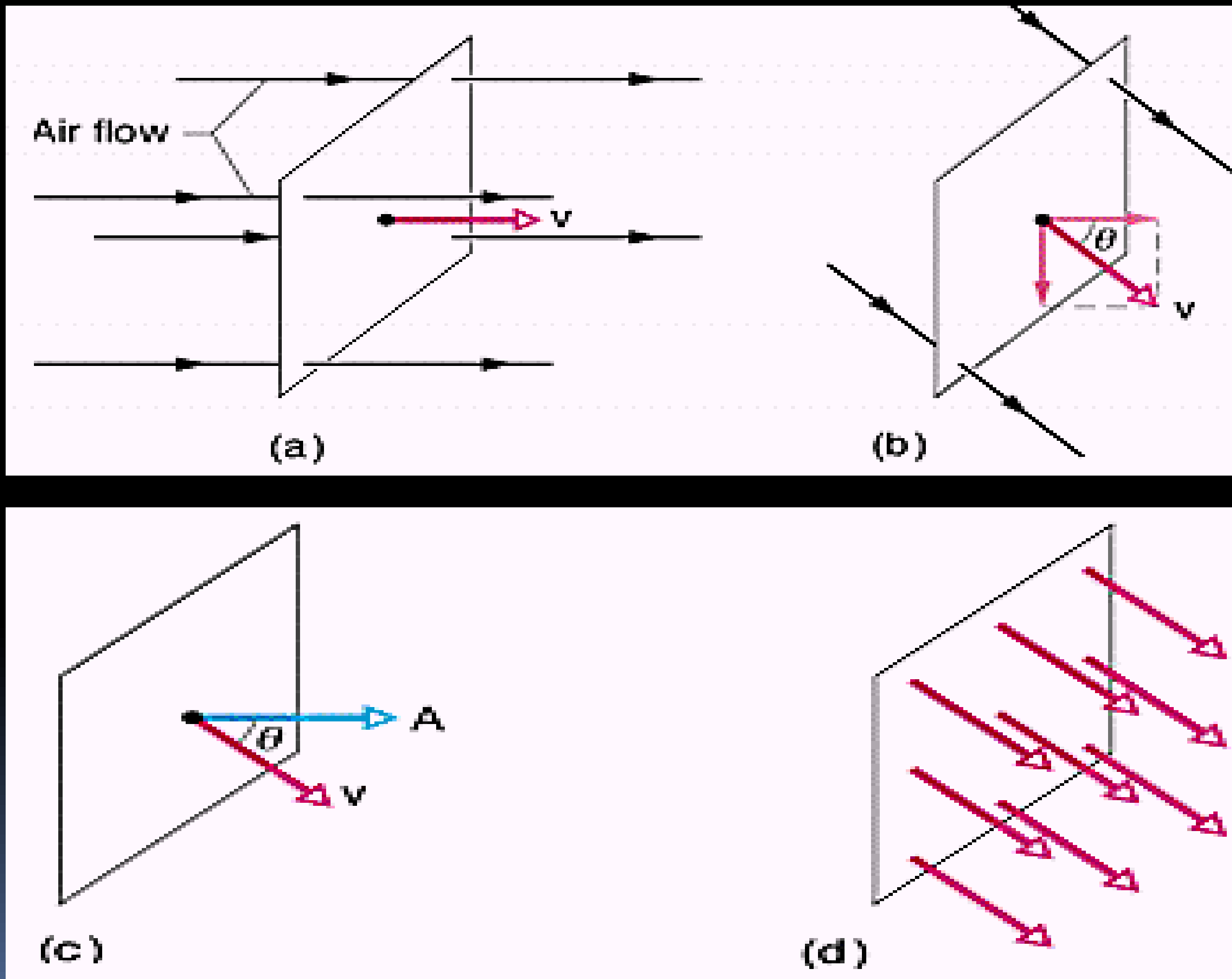
3-2 Flux (通量/流量)

- For a fluid

$m/s \times m^2 = m^3/s$

$$\Phi = (v\cos\theta)A = vA\cos\theta = \vec{v} \cdot \vec{A}$$




Air flow
v
(a)
θ
v
(b)
A
θ
v
(c)
(d)


Basking shark 姥鯊

3-3 Flux of an Electric Field

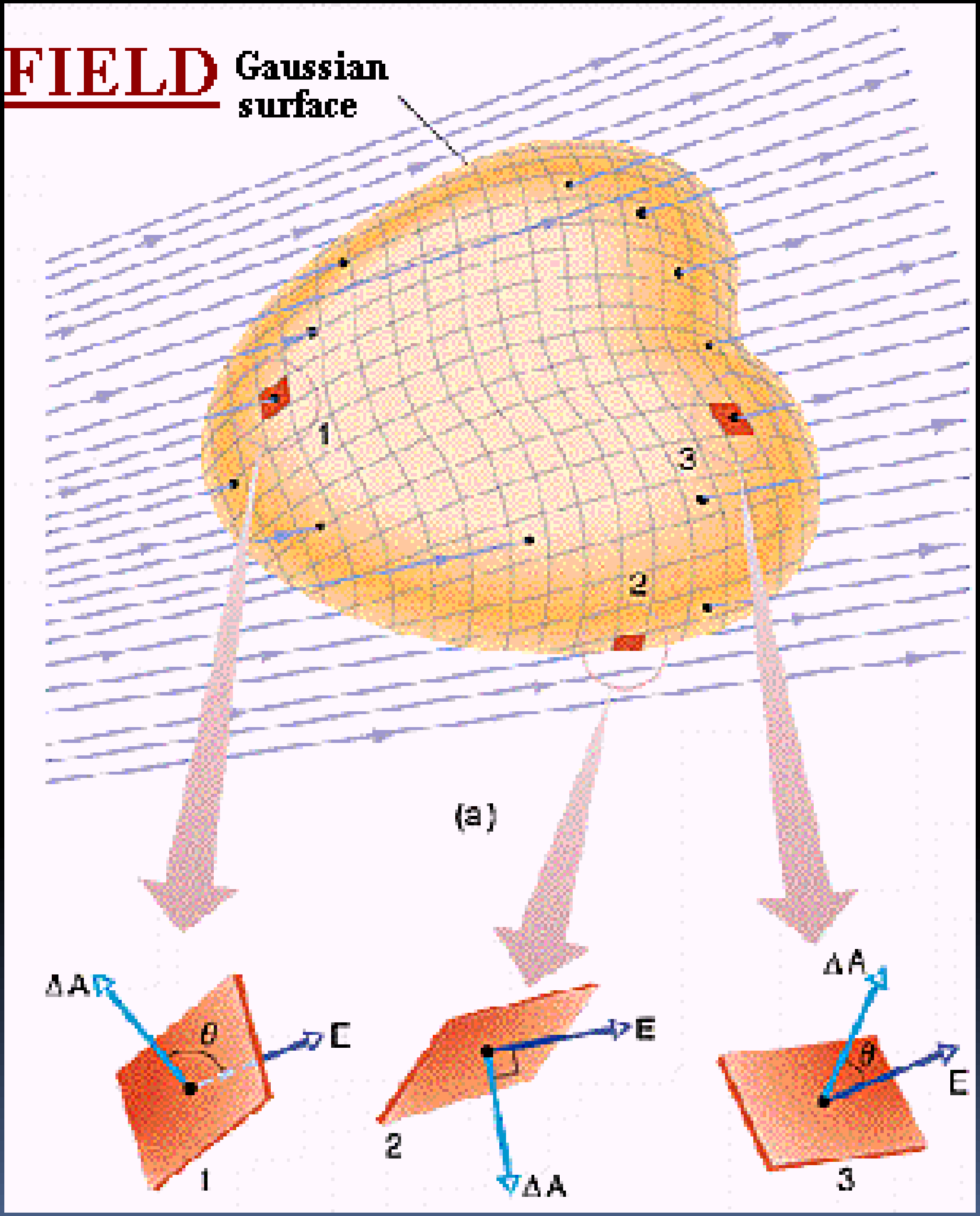
- For a flat surface

$$\Phi = (E\cos\theta)A = EA\cos\theta = \vec{E}\cdot\vec{A}$$

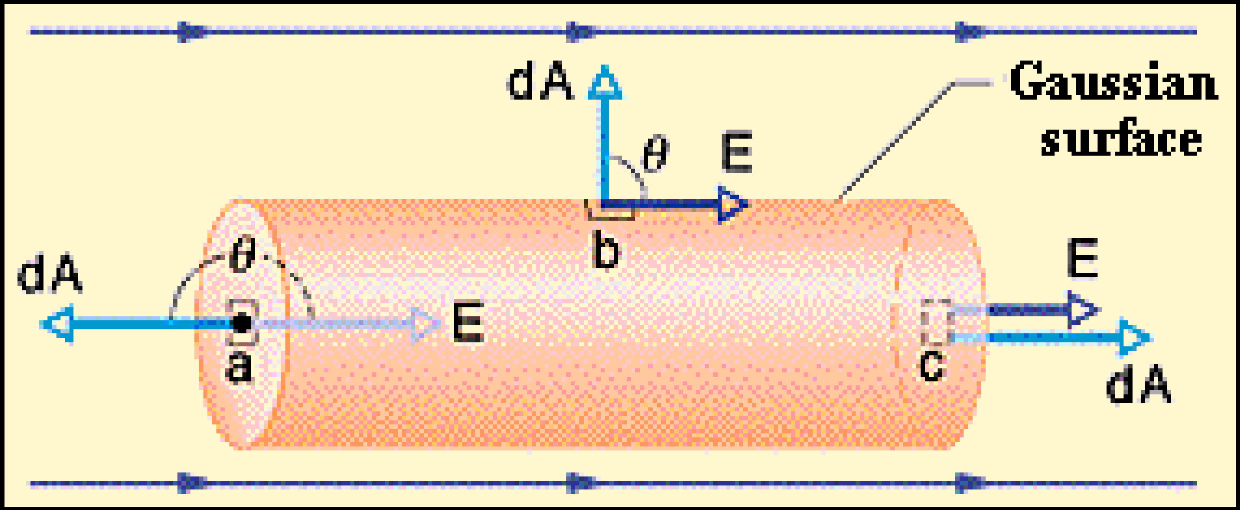
- For a arbitrary (asymmetric) surface

$$\Phi = \sum \vec{E}\cdot\Delta\vec{A} \rightarrow \oint \vec{E}\cdot d\vec{A}$$

- *An arbitrary Gaussian surface*



Ex.3-1 A cylindrical Gaussian surface



$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}$$

$$= -EA + 0 + EA = 0$$

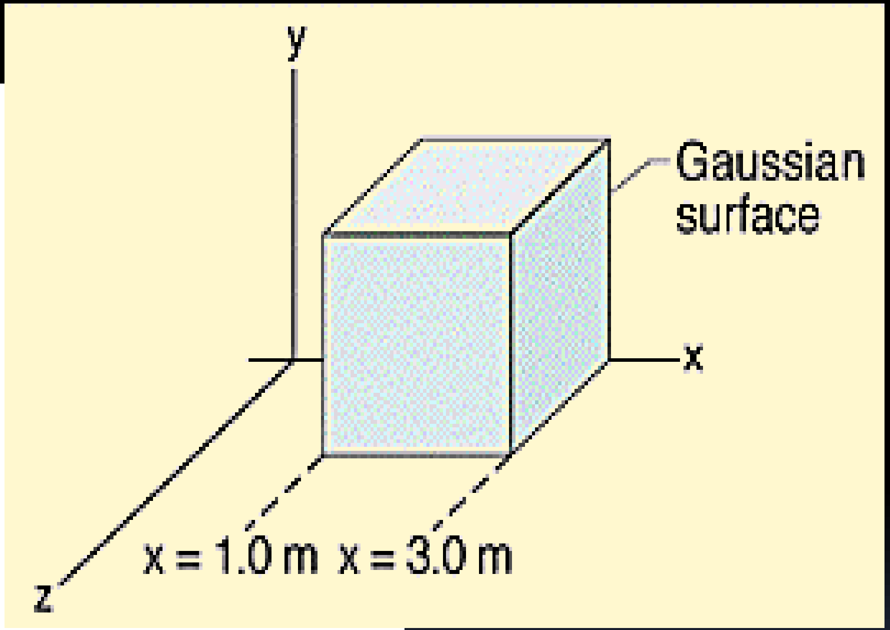
Ex.3-2 A nonuniform electric field and a Gaussian cube

$$\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$$

$$d\vec{A} = dA\hat{i}$$

$$\Phi_r = \int \vec{E} \cdot d\vec{A}$$

$$= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i})$$



Ex.3-2 right face

$$\Phi_r = \int \vec{E} \cdot d\vec{A}$$

$$= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i})$$

$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i})$$

$$= \int (3.0xdA + 0) = 3.0 \int xdA$$

$$= 3.0 \int 3.0dA = 9.0 \int dA = 36N \cdot m^2 / C$$

Ex.3-2 left and top faces

left face: $d\vec{A} = -dA\hat{i}$

$$\Phi_l = 3.0\int 1.0 dA = 3.0\int dA = -12 N\cdot m^2/C$$

$$\Phi_t = \int (3.0x\hat{i} + 4.0\hat{j})\cdot(dA\hat{j})$$

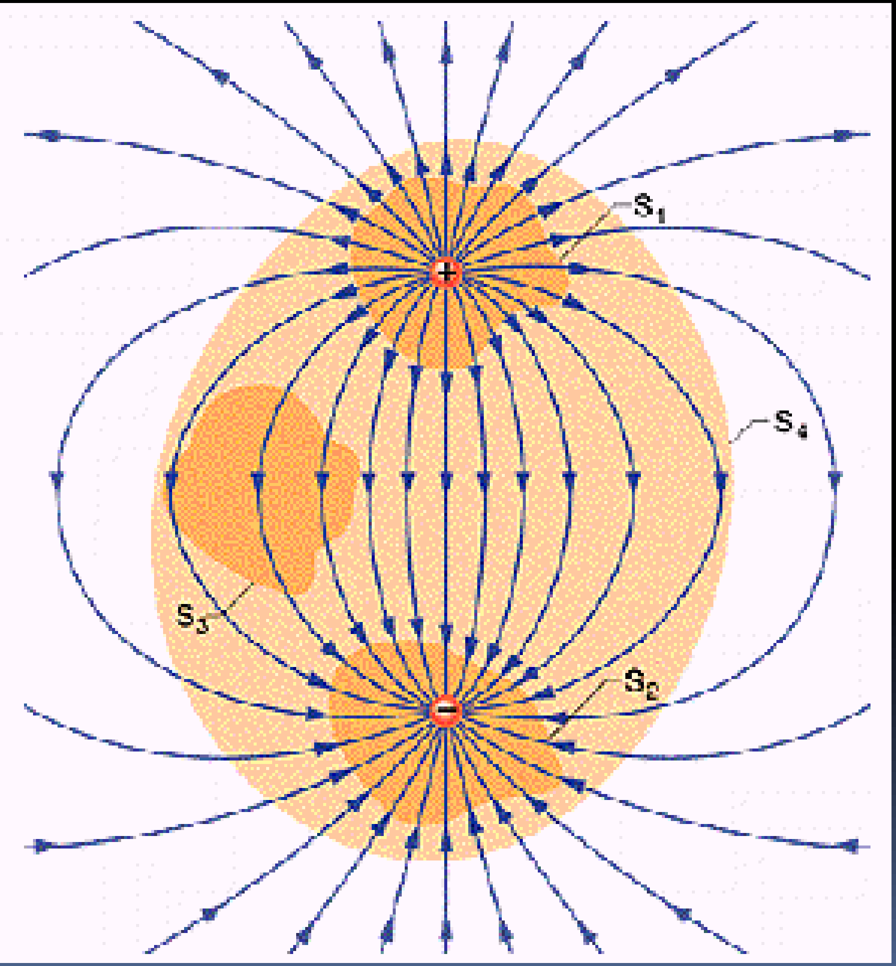
$$= \int [(3.0x)(dA)\hat{i}\cdot\hat{j} + (4.0)(dA)(\hat{j}\cdot\hat{j})]$$

$$= \int (0 + 4.0dA) = 4.0\int dA = 16 N\cdot m^2/C$$

3-4 Gauss' Law

- Flux ↔ enclosed charge

$$\varepsilon_0 \Phi = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$



Ex.3-3 bottom, front and back

$$\Phi_b = -16 N \cdot m^2 / C,\ \Phi_f = \Phi_b = 0$$

$$\Phi_t = 24 N \cdot m^2 / C$$

$$q_{enc} = \varepsilon_0 \Phi_t$$

$$= (8.85 \times 10^{-12} C^2 / N \cdot m^2)(24 N \cdot m^2 / C)$$

$$= 2.1 \times 10^{-10} C$$

3-5 Gauss' Law and Coulomb' Law

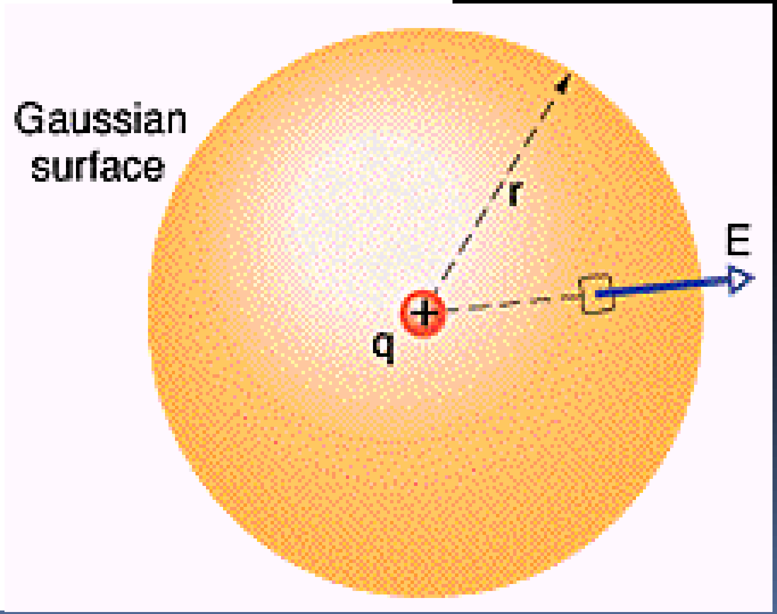
- From G.L. to C.L.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E dA = q$$

$$\varepsilon_0 E \oint dA = q$$

$$\varepsilon_0 E (4\pi r^2) = q$$

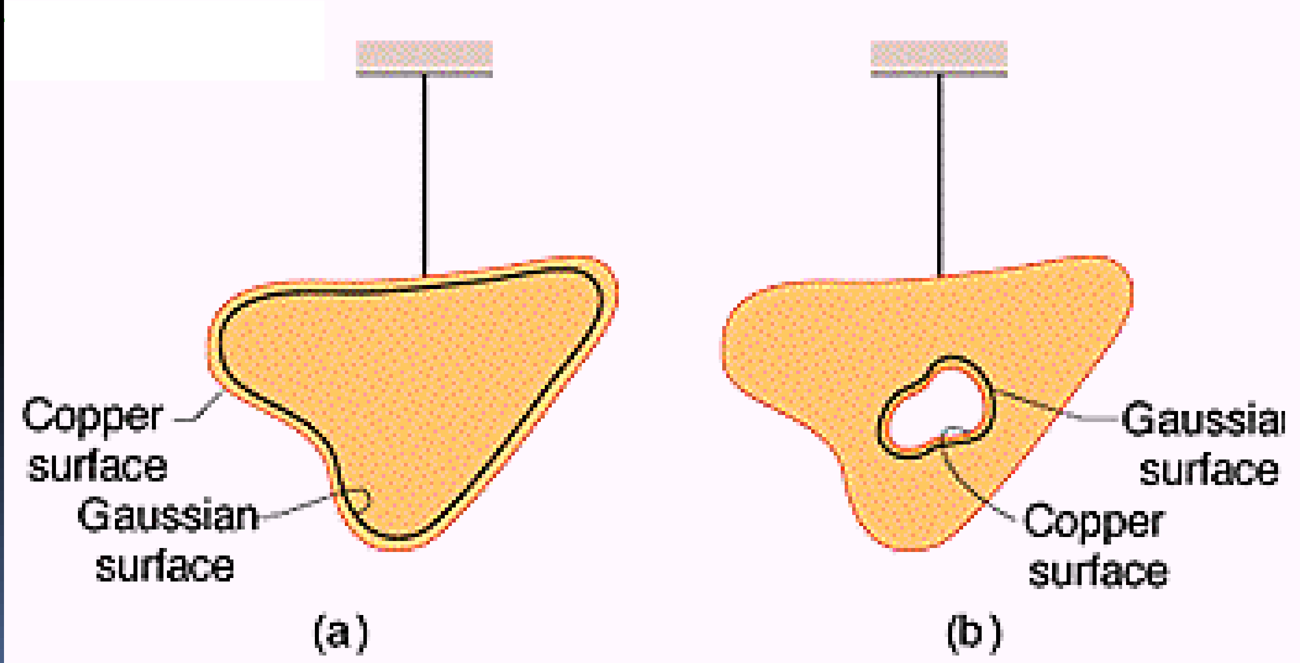
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$



3-6 A Charged Isolated Conductor

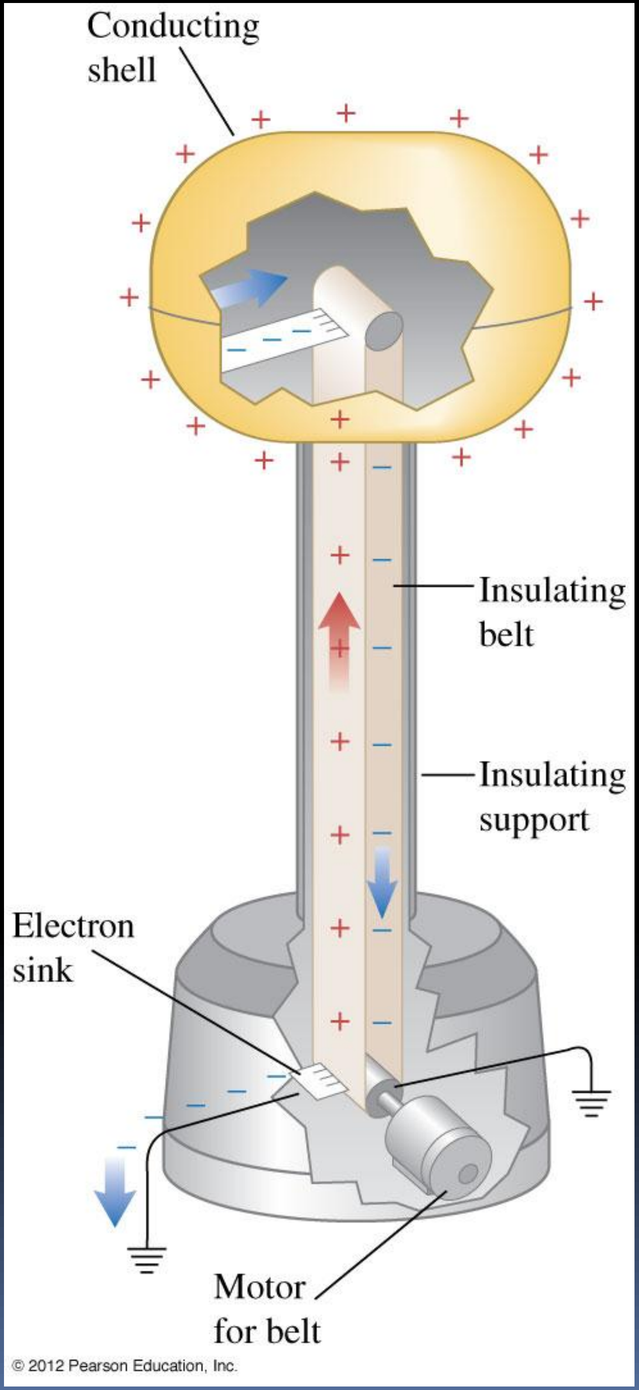
- 同號電荷相斥
- 導體內部無電場

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$



Human nerve cell

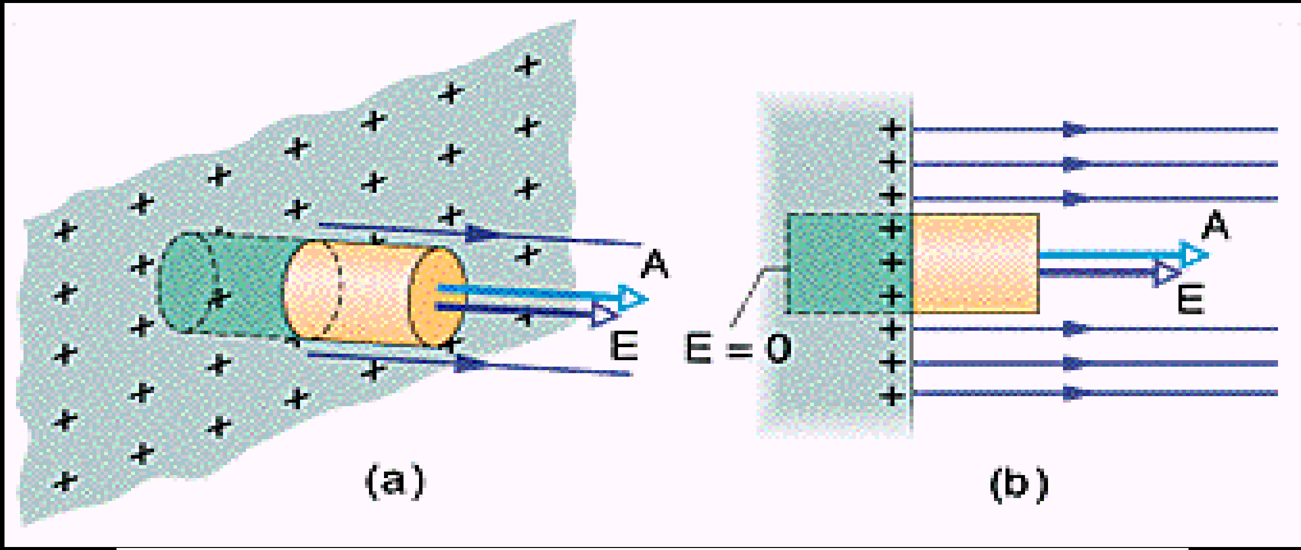
Van de Graaff electrostatic generator



ES shielding - Faraday Cage

(b)

The external electric field



$$q_{enc} = \sigma A, \Phi = EA$$

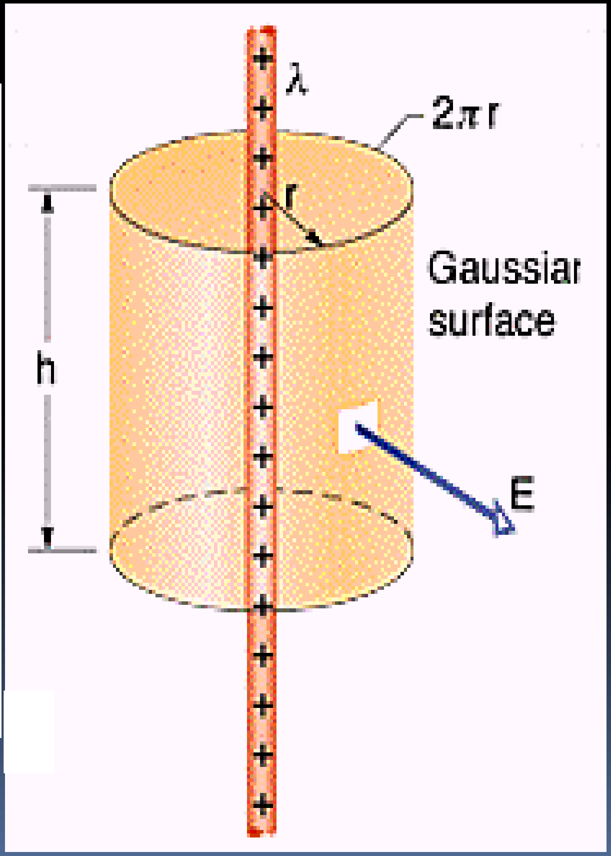
$$\varepsilon_0 EA = \sigma A \rightarrow E = \frac{\sigma}{\varepsilon_0}$$

3-7 Applying Gauss' Law: Cylindrical Symmetry

$$q_{enc} = \lambda h, A = 2\pi r h$$

$$\varepsilon_0 E 2\pi r h = \lambda h$$

$$\rightarrow E = \frac{\lambda}{2\pi \varepsilon_0 r}$$



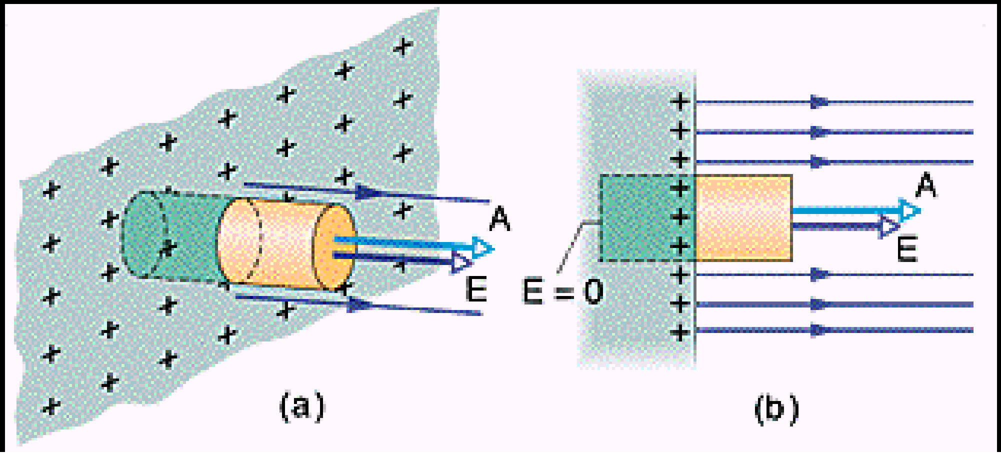
Ex.3-4 A lightning strike

$$r = \frac{\lambda}{2\pi\varepsilon_0 E}$$

$$= \frac{1\times10^{-3}}{2\pi\varepsilon_0(3\times10^{6})}$$

$$= 6m$$

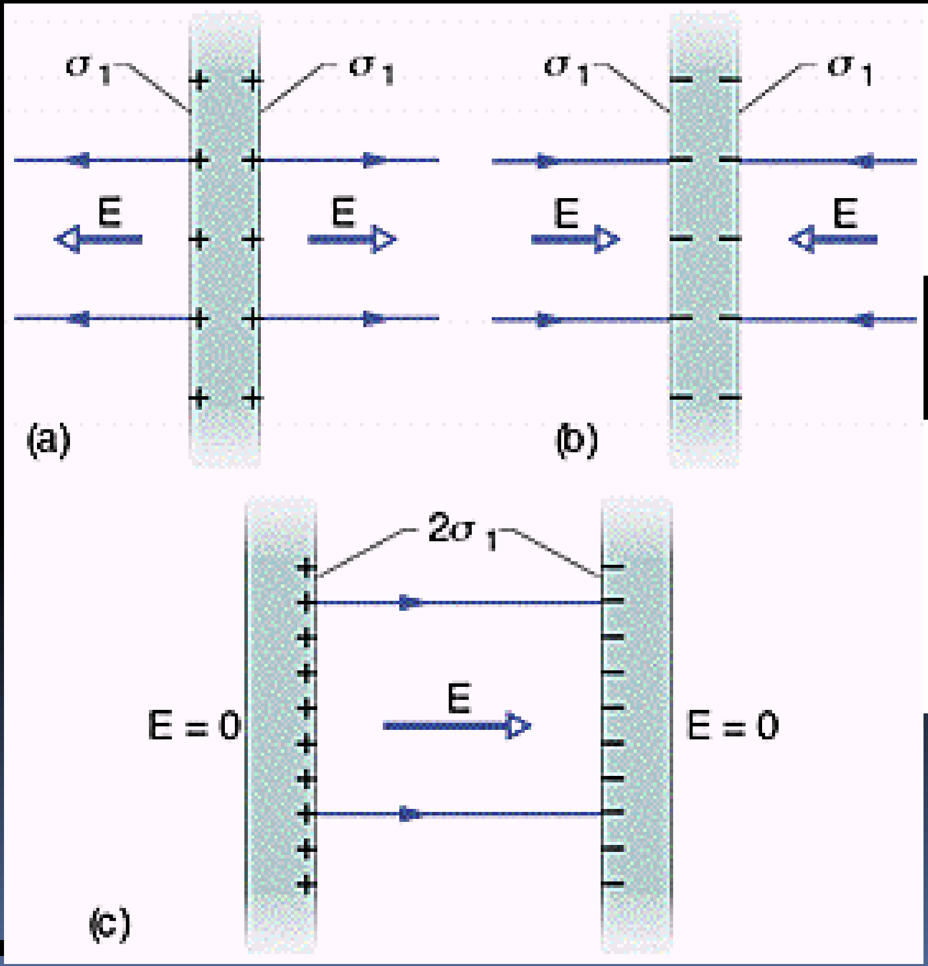
3-8 Applying Gauss' Law: Planar Symmetry



$$\vec{E}\cdot d\vec{A} = EdA$$

$$\varepsilon_0(EA+EA)=\sigma A \rightarrow E=\frac{\sigma}{2\varepsilon_0}$$

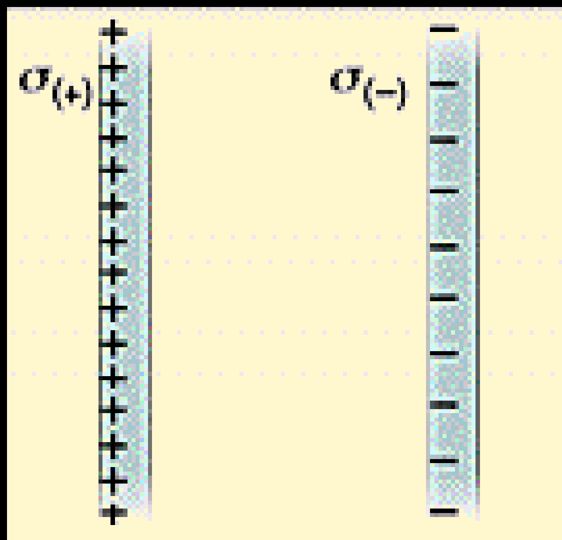
Two Conducting Plates

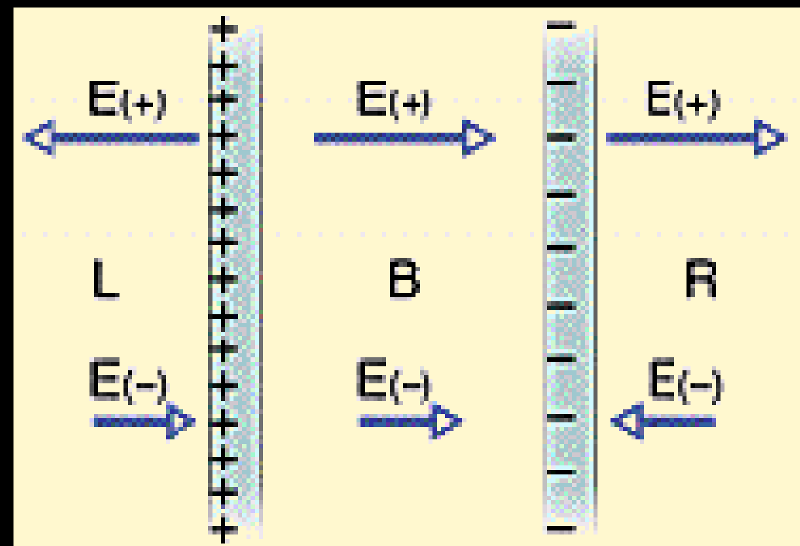


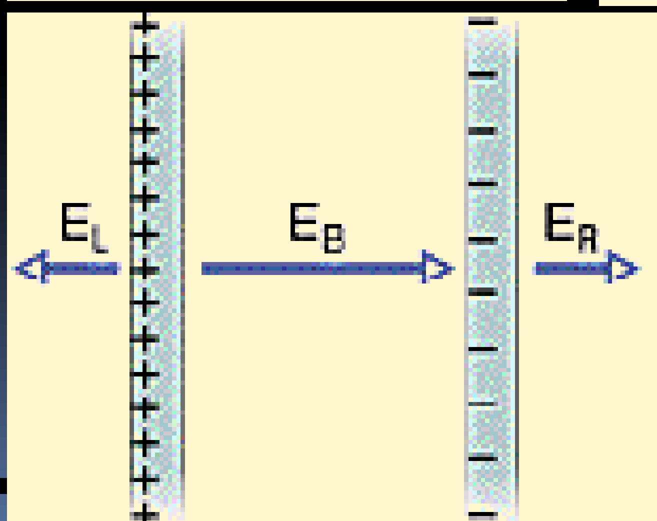
$$E = \frac{\sigma_1}{\varepsilon_0}$$

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

ex.3-5 Two || nonconducting sheets







$$E_{+} = \frac{\sigma_{+}}{2\varepsilon_0}$$

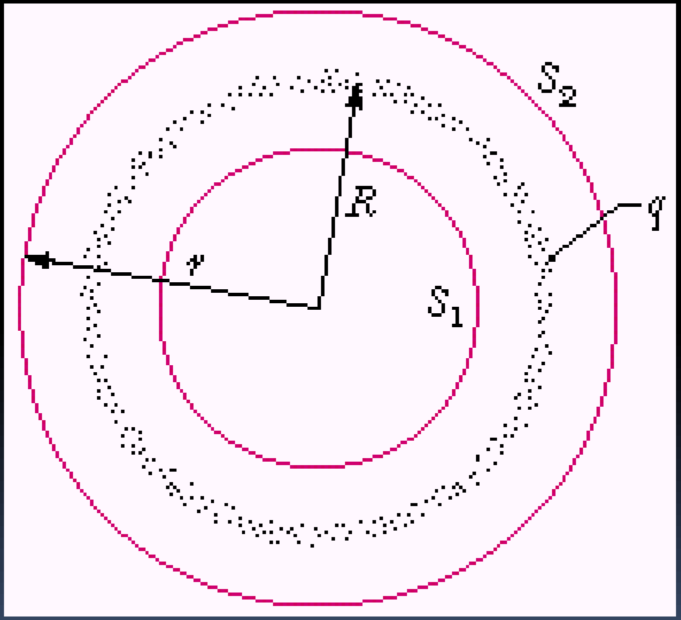
$$E_{-} = \frac{\sigma_{-}}{2\varepsilon_0}$$

3-9 Applying Gauss' Law: Spherical Symmetry

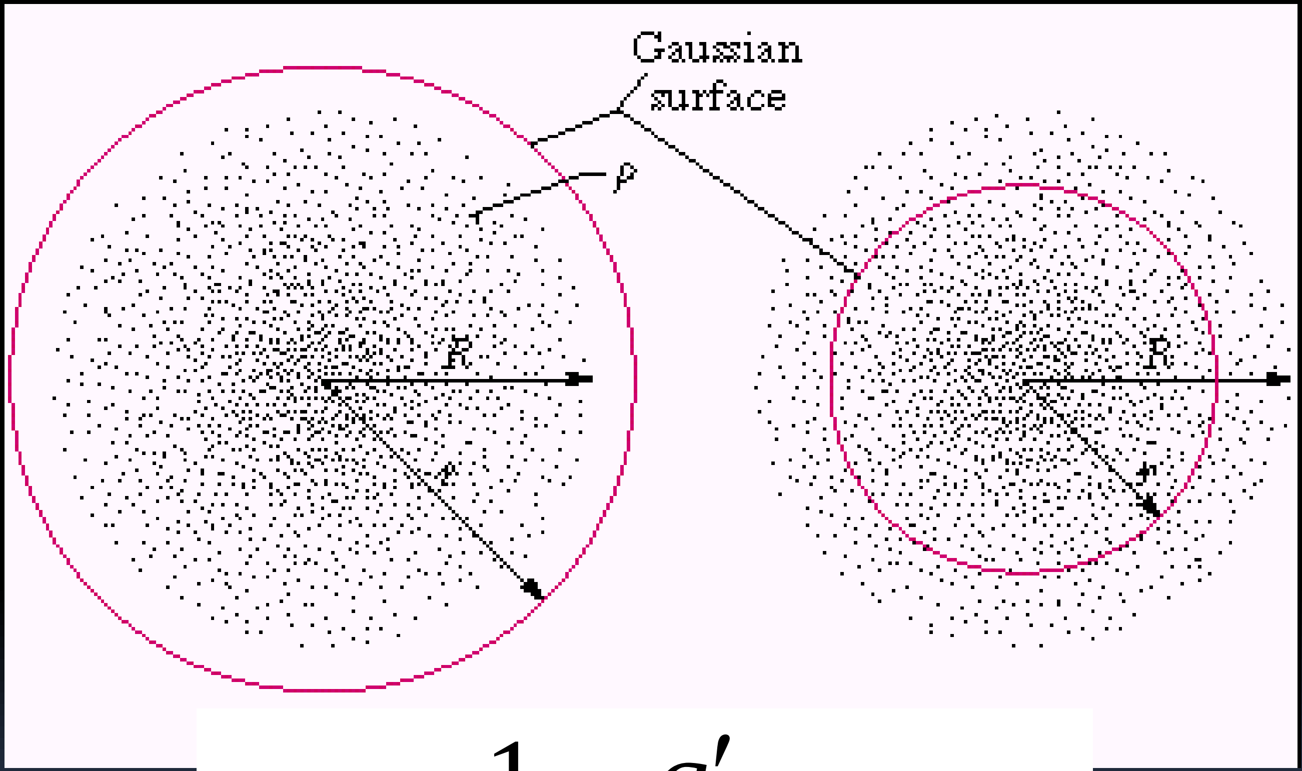
- The Shell Theorems

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,(r \geq R)$$

$$E = 0 \qquad (r < R)$$



A spherically symmetric charge distribution – ρ (r)



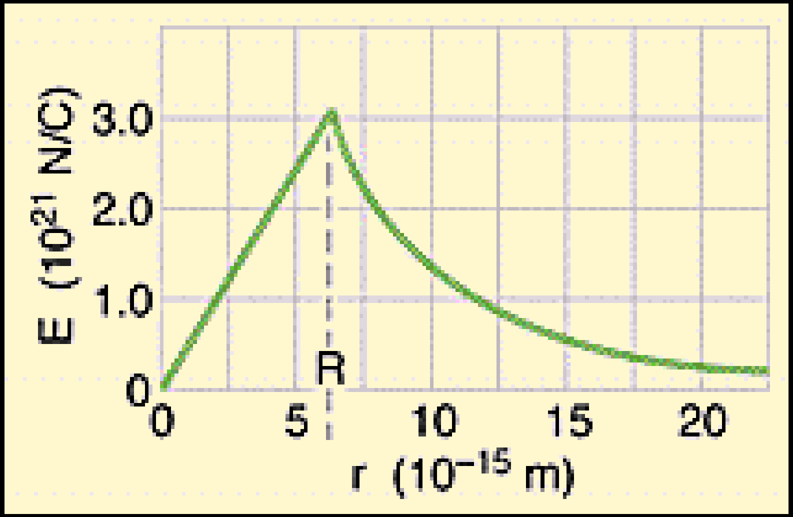
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2}(r \le R)$$

Uniform distribution

$$\frac{q'}{q} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} \rightarrow q' = q\frac{r^3}{R^3}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r$$

Ex.3-6 The electric field vs. r



$$q = Ze = (79)e = 1.264 \times 10^{-17} C$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 3.0 \times 10^{21} N/C$$

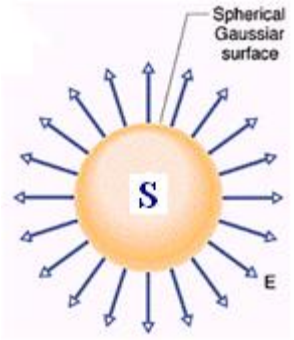
3-10 Gauss' Law in Differential Form

$$\varepsilon_0 \Phi = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

$$\rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Divergence Theorem

$$\oint_S \vec{E} \cdot d\vec{A} = \int_V \nabla \cdot \vec{E} dV$$



Total charge

$$\rightarrow \int \nabla \cdot \vec{E} dV = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho dV$$

Charge density

$$\rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$