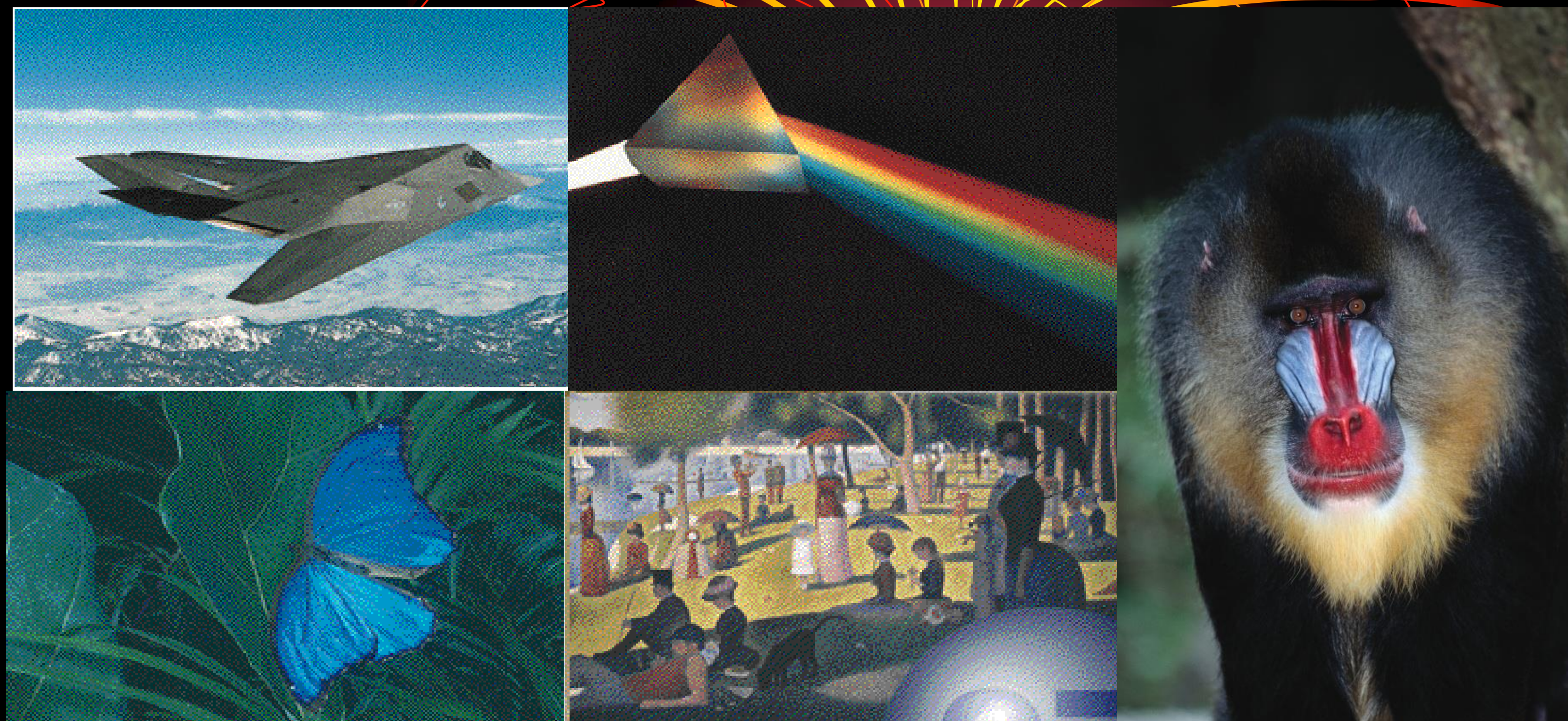
11 反射、折射、干涉、繞射



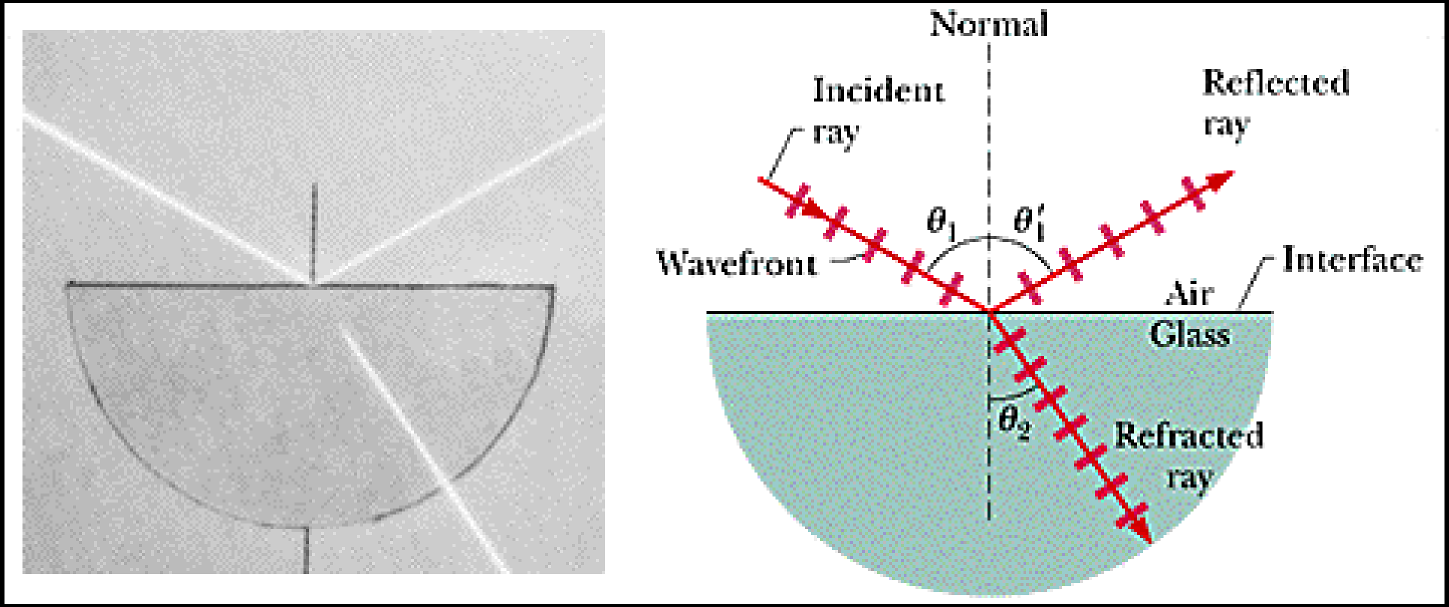
Sections

1. 反射 (reflection)與折射 (refraction)
2. 干涉 (interference)
3. 繞射 (diffraction)

11-1 Reflection and Refraction (反射與折射)

$$\theta_1' = \theta_1 \qquad \text{(reflection)}$$

$$n_2 \sin\theta_2 = n_1 \sin\theta_1 \qquad \text{(refraction)}$$



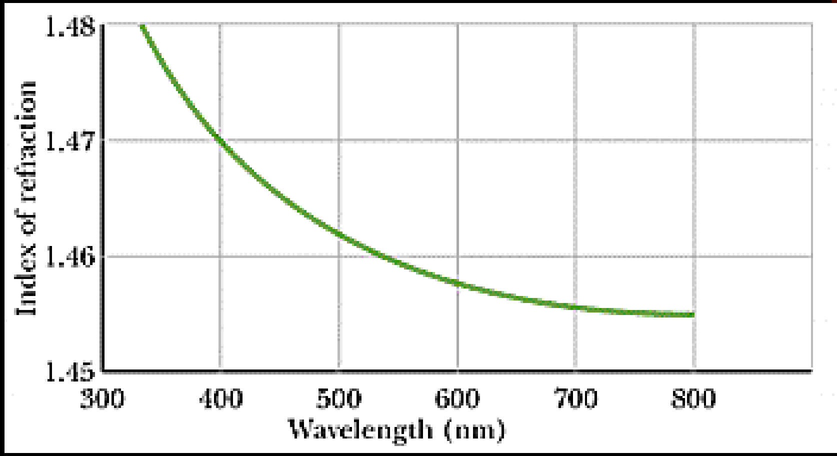
折射率

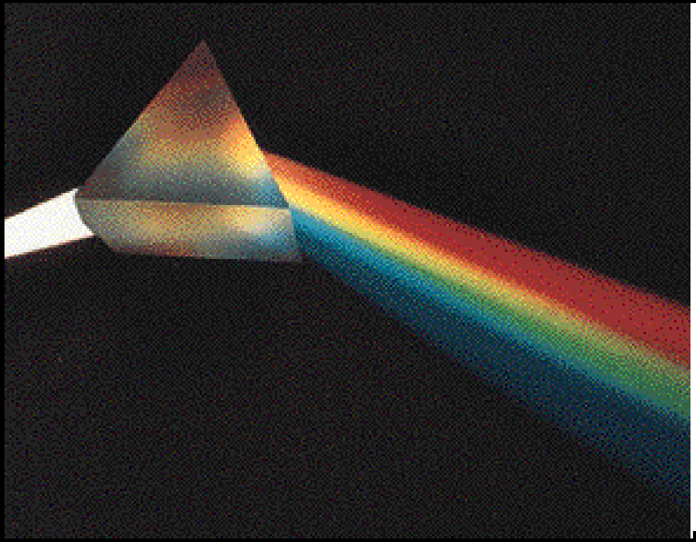
- The Index of Refraction
- The Stealth Aircraft F-117A

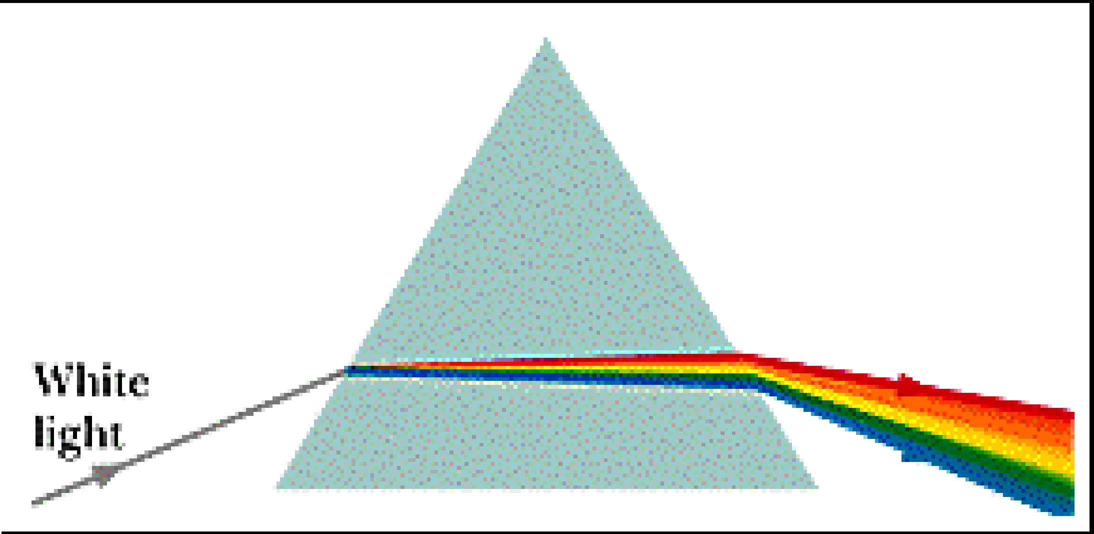
TABLE 34-1 SOME INDICES OF REFRACTION [a]

MEDIUM	INDEX	MEDIUM	INDEX
Vacuum	exactly 1	Typical crown glass	1.52
Air (STP) [b]	1.00029	Sodium chloride	1.54
Water (20° C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

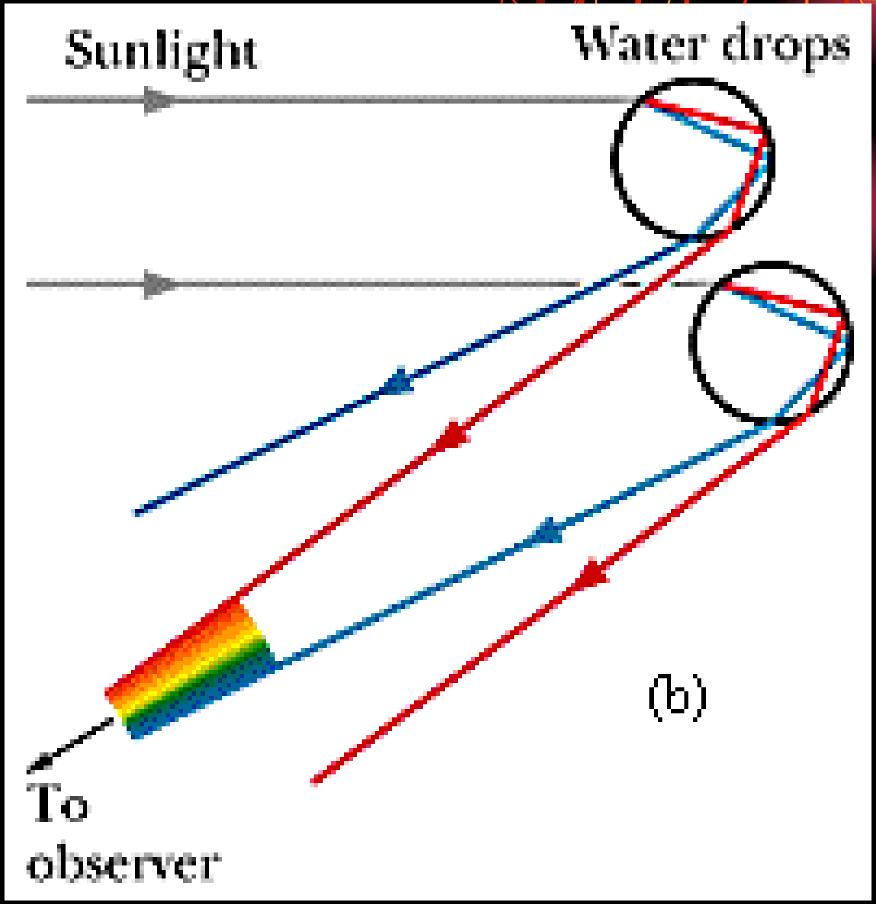
Chromatic Dispersion – prisms and gratings







Rainbows



11-2 Interference －（干涉）

What produces the blue-green of a Morpho's wing?

How do colorshifting inks shift colors?

Huygens' principle

All points on a wavefront serve as point sources of spherical secondary wavelets. After a time *t, the new position of the wavefront will be that of a surface tangent to* these secondary wavelets.

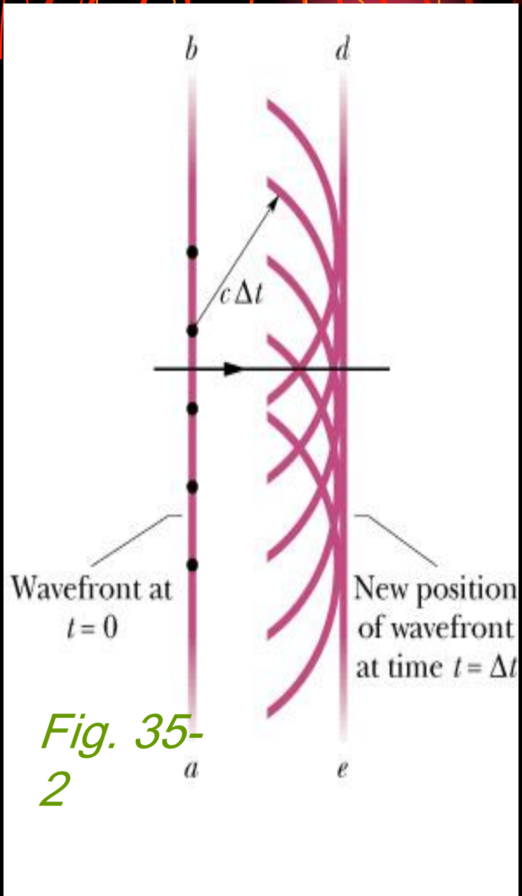


Fig. 35-2

Law of Refraction from Huygens' principle

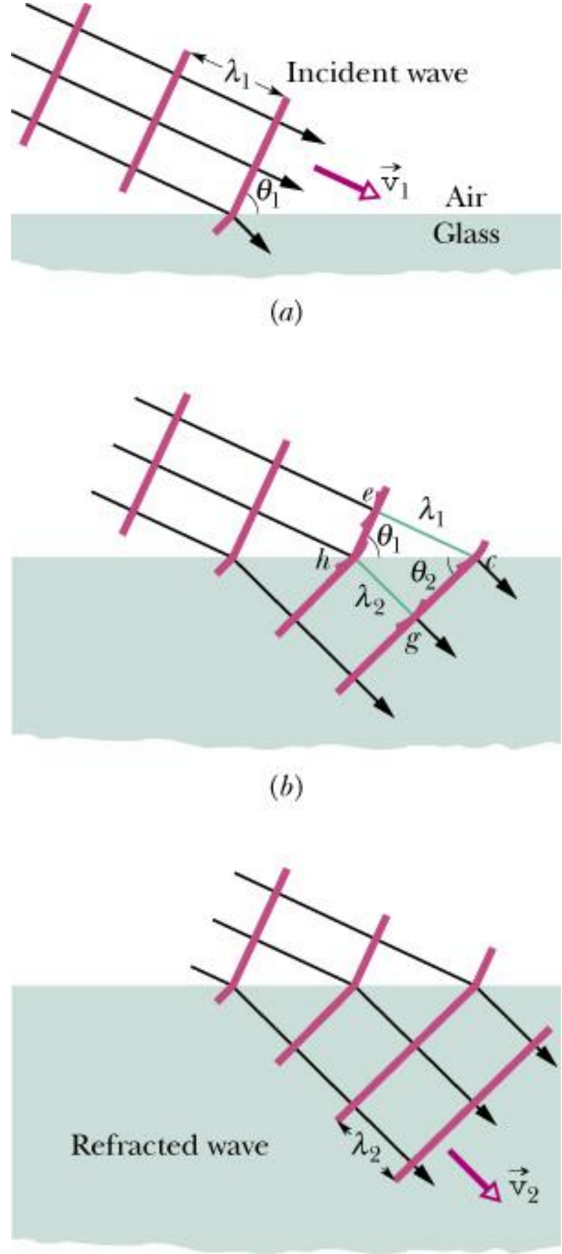
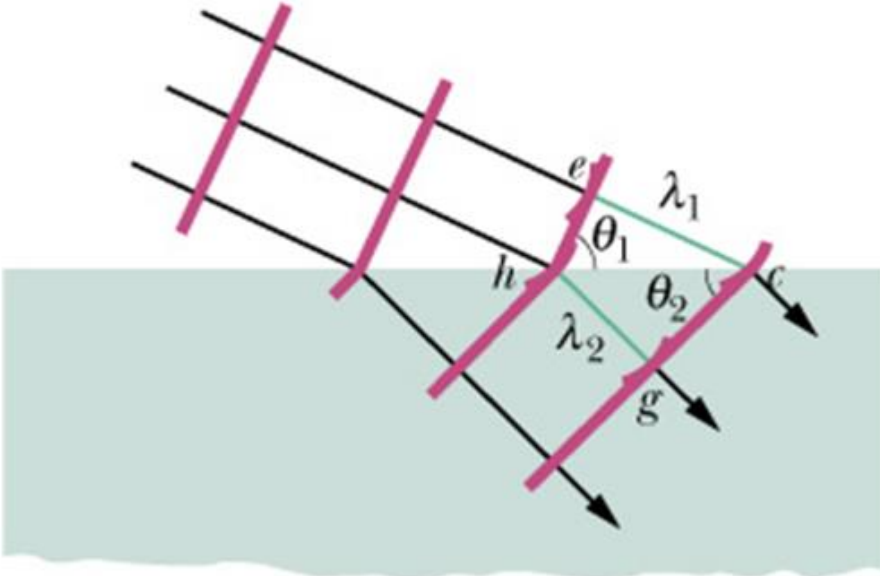

Incident wave
λ_1
$\vec{v}_1$
θ_1
Air
Glass
(a)
e
λ_1
θ_1
h
c
θ_2
λ_2
g
(b)
Refracted wave
λ_2
$\vec{v}_2$

Fig. 35-3

(c)



$$t_{ec} = t_{hg} = \frac{\lambda_1}{v_1} = \frac{\lambda_2}{v_2} \rightarrow \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\sin\theta_1 = \frac{\lambda_1}{hc} \quad \text{(for triangle hce)}$$

$$\sin\theta_2 = \frac{\lambda_2}{hc} \quad \text{(for triangle hcg)}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

Index of Refraction: $n = \dfrac{c}{v}$

$$n_1 = \frac{c}{v_1} \quad \text{and} \quad n_2 = \frac{c}{v_2}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

Law of Refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Phase Difference, Wavelength and Index of Refraction

Wavelength and Index of Refraction

$$\frac{\lambda_n}{\lambda} = \frac{v}{c} \rightarrow \lambda_n = \lambda \frac{v}{c} \rightarrow \lambda_n = \frac{\lambda}{n}$$

$$f_n = \frac{v}{\lambda_n} = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f$$

The frequency of light in a medium is the same as it is in vacuum

Phase Difference

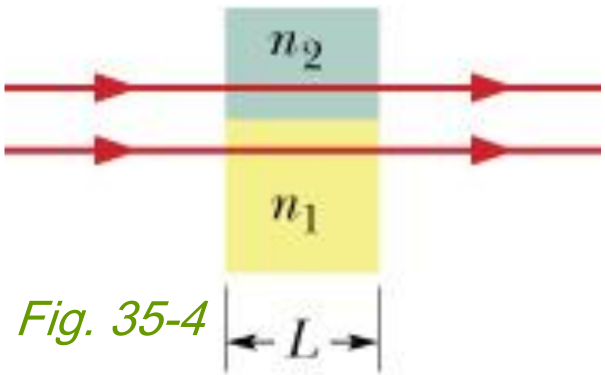


Fig. 35-4

Since wavelengths in n1 and n2 are different, the two beams may no longer be in phase

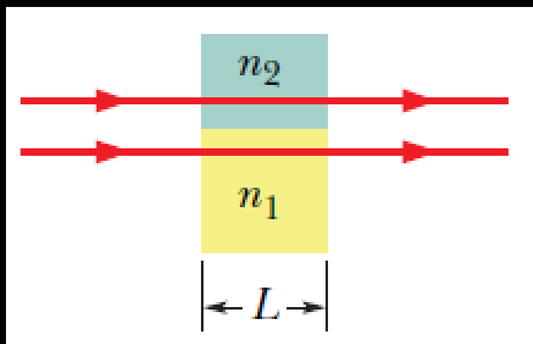
$$\text{Number of wavelengths in } n_1\text{: } N_1 = \frac{L}{\lambda_{n1}} = \frac{L}{\lambda/n_1} = \frac{Ln_1}{\lambda}$$

$$\text{Number of wavelengths in } n_2\text{: } N_2 = \frac{L}{\lambda_{n2}} = \frac{L}{\lambda/n_2} = \frac{Ln_2}{\lambda}$$

$$\text{Assuming } n_2 > n_1\text{: } N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_2}{\lambda} = \frac{L}{\lambda}\left(n_2 - n_1\right)$$

$$N_2 - N_1 = 1/2 \text{ wavelength} \rightarrow \text{destructive interference}$$

Ex.11-1 35-1



wavelength 550.0 nm

n_2=1.600 and

L = 2.600 m

$$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$$

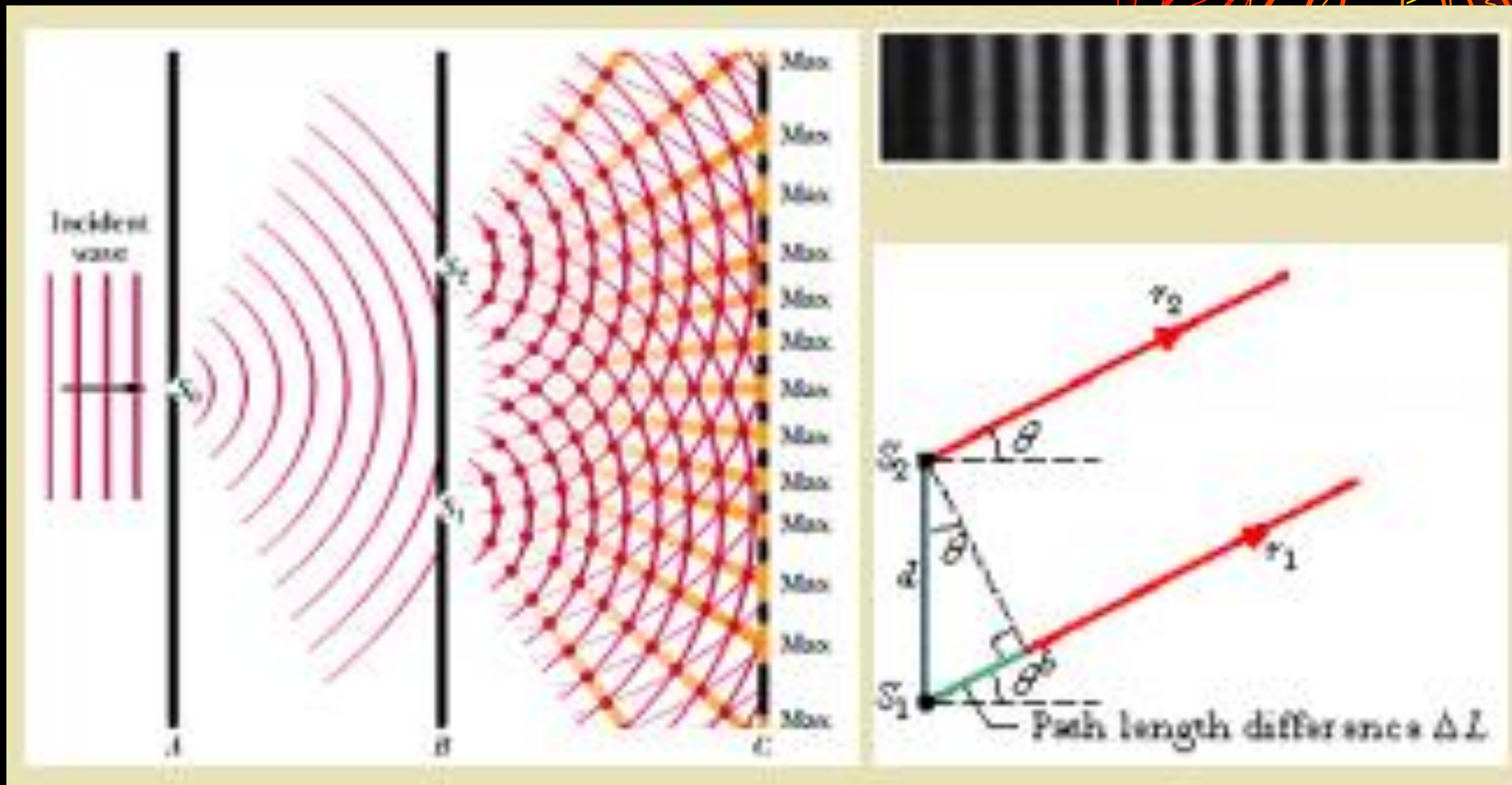
$$= \frac{2.600 \times 10^{-6}\ \mathrm{m}}{5.500 \times 10^{-7}\ \mathrm{m}}(1.600 - 1.000)$$

$$= 2.84. \qquad \text{(Ans}$$

phase difference = 17.8 rad ≈ 1020°

effective phase difference = 0.84 wavelength

Young's Experiment



$d \sin\theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \qquad \text{(maxima—bright fringes).}$

$d \sin\theta = (m + \frac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \qquad \text{(minima—dark fringes).}$

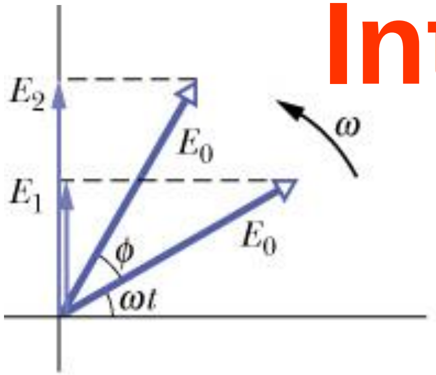
Coherence

Two sources to produce an interference that is stable over time, if their light has a *phase relationship* that does not change with time: $E(t)=E_0\cos(\omega t+\phi)$

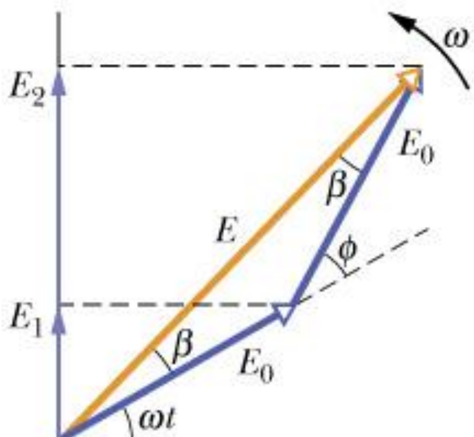
Coherent sources: Phase ϕ must be well defined and constant. When waves from coherent sources meet, stable interference can occur — laser light (produced by cooperative behavior of atoms)

Incoherent sources: ϕ jitters randomly in time, no stable interference occurs — sunlight

Intensity and phase



(a)



$$\beta + \beta = \phi$$

Fig. 35-13 (b)

$$E(t) = E_0 \sin \omega t + E_0 \sin(\omega t + \phi) = ?$$

$$E = 2(E_0 \cos \beta) = 2E_0 \cos \tfrac{1}{2}\phi$$

$$E^2 = 4E_0^2 \cos^2 \tfrac{1}{2}\phi$$

$$\frac{I}{I_0} = \frac{E^2}{E_0^2} = 4\cos^2 \tfrac{1}{2}\phi \rightarrow I = 4I_0 \cos^2 \tfrac{1}{2}\phi$$

Eq. 35-22

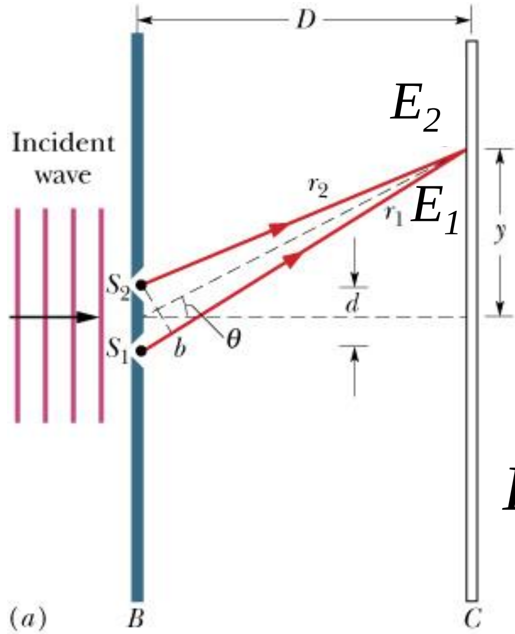
$$\frac{\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix}}{2\pi} = \frac{\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}}{\lambda}$$

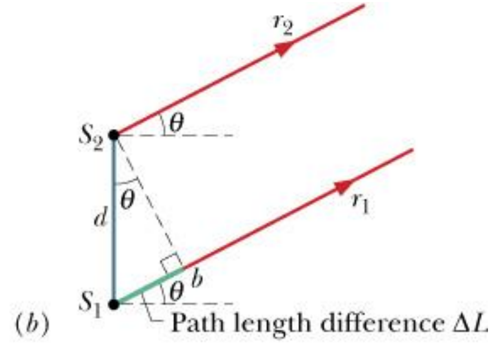
$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \frac{2\pi}{\lambda}\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}$$

$$\phi = \frac{2\pi}{\lambda}(d \sin\theta)$$

Eq. 35-23

Intensity in Double-Slit Interference





$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \phi)$$

$$\boxed{I = 4I_0 \cos^2 \tfrac{1}{2}\phi} \qquad \boxed{\phi = \frac{2\pi d}{\lambda}\sin\theta}$$

maxima when: $\tfrac{1}{2}\phi = m\pi$ for $m = 0,1,2,\ldots \rightarrow \phi = 2m\pi = \dfrac{2\pi d}{\lambda}\sin\theta$

$\rightarrow d\sin\theta = m\lambda$ for $m = 0,1,2,\ldots$ (maxima)

minima when: $\tfrac{1}{2}\phi = \left(m + \tfrac{1}{2}\right)\pi \rightarrow d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$ for $m = 0,1,2,\ldots$ (minima)

Intensity in Double-Slit Interference

$$I_{avg} = 2I_0$$

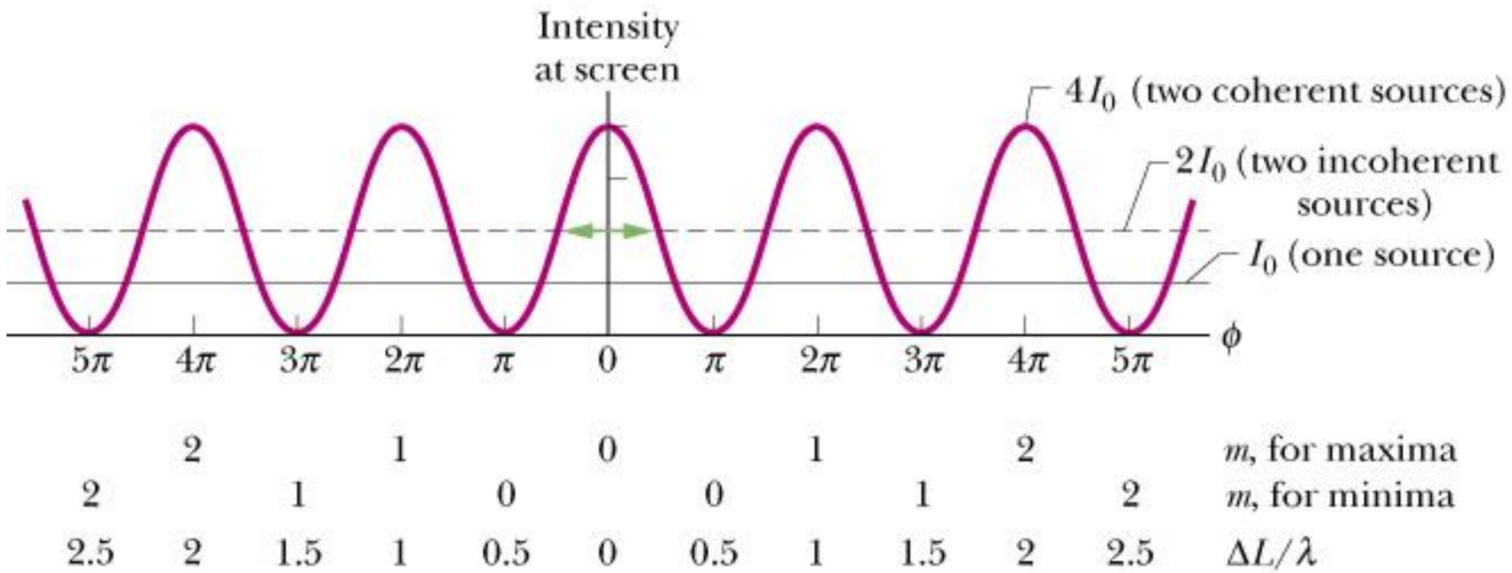
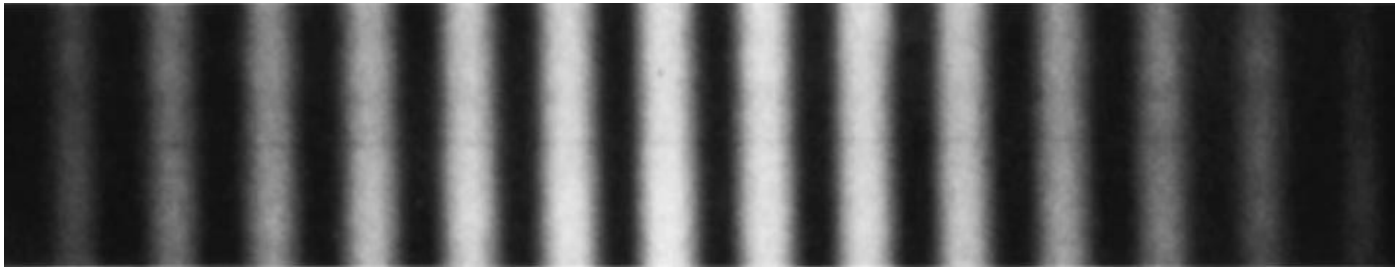
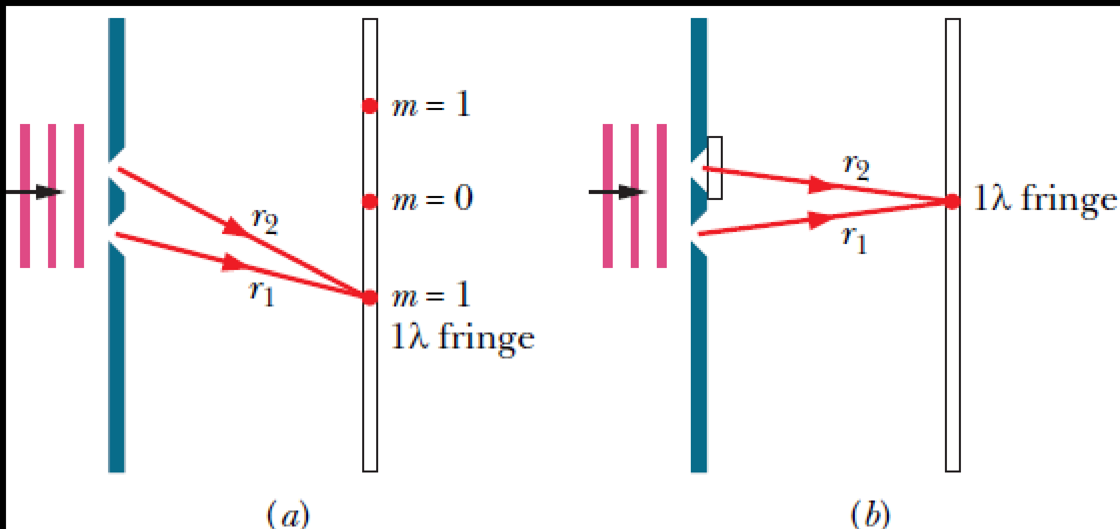


Fig. 35-12



Ex.11-2 35-2



$$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$$

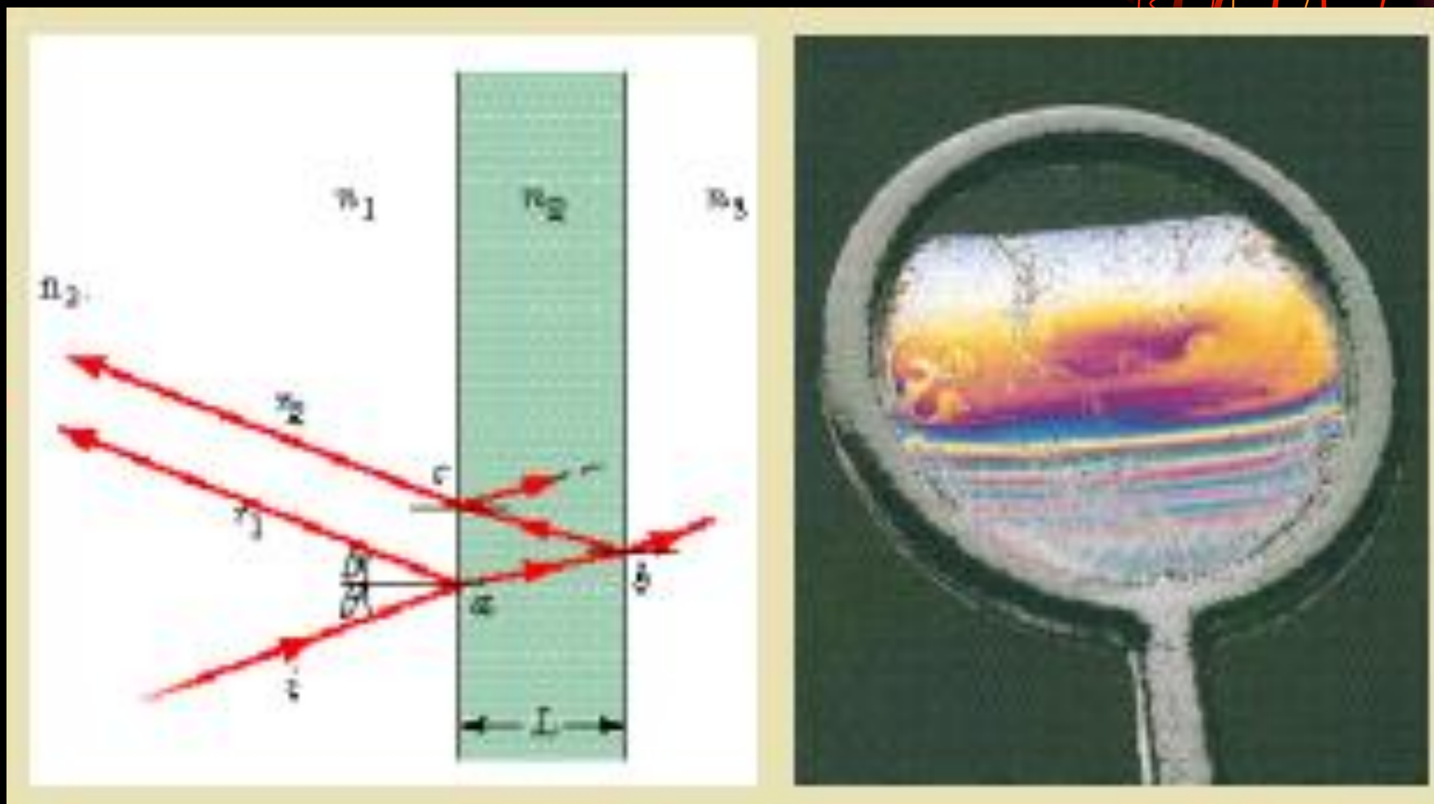
$$L = \frac{\lambda(N_2 - N_1)}{n_2 - n_1} = \frac{(600 \times 10^{-9}\ \mathrm{m})(1)}{1.50 - 1.00}$$
$$= 1.2 \times 10^{-6}\ \mathrm{m}. \qquad \text{(An}$$

wavelength 600 nm

n_2=1.5 and

m = 1 $\longrightarrow$ m = 0

Interference form Thin Films



Reflection Phase Shifts

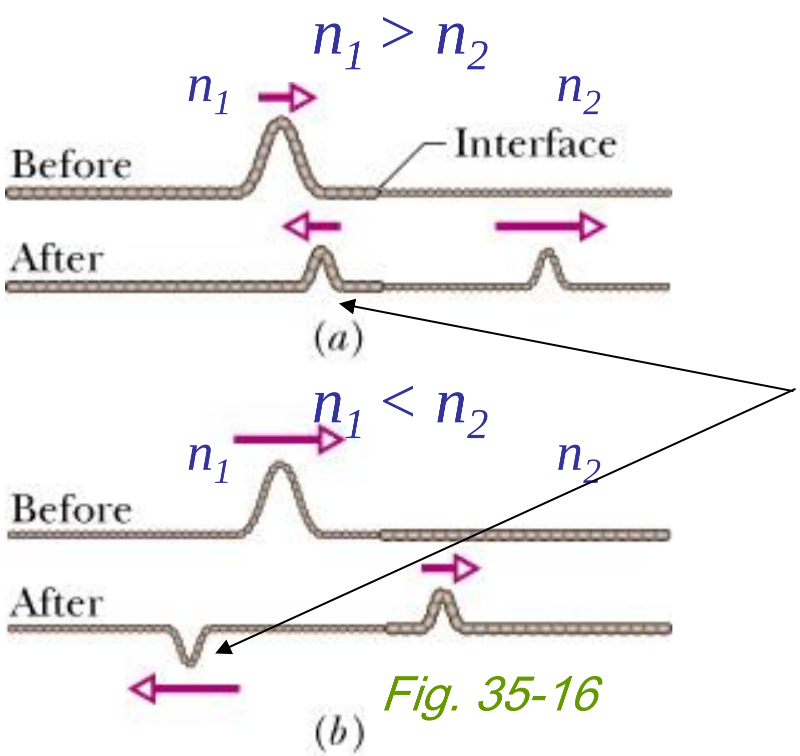


Fig. 35-16

Reflection	Reflection Phase Shift
Off lower index	0
Off higher index	0.5 wavelength

Phase Difference in Thin-Film Interference

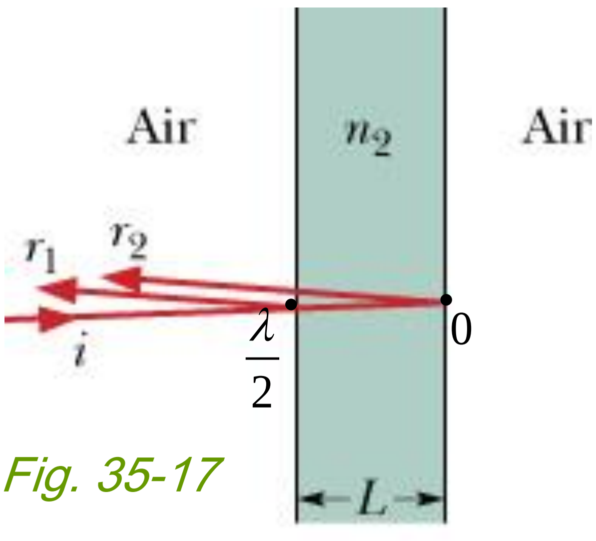


Fig. 35-17

Three effects can contribute to the phase difference between r_1 and r_2.

1. Differences in reflection conditions
2. Difference in path length traveled.
3. Differences in the media in which the waves travel. One must use the wavelength in each medium (λ / n), to calculate the phase.

Equations for Thin-Film Interference

½ wavelength phase difference to difference in reflection of r_1 and r_2

$$2L = \frac{\text{odd number}}{2} \times \text{ wavelength} = \frac{\text{odd number}}{2} \times \lambda_{n2} \quad \text{(in-phase waves)}$$

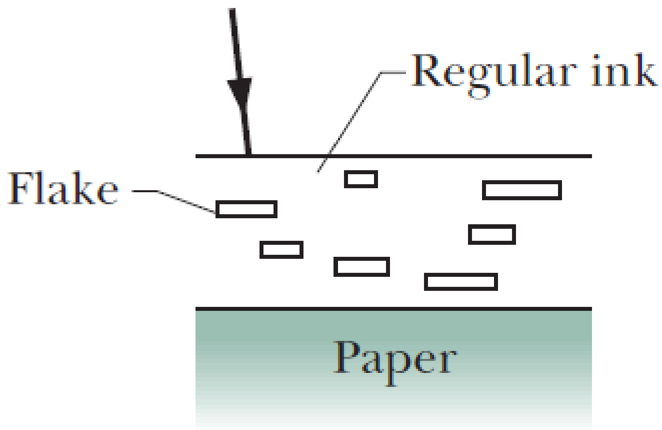
$$2L = \text{ integer } \times \text{ wavelength} = \text{integer} \times \lambda_{n2} \quad \text{(out-of-phase waves)}$$

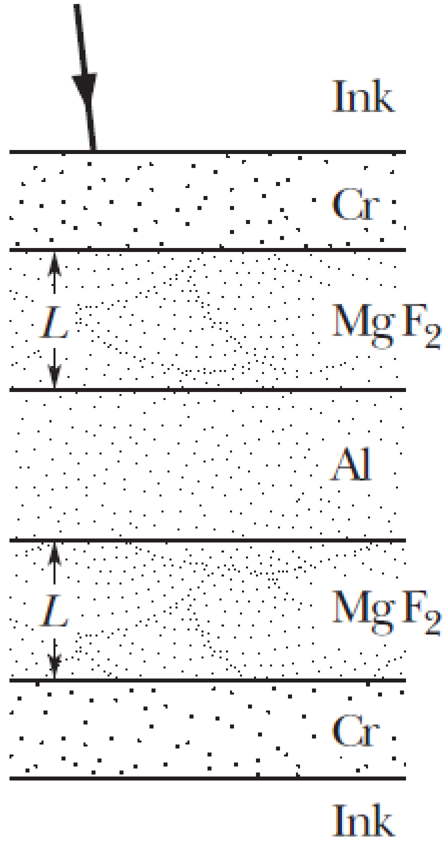
$$\lambda_{n2} = \frac{\lambda}{n_2}$$

$$\boxed{2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{for } m = 0, 1, 2, \ldots \text{ (maxima-- bright film in air)}}$$

$$\boxed{2L = m\frac{\lambda}{n_2} \quad \text{for } m = 0, 1, 2, \ldots \text{ (minima-- dark film in air)}}$$

Color Shifting by Paper Currencies,paints and Morpho Butterflies





weak mirror

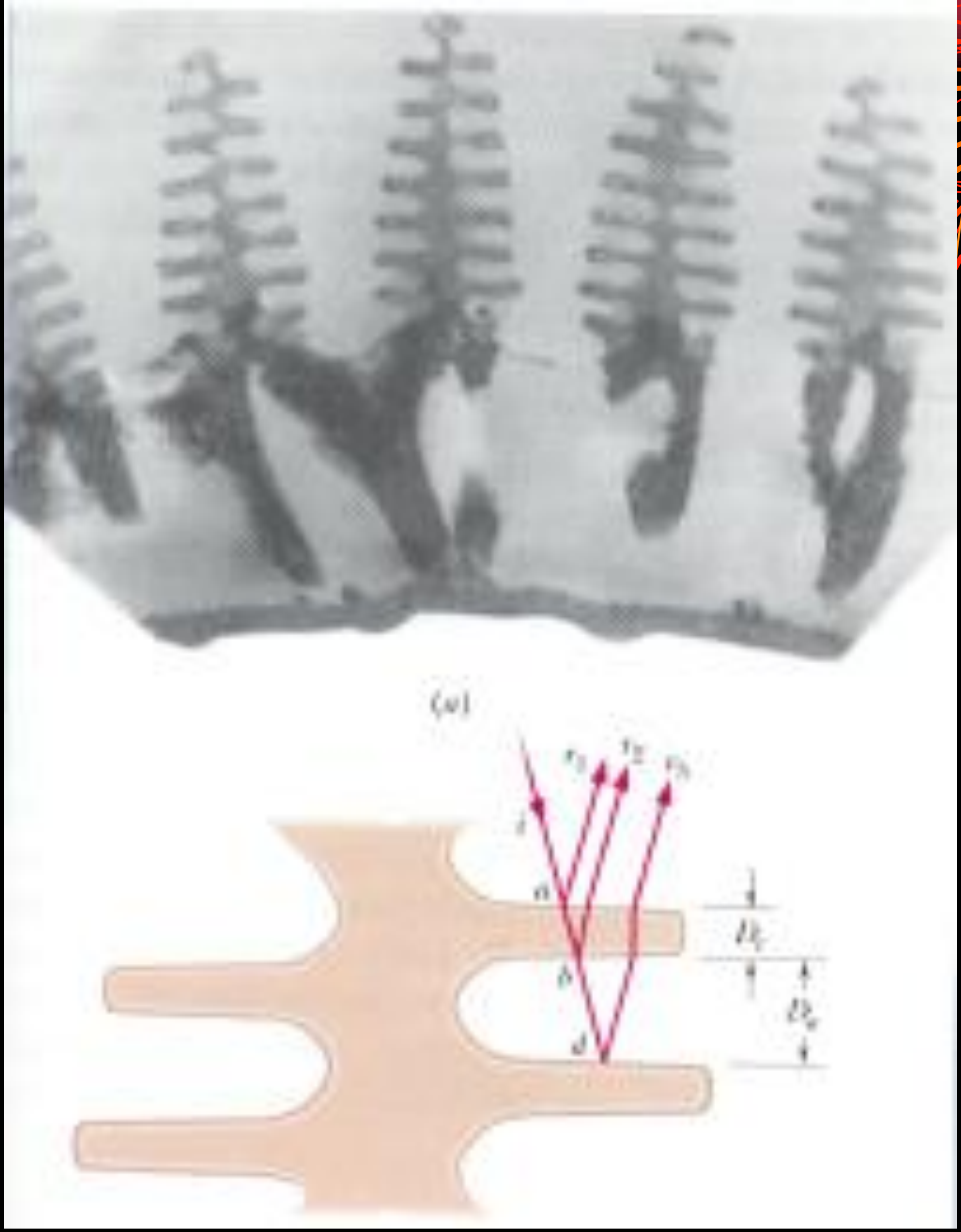
soap film

better mirror

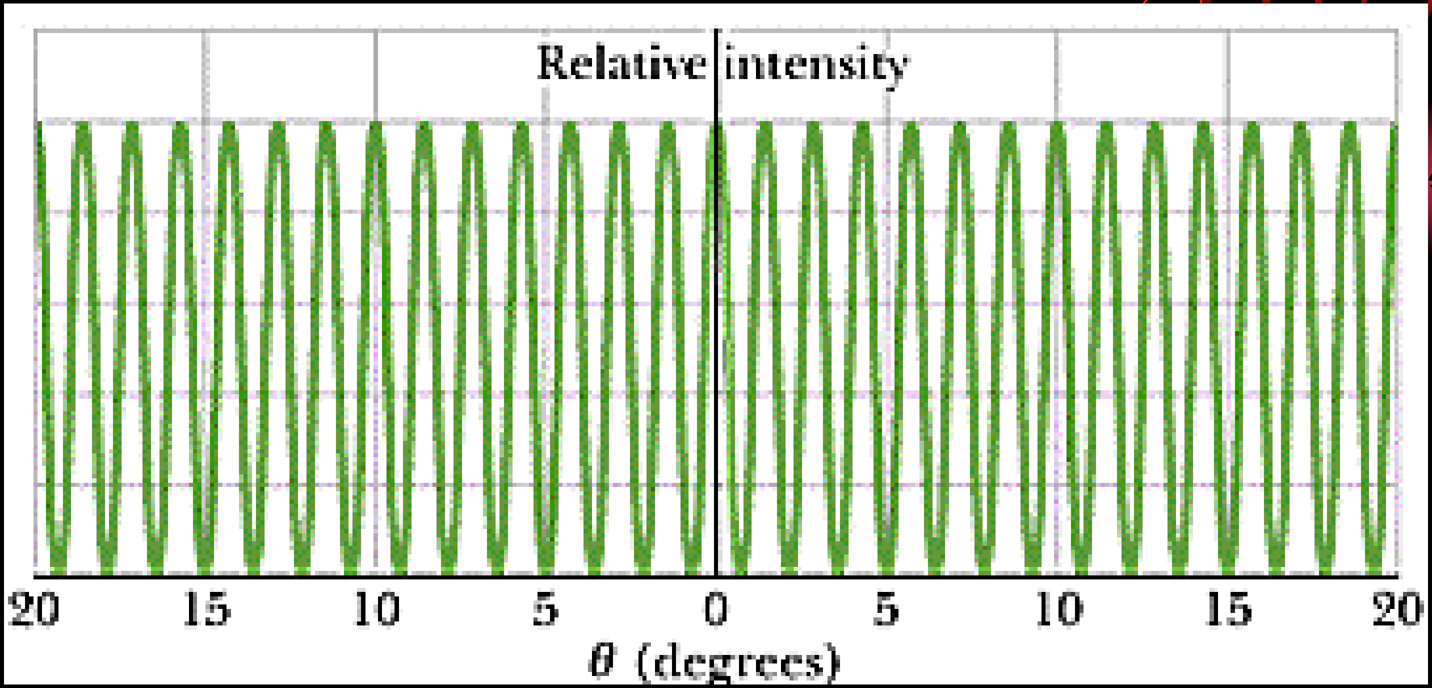
looking directly down ： red or red-yellow

tilting ：green

大藍魔爾蝴蝶



雙狹縫干涉之強度



Ex.11-3 35-3 Brighted reflected light from a water film

$$2L = \frac{\text{odd number}}{2} \times \frac{\lambda}{n_2},$$

$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}.$$

thickness 320 nm

n_2=1.33

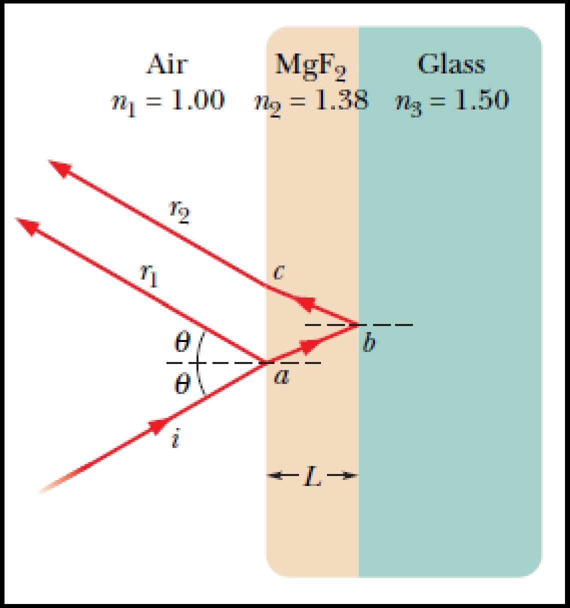
$$\lambda = \frac{2n_2L}{m+\frac{1}{2}} = \frac{(2)(1.33)(320\text{ nm})}{m+\frac{1}{2}} = \frac{851\text{ nm}}{m+\frac{1}{2}}.$$

m = 0, 1700 nm, infrared

m = 1, 567 nm, yellow-green

m = 2, 340 nm, ultraviolet

Ex.11-4 35-4 anti-reflection coating

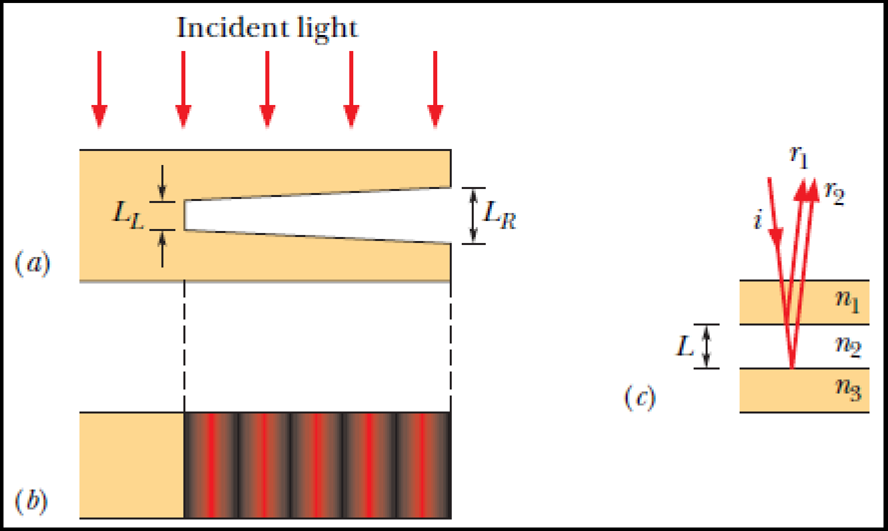


$$2L = \frac{\text{odd number}}{2} \times \frac{\lambda}{n_2}.$$

$$L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2n_2}, \qquad \text{for } m = 0, 1, 2, \ldots .$$

$$L = \frac{\lambda}{4n_2} = \frac{550\ \text{nm}}{(4)(1.38)} = 99.6\ \text{nm}.$$

Ex.11-5 35-5 thin air wedge



$$2L = \text{integer} \times \frac{\lambda}{n_2},$$

$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots .$$

$$L_L = (m_L)\frac{\lambda}{2n_2}, \qquad L_R = (m_L + 5)\frac{\lambda}{2n_2}.$$

$$\Delta L = L_R - L_L = \frac{(m_L + 5)\lambda}{2n_2} - \frac{m_L\lambda}{2n_2} = \frac{5}{2}\frac{\lambda}{n_2}$$

$$= 1.58 \times 10^{-6}\ \text{m}. \qquad \text{(Answer)}$$

Michelson Interferometer

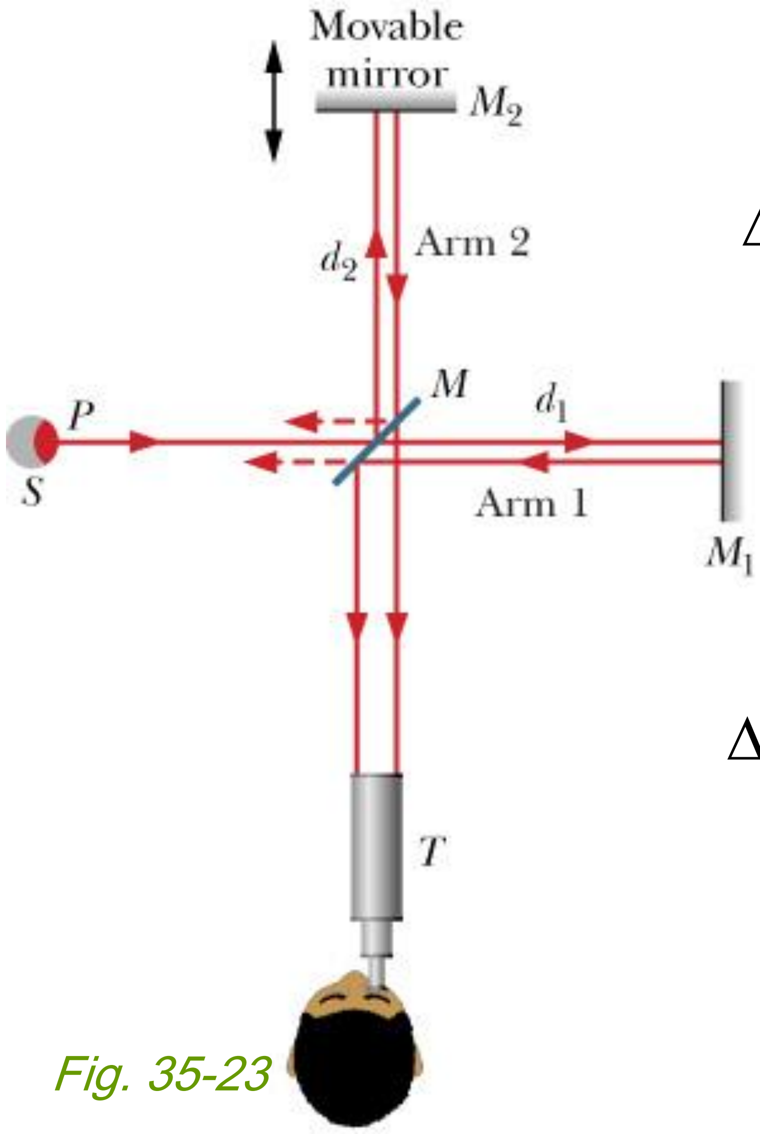


Fig. 35-23

$$\Delta L = 2d_1 - 2d_2 \quad \text{(interferometer)}$$

$$\Delta L_m = 2L \quad \text{(slab of material of thickness } L \text{ placed in front of } M_1\text{)}$$

Determining Material thickness L

$$N_m = \frac{2L}{\lambda_m} = \frac{2Ln}{\lambda}$$ (number of wavelengths

in slab of material)

$$N_a = \frac{2L}{\lambda}$$ (number of wavelengths

in same thickness of air)

$$N_m \text{-} N_a = \frac{2Ln}{\lambda} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(\text{n-1})$$ (difference in wavelengths

for paths with and without

thin slab)

Problem 35-81

In Fig. 35-49, an airtight chamber of length $d = 5.0$ *cm* is placed in one of the arms of a Michelson interferometer. (The glass window on each end of the chamber has negligible thickness.) Light of wavelength $\lambda = 500$ nm is used. Evacuating the air from the chamber causes a shift of 60 bright fringes. From these data and to six significant figures, find the index of refraction of air at atmospheric pressure.

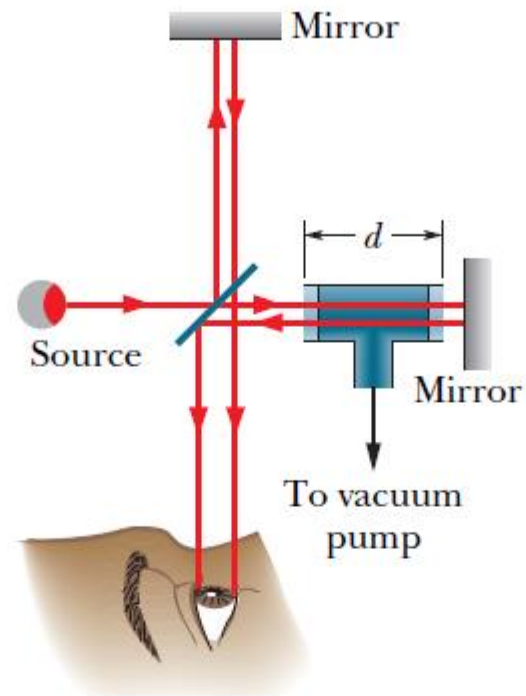


FIG. 35-49 Problem 81.

Solution to Problem 35-81

φ_1 the phase difference with air ； 2 ：vacuum

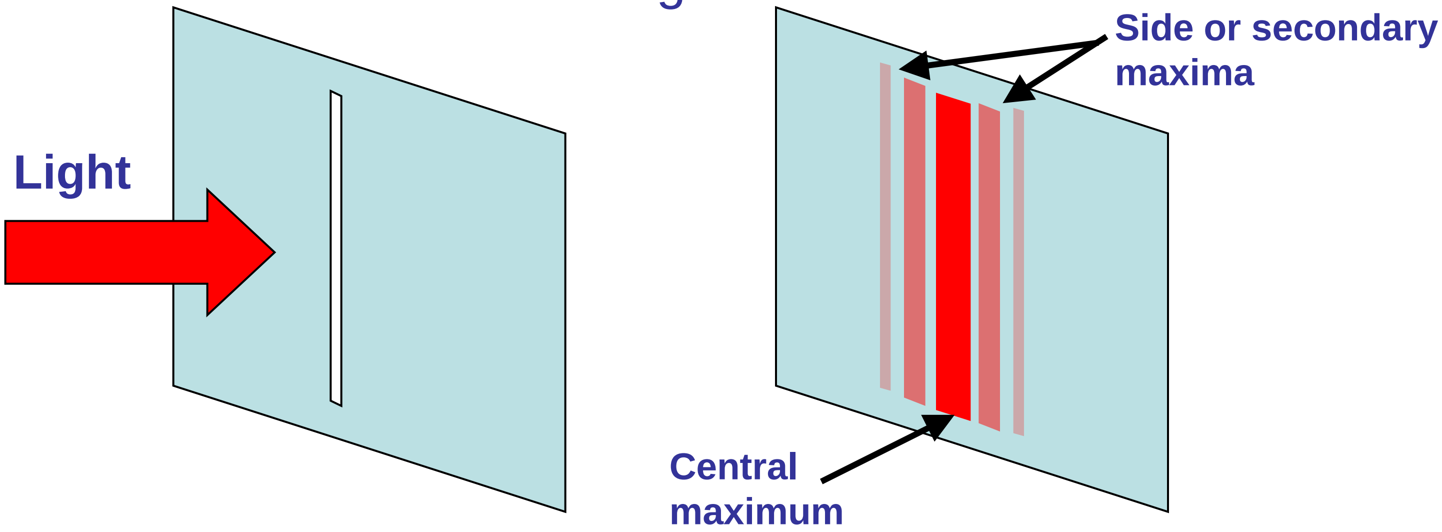
$$\phi_1 - \phi_2 = 2L\left[\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda}\right] = \frac{4\pi(n-1)L}{\lambda}$$

$$\frac{4\pi(n-1)L}{\lambda} = 2N\pi$$ *N* fringes

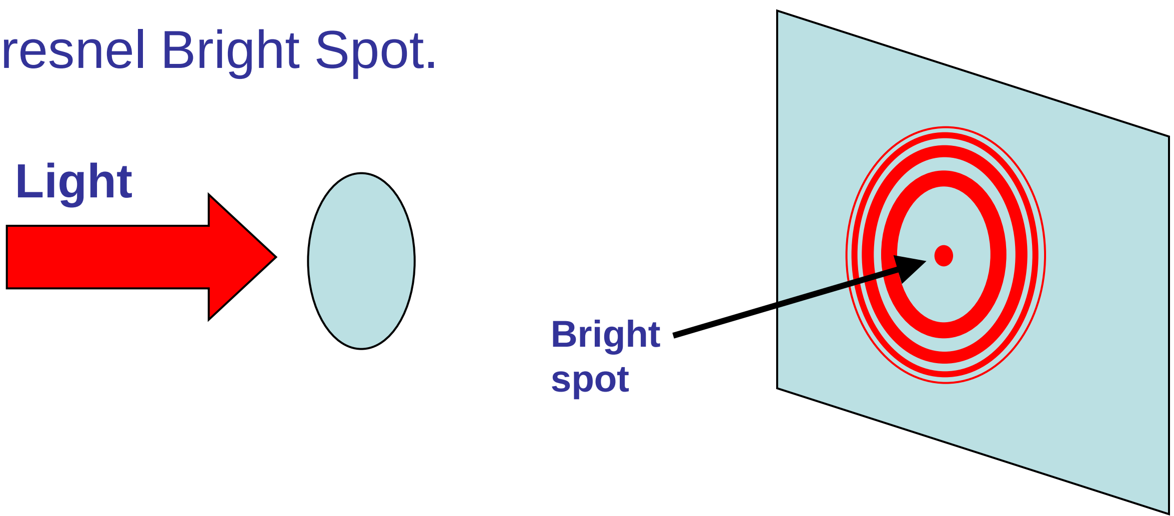
$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500\times 10^{-9}\ \mathrm{m})}{2(5.0\times 10^{-2}\ \mathrm{m})} = 1.00030\ .$$

11-3 Diffraction
and the Wave Theory of Light

Diffraction Pattern from a single narrow slit.

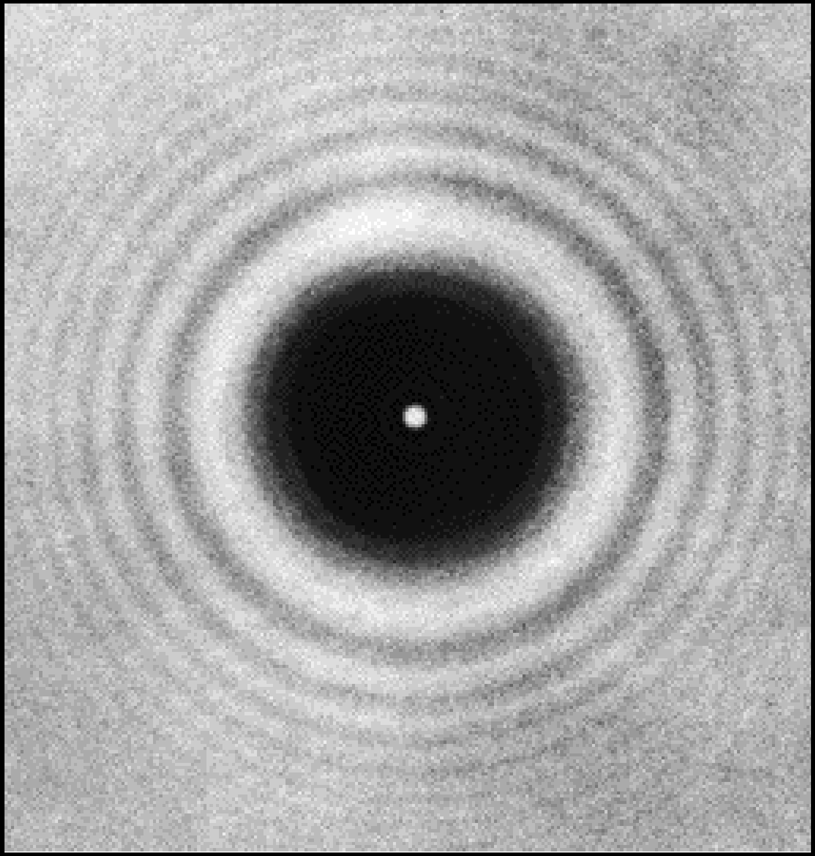


Fresnel Bright Spot.



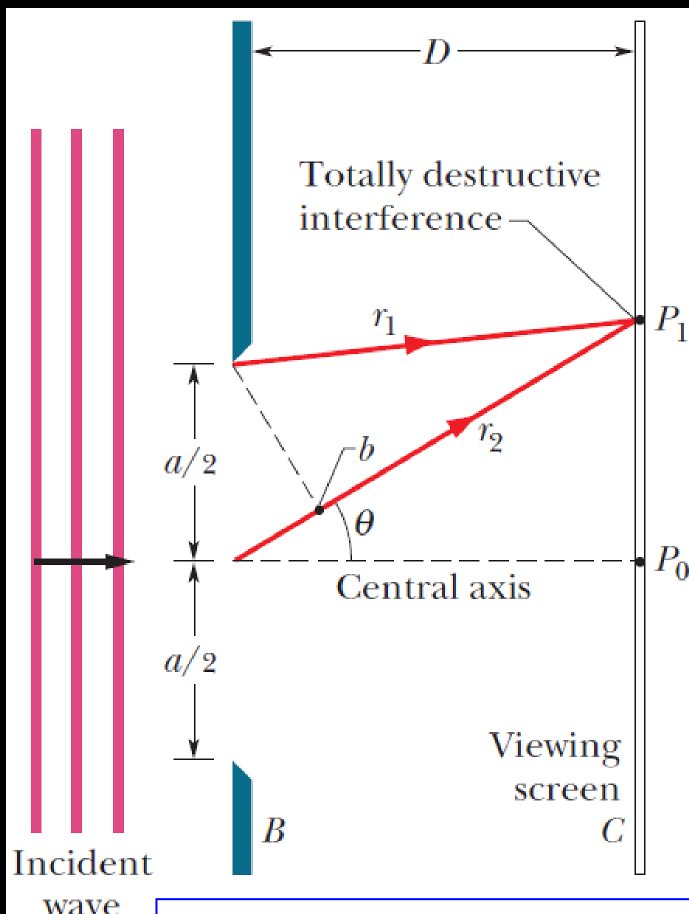
These patterns cannot be explained using geometrical optics (Ch. 34)!

The Fresnel Bright Spot (1819)

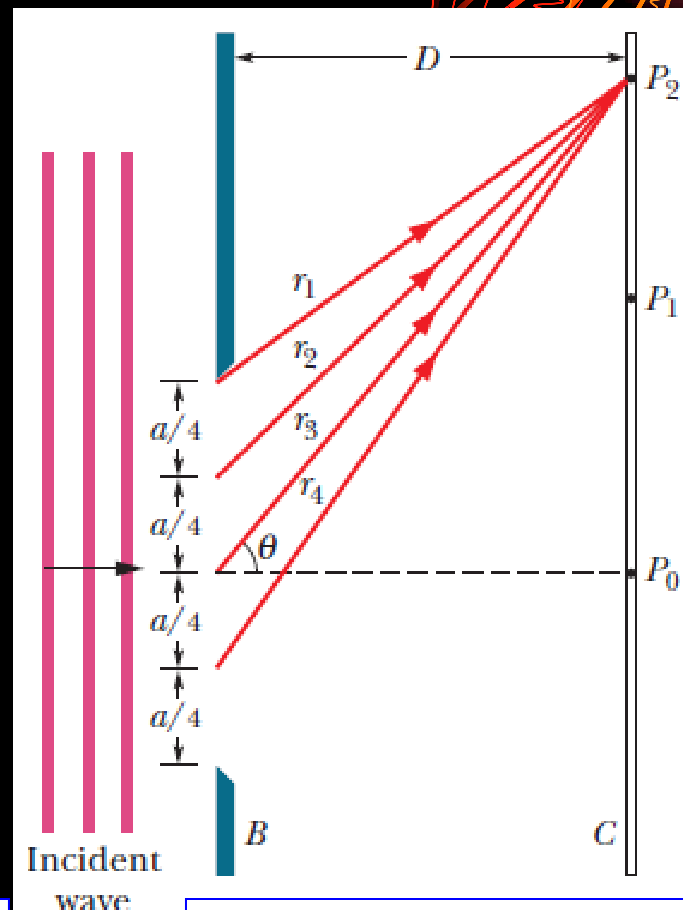


- Newton
  - corpuscle
- Poisson
- Fresnel
  - wave

Diffraction by a single slit



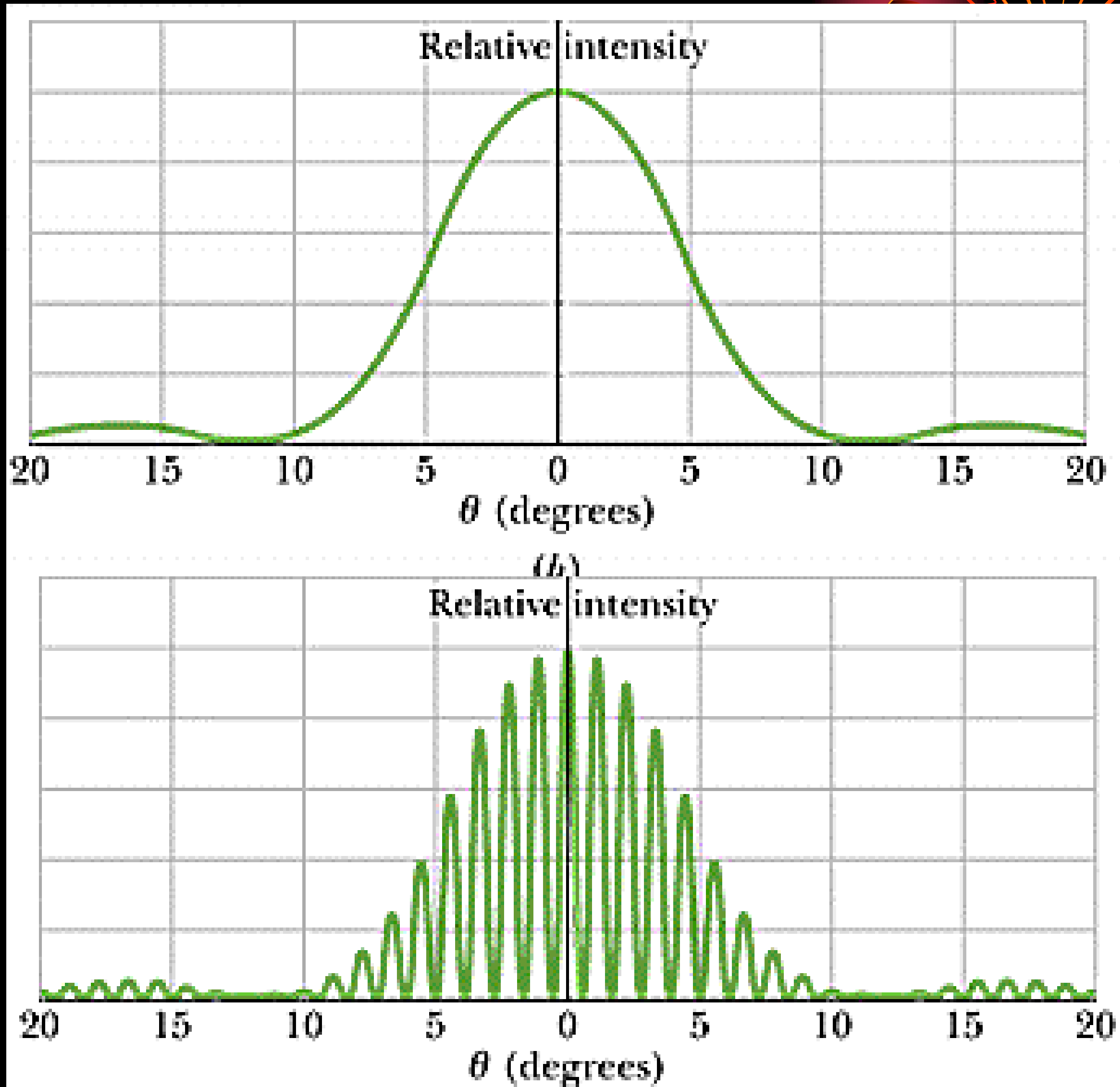
$$a \sin\theta = \lambda \quad (1^{\text{st}} \text{ minima})$$



$$a \sin\theta = 2\lambda \quad (2^{\text{nd}} \text{ minima})$$

$$a \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \qquad \text{(minima—dark fringes).}$$

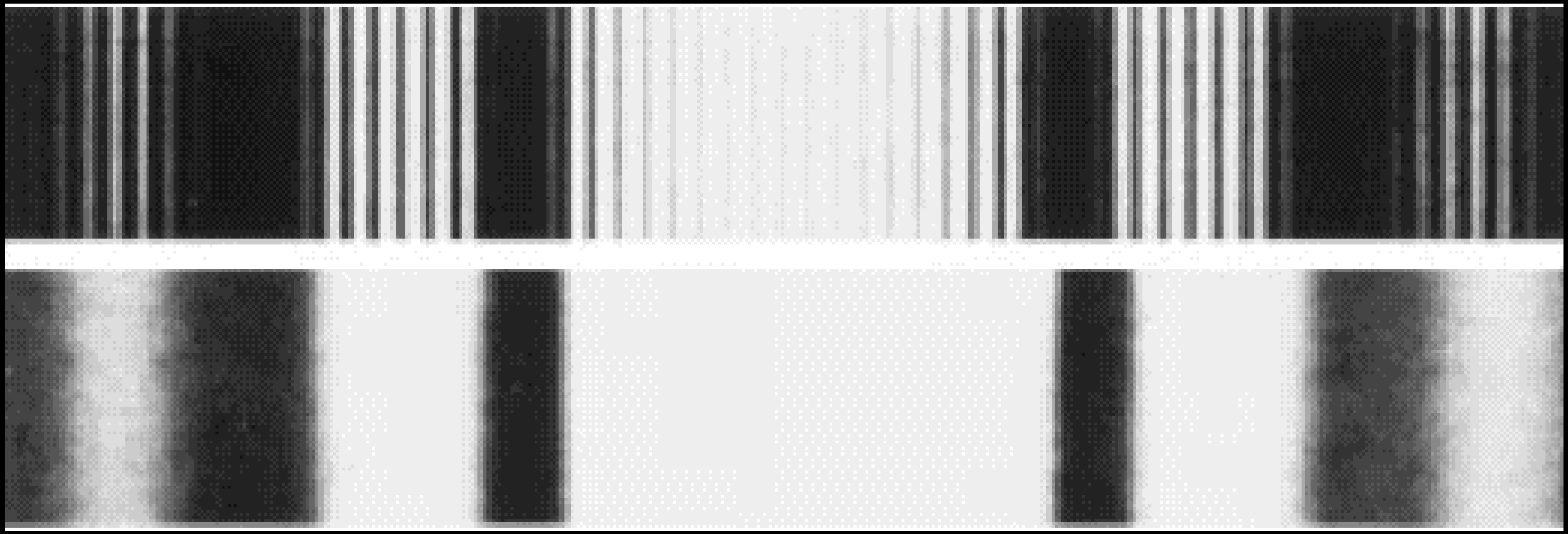
單狹縫繞射之強度



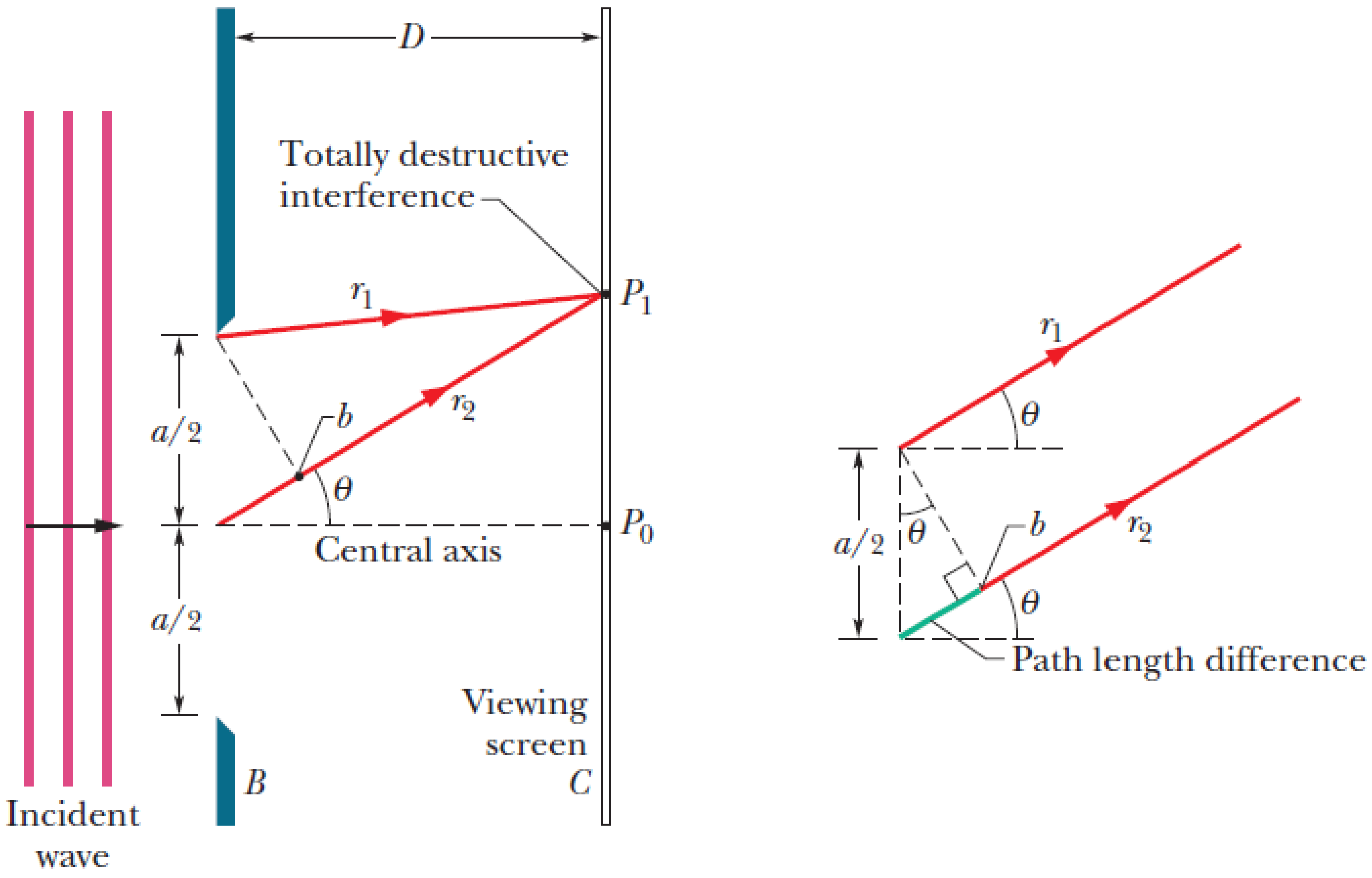
Relative intensity
20 15 10 5 0 5 10 15 20
θ (degrees)
(b)
Relative intensity
20 15 10 5 0 5 10 15 20
θ (degrees)

雙狹縫與單狹縫

- Double-slit diffraction (with interference)
- Single-slit diffraction



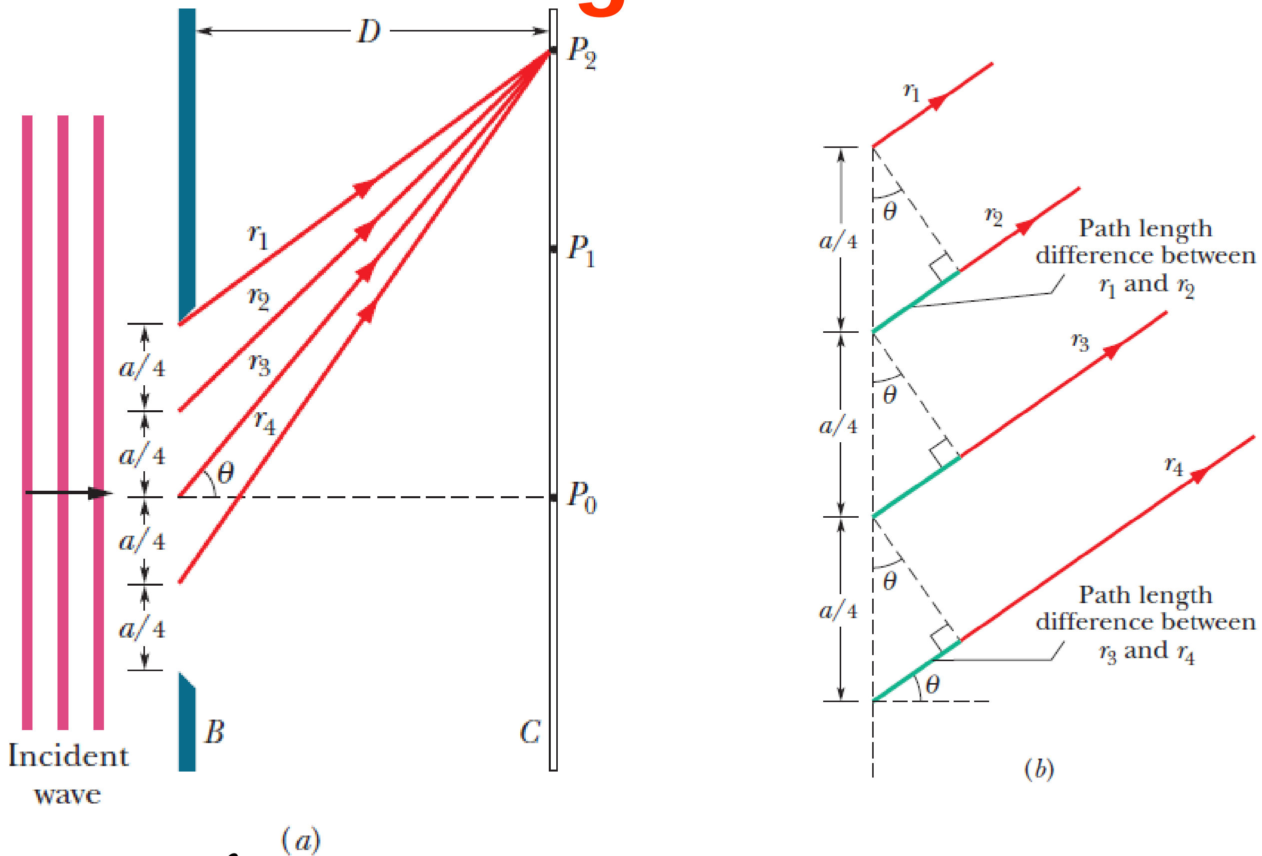
Diffraction by a Single Slit: Locating the first minimum



$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \rightarrow a\sin\theta = \lambda$$

(first minimum)

Diffraction by a Single Slit: Locating the Minima



$$\frac{a}{4}\sin\theta = \frac{\lambda}{2} \rightarrow a\sin\theta = 2\lambda$$ (second minimum)

$$\boxed{a\sin\theta = m\lambda, \quad \text{for } m = 1,2,3\ldots}$$ (minima-dark fringes)

Ex.11-6 36-1 Slit width

(a) For what value of a will the first minimum for red light of wavelength $\lambda = 650$ nm appear at $\theta = 15°$?

$$a = \frac{m\lambda}{\sin\theta} = \frac{(1)(650\text{ nm})}{\sin 15°}$$
$$= 2511\text{ nm} \approx 2.5\ \mu\text{m}.$$

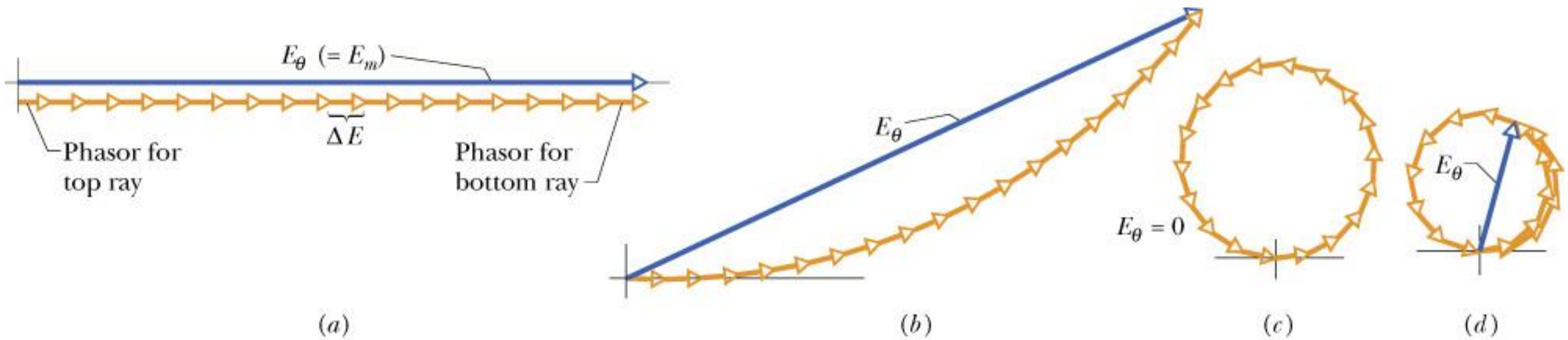
(b) What is the wavelength λ' of the light whose first side diffraction maximum is at 15°, thus coinciding with the first minimum for the red light?

$$a \sin\theta = 1.5\lambda'.$$

$$\lambda' = \frac{a\sin\theta}{1.5} = \frac{(2511\text{ nm})(\sin 15°)}{1.5}$$
$$= 430\text{ nm}.$$

Intensity in Single-Slit Diffraction, Qualitatively

$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \left(\frac{2\pi}{\lambda}\right)\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix} \qquad \Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin\theta)$$



N=18 $\theta = 0$ θ small 1st min. 1st side max.

Fig. 36-7

Intensity and path length difference

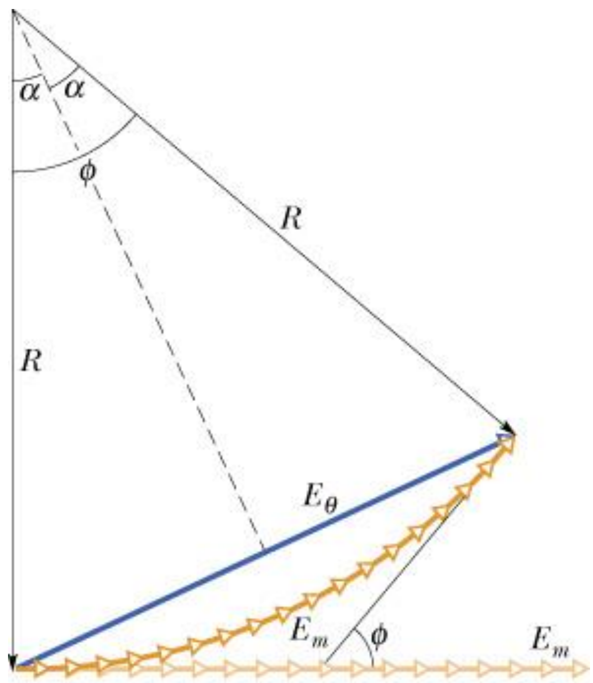


Fig. 36-9

$$\sin \tfrac{1}{2}\phi = \frac{E_\theta}{2R} \qquad \phi = \frac{E_m}{R}$$

$$E_\theta = \frac{E_m}{\frac{1}{2}\phi}\sin \tfrac{1}{2}\phi$$

$$\frac{I(\theta)}{I_m} = \frac{E_\theta^2}{E_m^2} \rightarrow I(\theta) = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\phi = \left(\frac{2\pi}{\lambda}\right)(a\sin\theta)$$

Intensity in Single-Slit Diffraction, Quantitatively

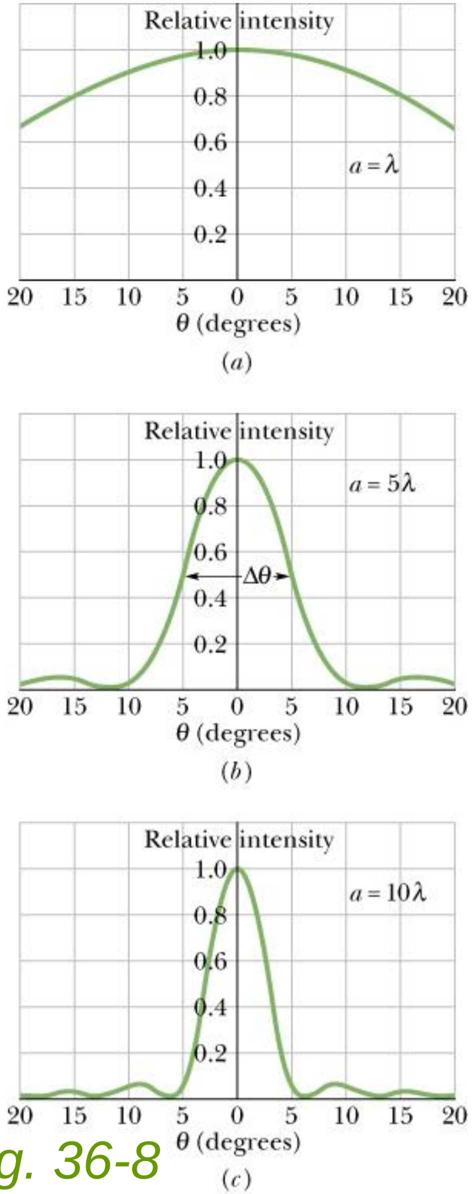


Fig. 36-8

Here we will show that the intensity at the screen due to a single slit is:

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2 \qquad (36\text{-}5)$$

$$\text{where} \quad \alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta \qquad (36\text{-}6)$$

In Eq. 36-5, minima occur when:

$$\alpha = m\pi, \qquad \text{for } m = 1, 2, 3 \ldots$$

If we put this into Eq. 36-6 we find:

$$m\pi = \frac{\pi a}{\lambda} \sin \theta, \qquad \text{for } m = 1, 2, 3 \ldots$$

$$\text{or} \qquad a \sin \theta = m\lambda, \qquad \text{for } m = 1, 2, 3 \ldots$$

(minima-dark fringes)

Ex.11-7 36-2

Find the intensities of the first three secondary maxima (side maxima) in the single-slit diffraction pattern of Fig. 36-1, measured as a percentage of the intensity of the central maximum.

$$\alpha = \left(m + \frac{1}{2}\right)\pi, \quad m = 1, 2, 3, \cdots$$

$$\frac{I}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin(m+\frac{1}{2})\pi}{(m+\frac{1}{2})\pi}\right)^2, \quad \text{for } m = 1, 2, 3, \ldots.$$

$$\frac{I_1}{I_m} = \left(\frac{\sin(1+\frac{1}{2})\pi}{(1+\frac{1}{2})\pi}\right)^2 = \left(\frac{\sin 1.5\pi}{1.5\pi}\right)^2$$

$$= 4.50 \times 10^{-2} \approx 4.5\%.$$

$$\frac{I_2}{I_m} = 1.6\% \quad \text{and} \quad \frac{I_3}{I_m} = 0.83\%$$

Diffraction by a Circular Aperture

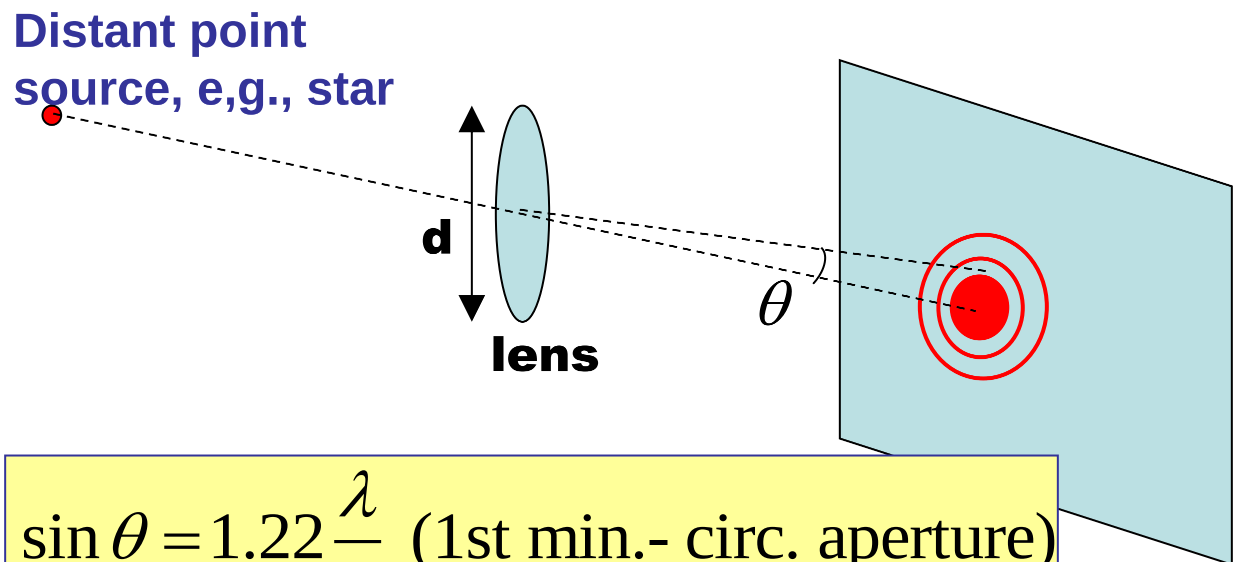
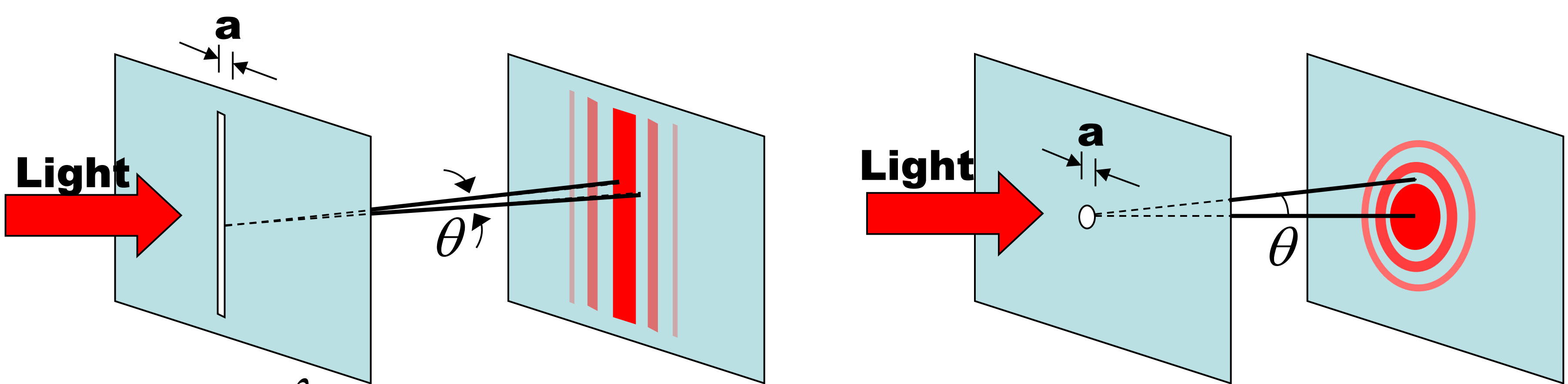


Image is not a point, as expected from geometrical optics! Diffraction is responsible for this image pattern

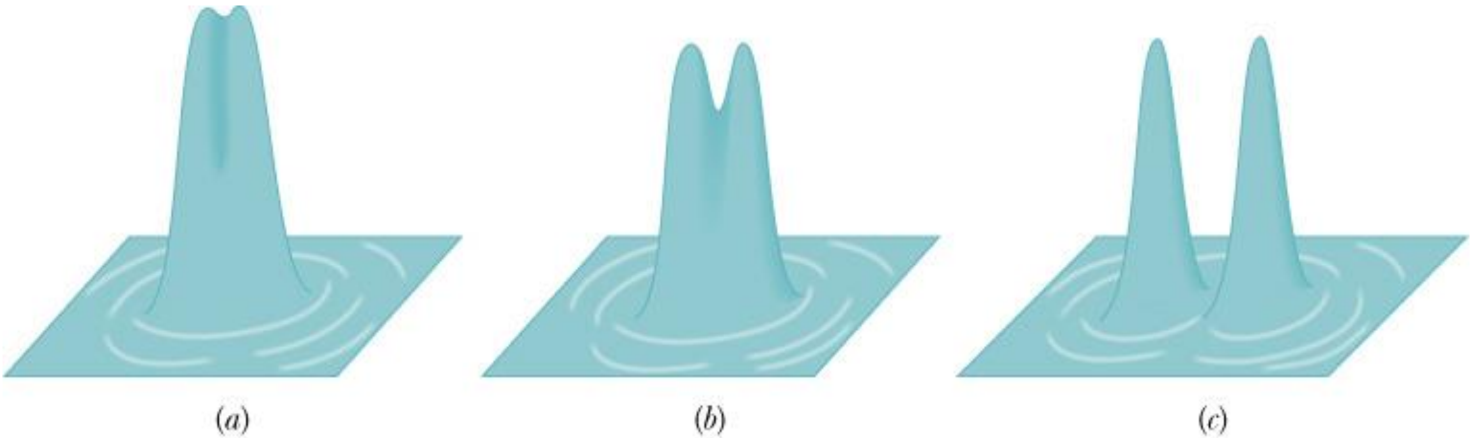
$$\sin\theta = 1.22\frac{\lambda}{d} \quad \text{(1st min.- circ. aperture)}$$



$$\sin\theta = 1.22\frac{\lambda}{a} \quad \text{(1st min.- single slit)}$$

Resolvability

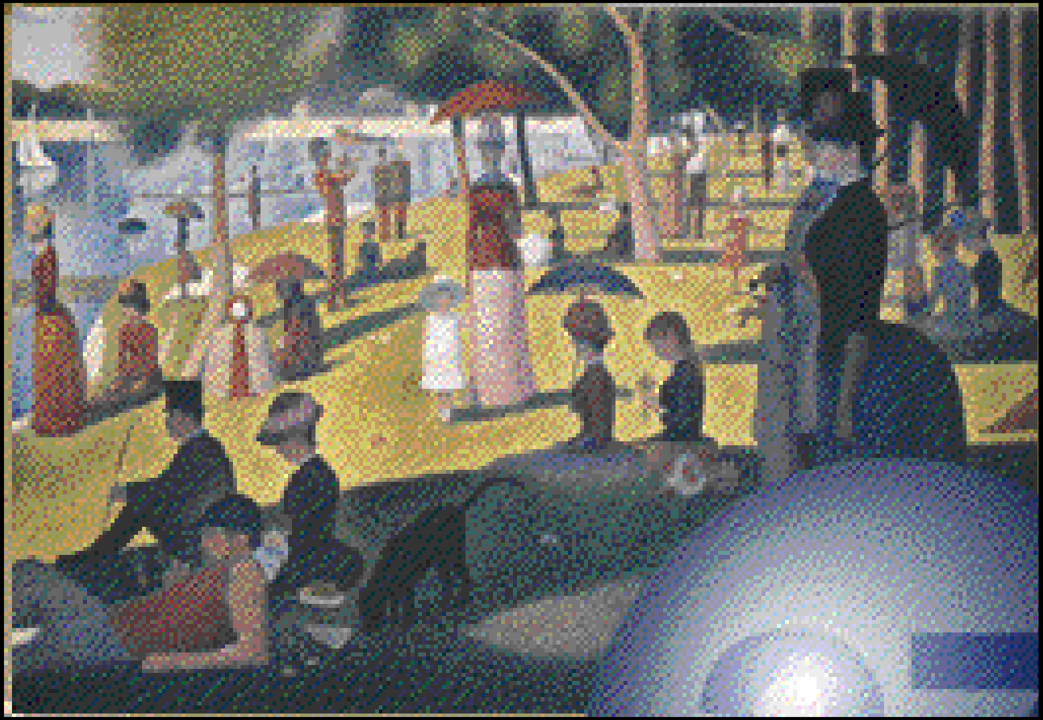
Rayleigh's Criterion: two point sources are barely resolvable if their angular separation θ_R results in the central maximum of the diffraction pattern of one source's image is centered on the first minimum of the diffraction pattern of the other source's image.



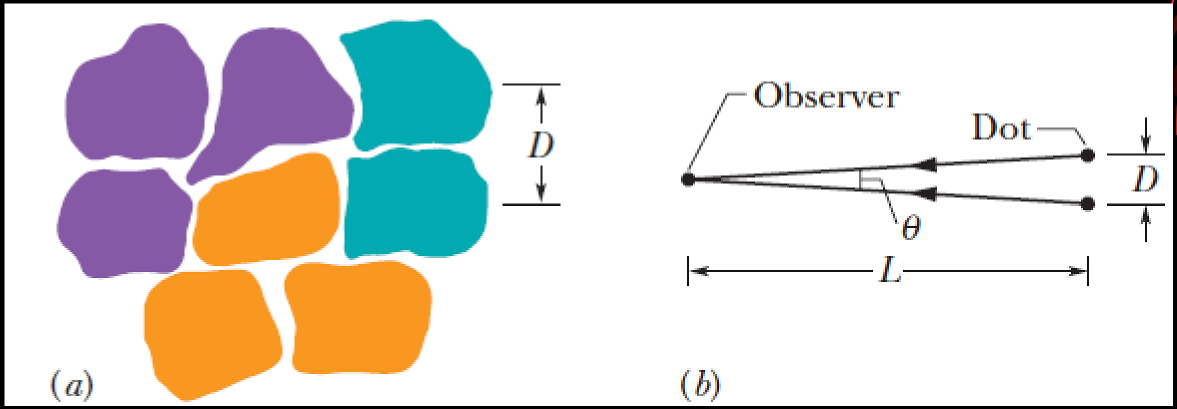
$$\theta_R = \sin^{-1}\left(1.22\frac{\lambda}{d}\right) \overset{\theta_R \text{ small}}{\approx} 1.22\frac{\lambda}{d} \quad \text{(Rayleigh's criterion)}$$

11-4.9 Diffraction – (繞射)

Why do the colors in a pointillism painting change with viewing distance?



Ex.11-8 36-3 pointillism



D = 2.0 mm

d = 1.5 mm

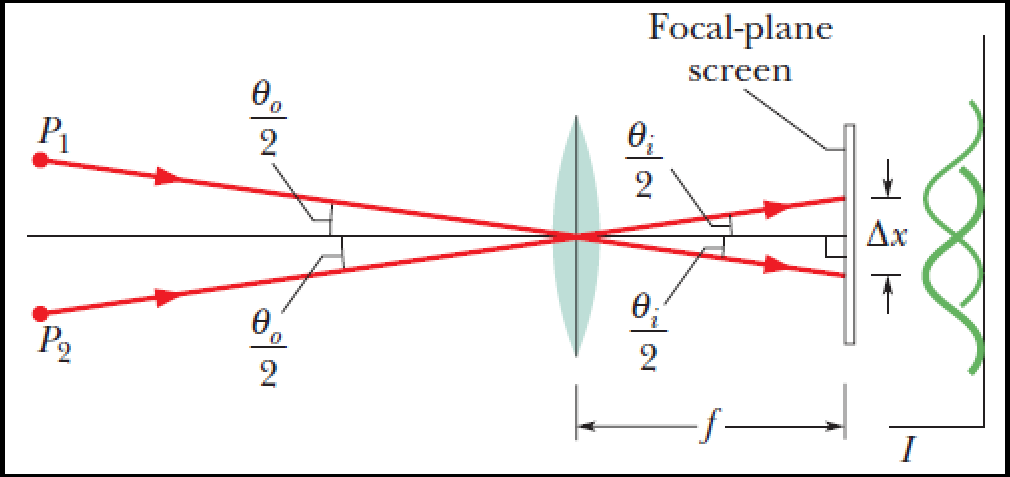
$$\theta_R = 1.22\,\frac{\lambda}{d}.$$

$$\theta = \frac{D}{L}.$$

$$L = \frac{Dd}{1.22\lambda}.$$

$$L = \frac{(2.0 \times 10^{-3}\ \mathrm{m})(1.5 \times 10^{-3}\ \mathrm{m})}{(1.22)(400 \times 10^{-9}\ \mathrm{m})} = 6.1\ \mathrm{m}.$$

Ex.11-9 36-4



d = 32 mm

f = 24 cm

λ= 550 nm

$$\theta_o = \theta_i = \theta_R = 1.22\frac{\lambda}{d}$$

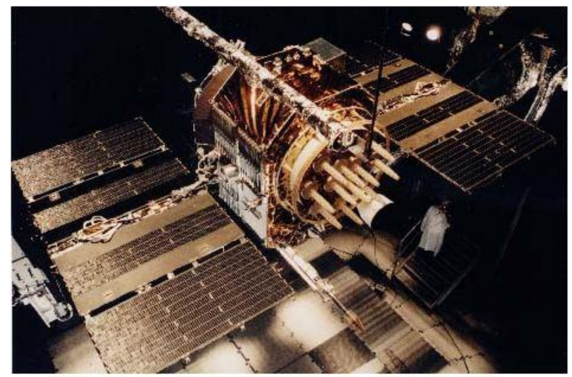
$$= \frac{(1.22)(550 \times 10^{-9}\text{ m})}{32 \times 10^{-3}\text{ m}} = 2.1 \times 10^{-5}\text{ rad.}$$

$$\Delta x = f\theta_i,$$

$$\Delta x = (0.24\text{ m})(2.1 \times 10^{-5}\text{ rad}) = 5.0\ \mu\text{m.}$$

The telescopes on some commercial and military surveillance satellites

Resolution of 85 cm and 10 cm respectively

$$\frac{D}{L} = \theta_{\mathrm{R}} = 1.22\frac{\bullet}{d}$$

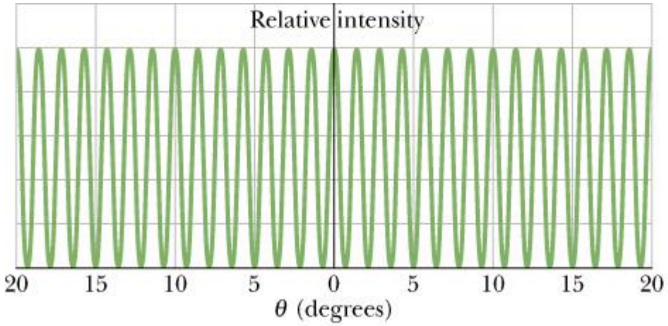
$\lambda = 550 \times 10^{-9}$ m.

(a) $L = 400 \times 10^{3}$ m ， $D = 0.85$ m $\rightarrow d = 0.32$ m.

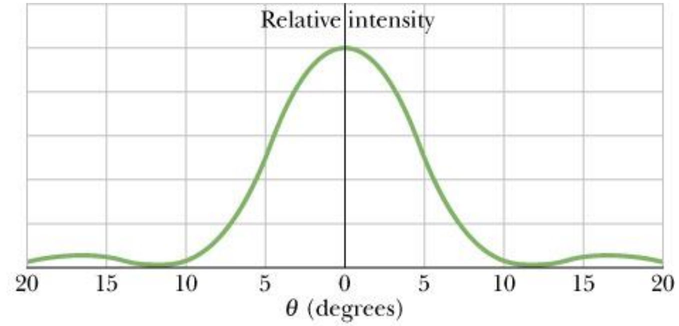
(b) $D = 0.10$ m $\rightarrow d = 2.7$ m.

Diffraction by a Double Slit

Single slit $a \sim \lambda$



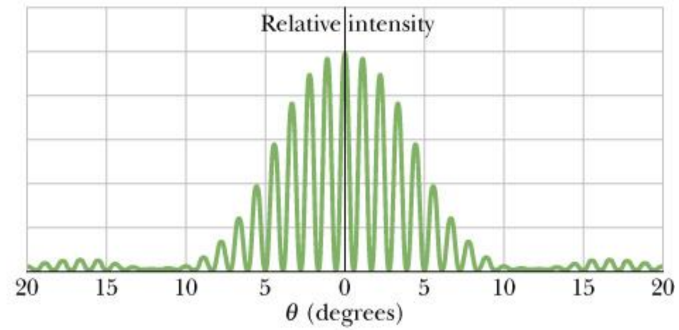
(a)



(b)

Two vanishingly narrow slits $a \ll \lambda$

Two Single slits $a \sim \lambda$



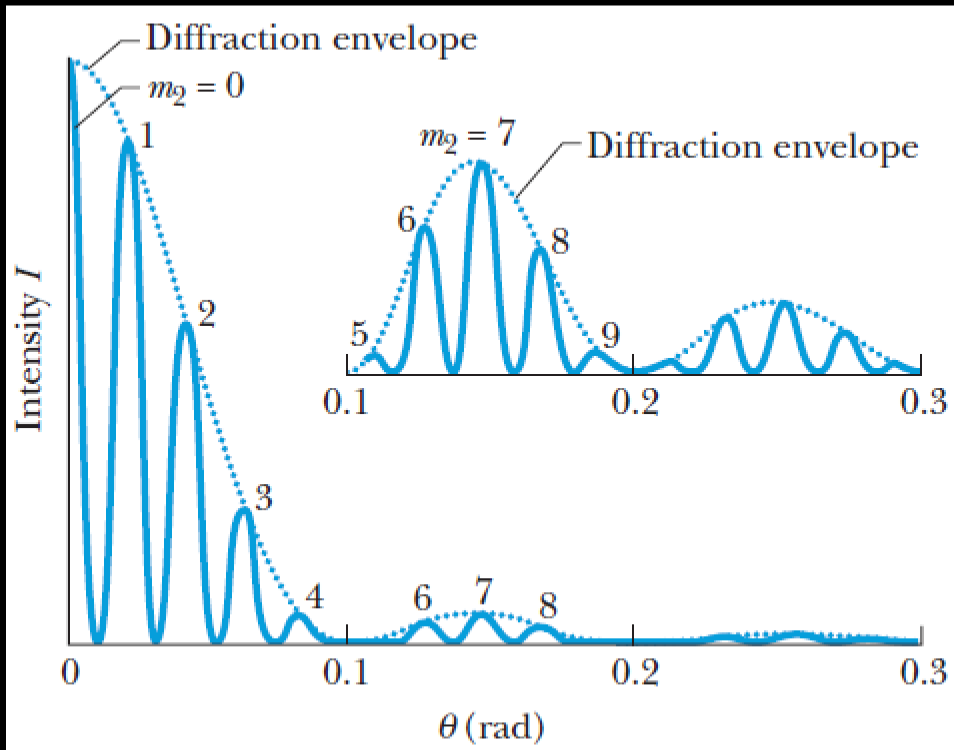
(c)

$$\boxed{I(\theta) = I_m \left(\cos^2 \beta\right)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{(double slit)}}$$

$$\beta = \frac{\pi d}{\lambda}\sin\theta$$

$$\alpha = \frac{\pi a}{\lambda}\sin\theta$$

Ex.11-10 36-5



d = 32 µm

a = 4.050 µm

λ= 405 nm

$$a \sin\theta = \lambda$$

$$d \sin\theta = m_2\lambda \quad \text{for } m = 0, 1, 2, \ldots$$

$$a \sin\theta = 2\lambda.$$

$$m_2 = \frac{d}{a} = \frac{19.44\ \mu\text{m}}{4.050\ \mu\text{m}} = 4.8.$$

$$m_2 = \frac{2d}{a} = \frac{(2)(19.44\ \mu\text{m})}{4.050\ \mu\text{m}} = 9.6.$$

Diffraction Gratings

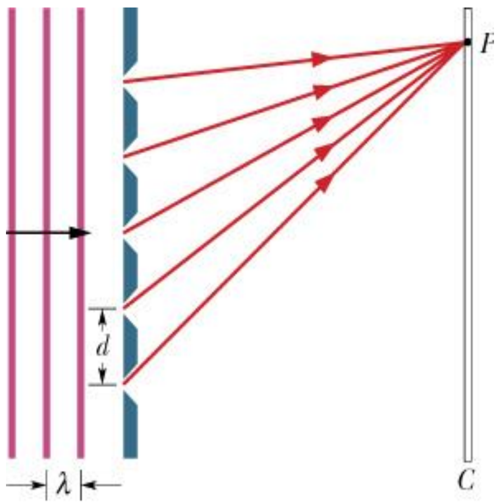


Fig. 36-18

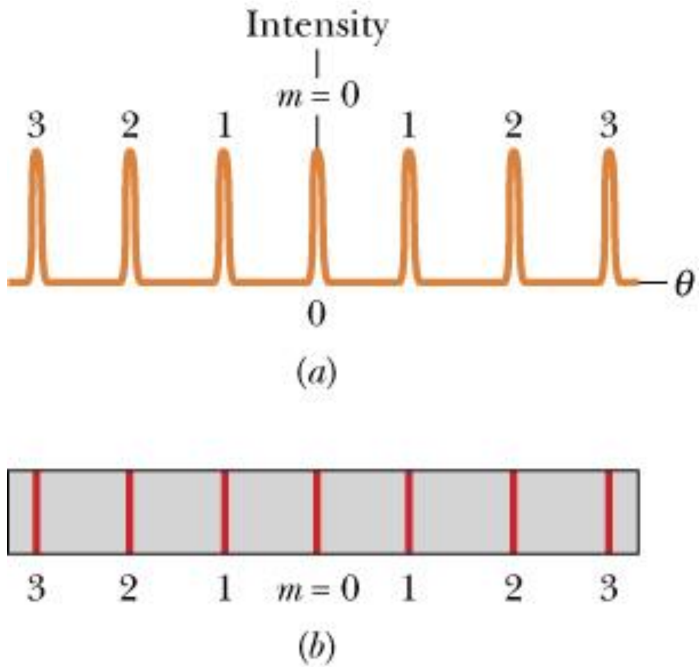


Fig. 36-19

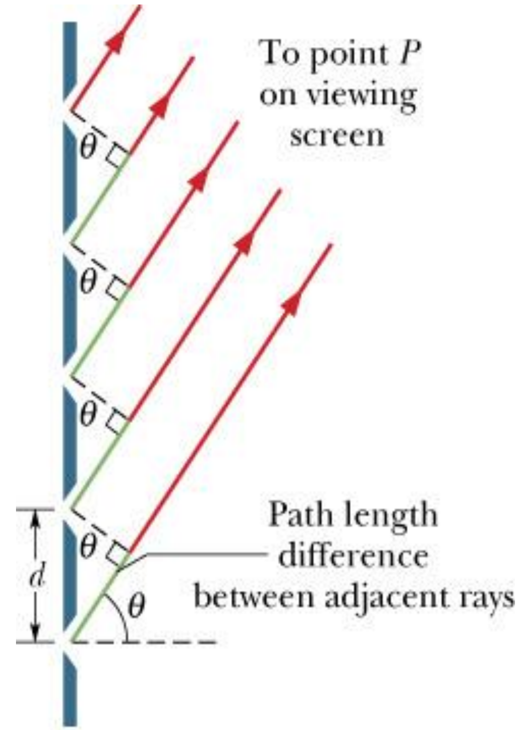


Fig. 36-20

$$d \sin\theta = m\lambda \quad \text{for } m = 0, 1, 2\ldots \quad \text{(maxima-lines)}$$

Width of Lines

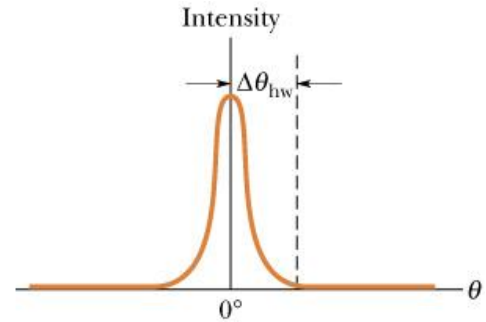


Fig. 36-21

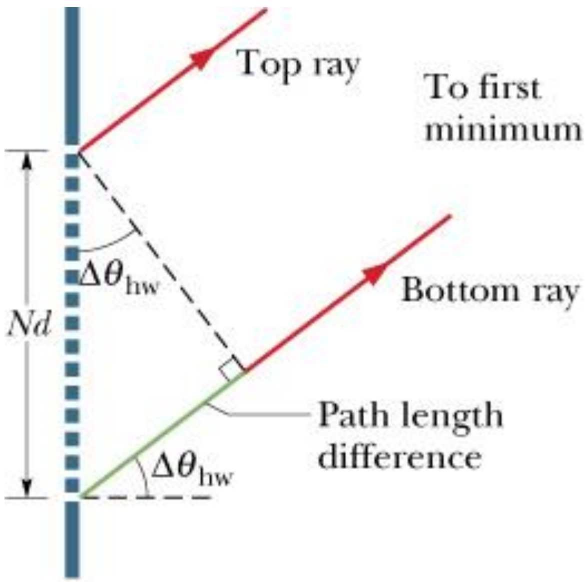


Fig. 36-22

$$Nd\sin\Delta\theta_{\text{hw}} = \lambda \quad , \qquad \sin\Delta\theta_{hw} \approx \Delta\theta_{hw}$$

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd} \quad \text{(half width of central line)}$$

$$\boxed{\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd\cos\theta} \quad \text{(half width of line at } \theta\text{)}}$$

Grating Spectroscope

Separates different wavelengths (colors) of light into distinct diffraction lines

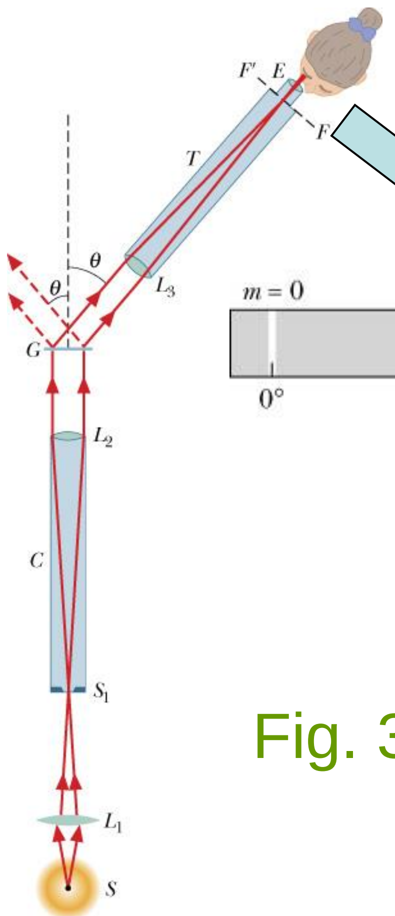


Fig. 36-23

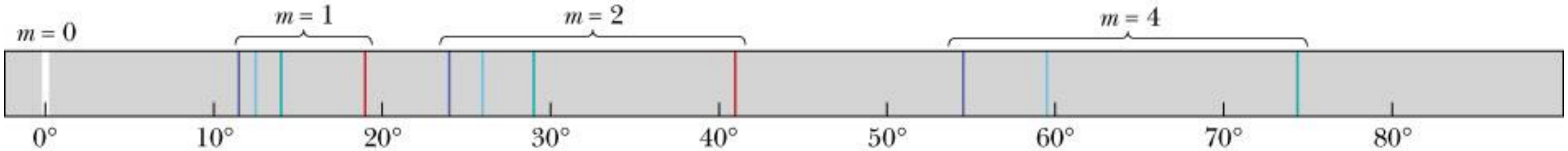
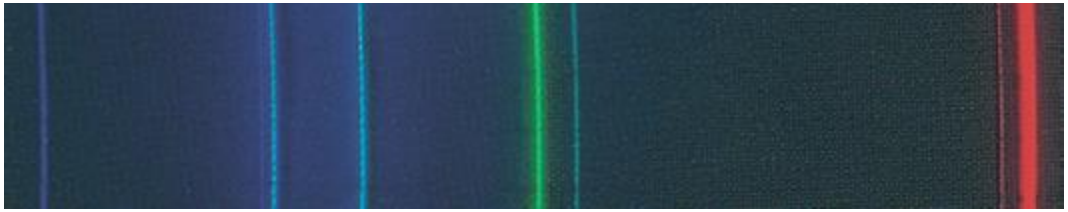
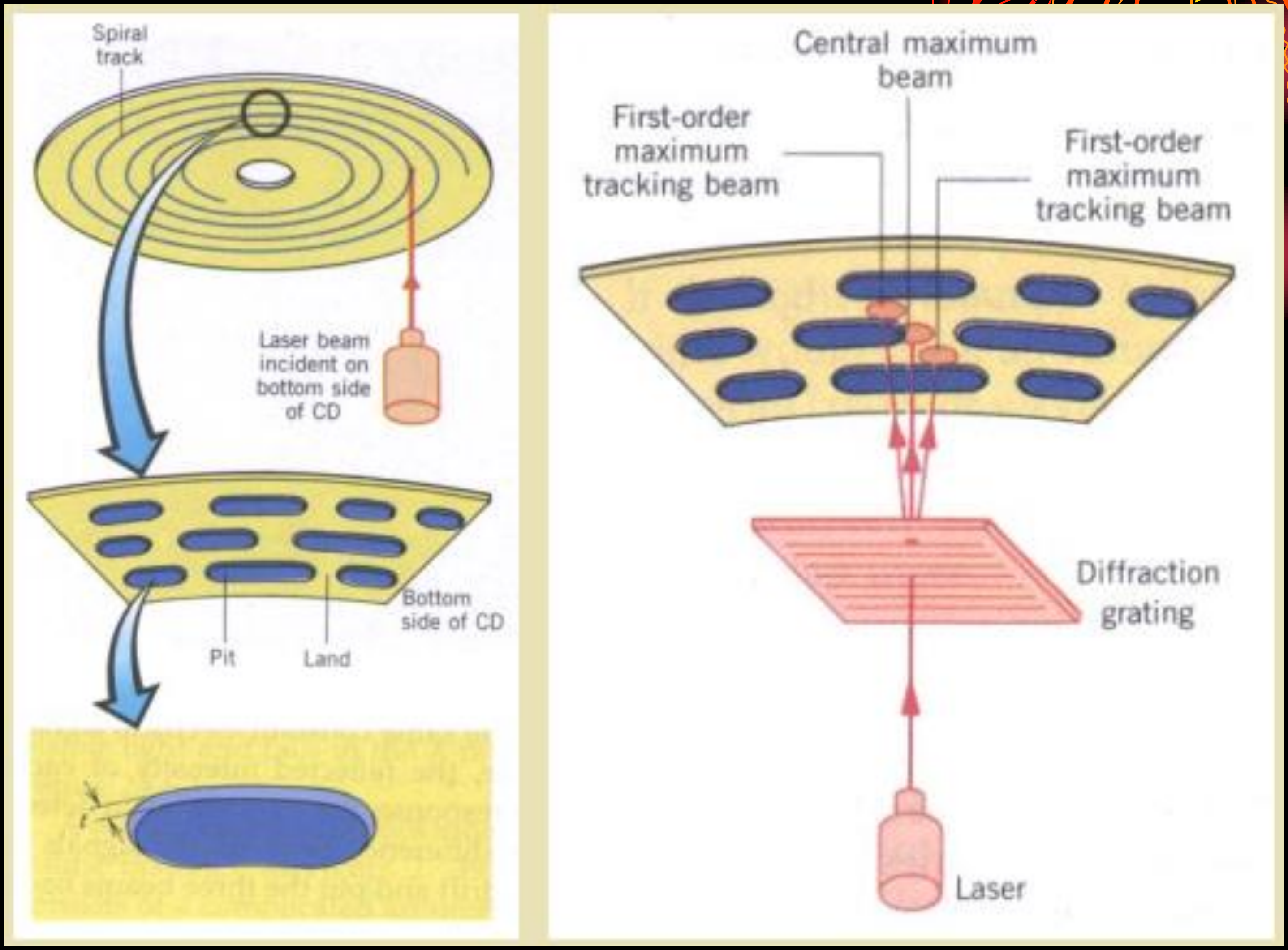


Fig. 36-24



Compact Disc



Optically Variable Graphics

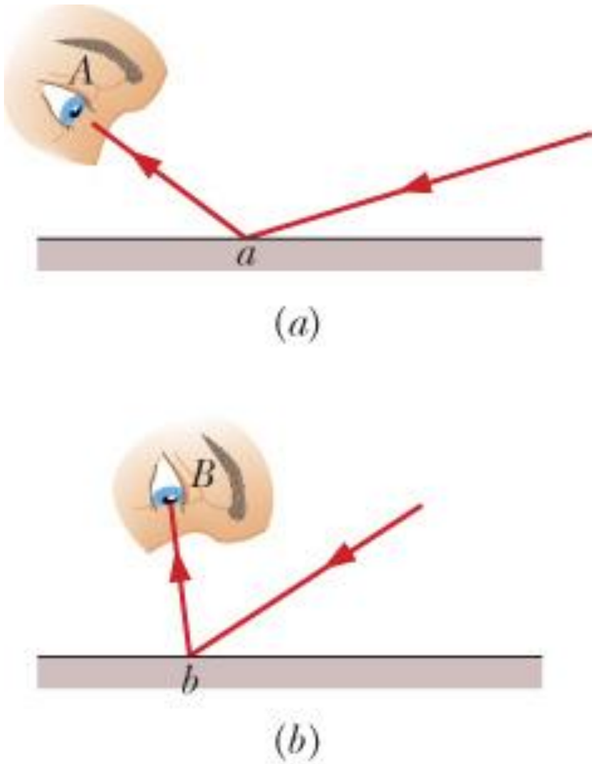
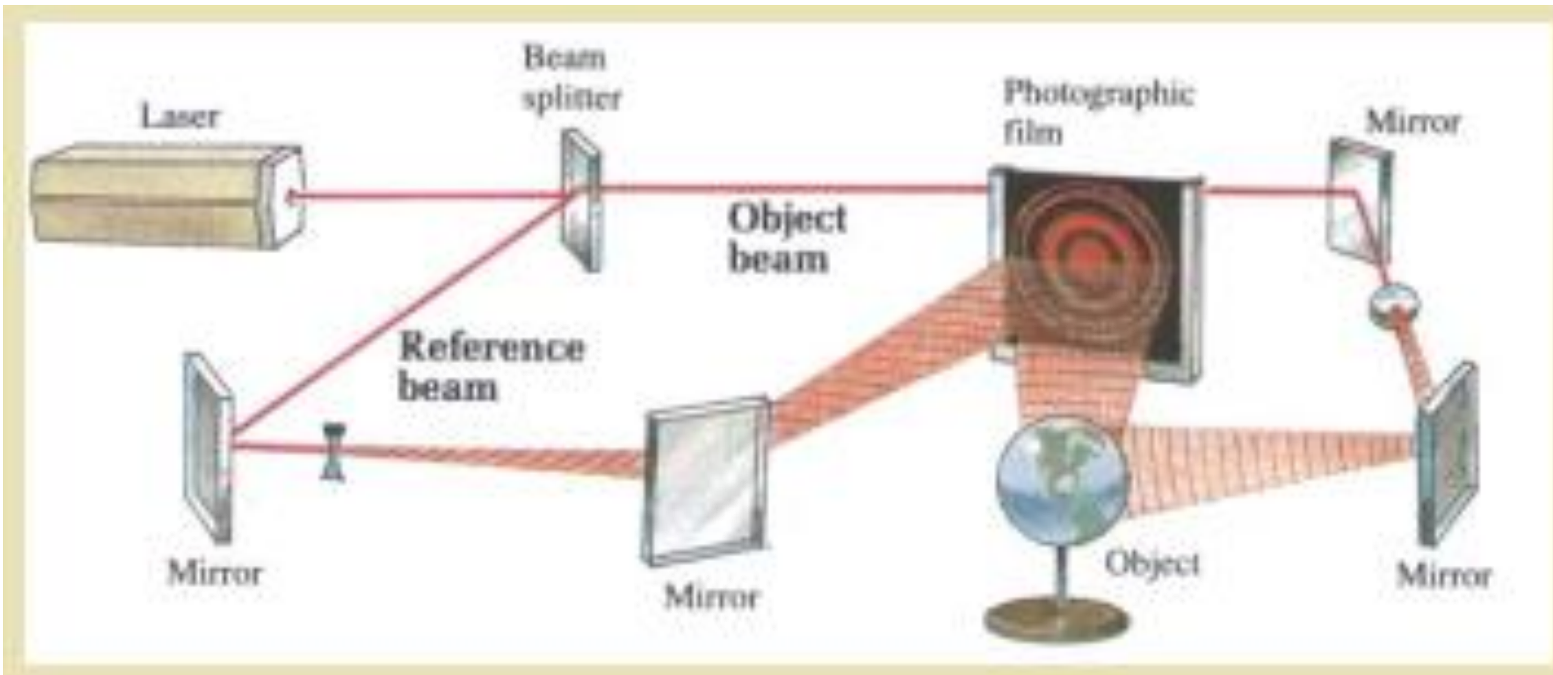
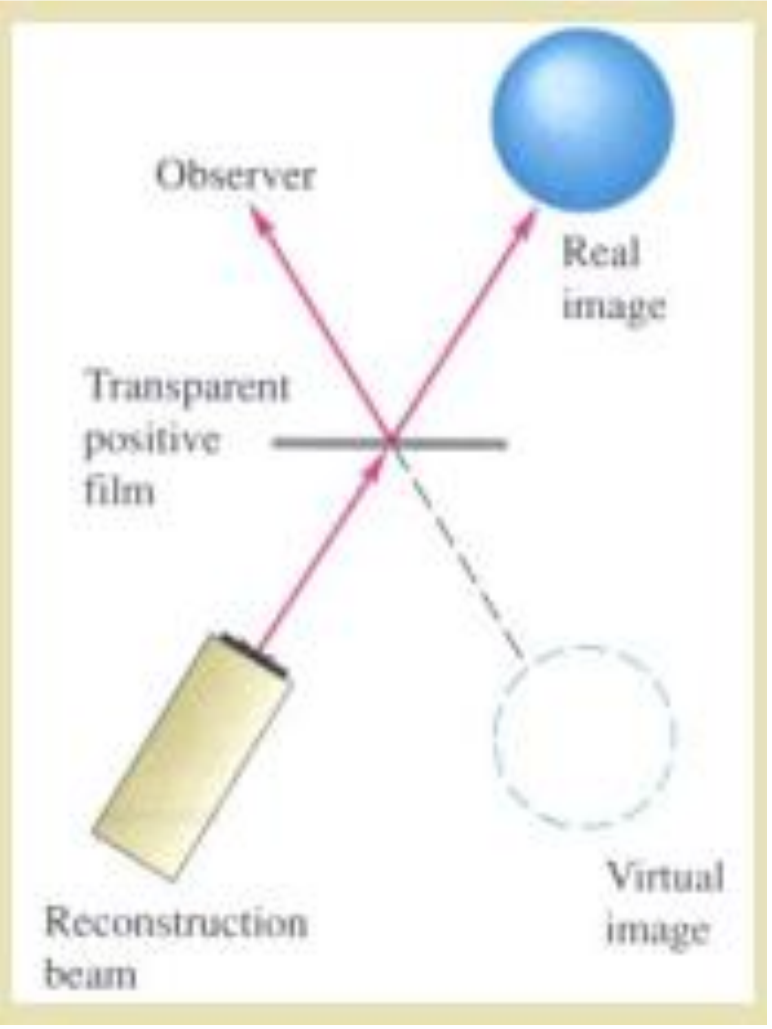


Fig. 36-27

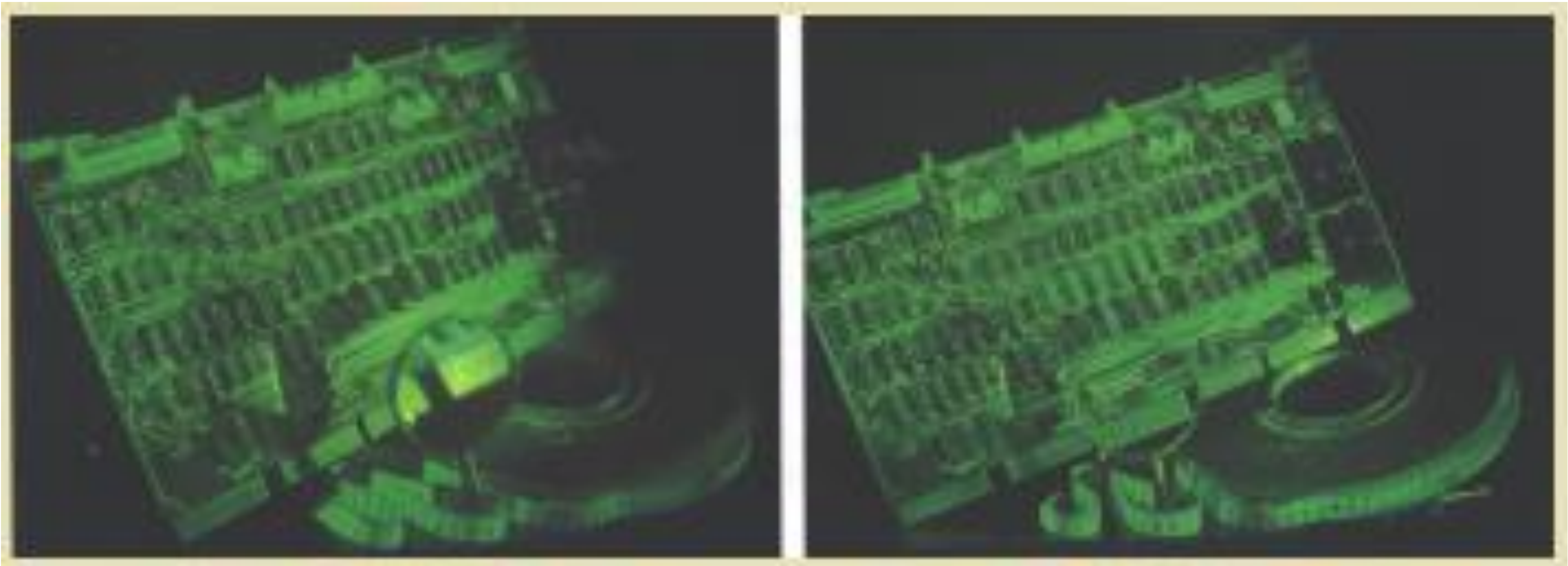
全像術


Laser
Beam
splitter
Photographic
film
Mirror
Object
beam
Reference
beam
Mirror
Mirror
Object
Mirror

Viewing a holograph



A Holograph



Gratings: Dispersion

$$D = \frac{\Delta\theta}{\Delta\lambda} \quad \text{(dispersion defined)}$$

$$\boxed{D = \frac{m}{d\cos\theta} \quad \text{(dispersion of a grating)} \quad \text{(36-30)}}$$

Angular position of maxima $\quad d\sin\theta = m\lambda$

Differential of first equation (what change in angle does a change in wavelength produce?)

$$d\left(\cos\theta\right)d\theta = md\lambda$$

For small angles

$$d\theta \rightarrow \Delta\theta \quad \text{and} \quad d\lambda \rightarrow \Delta\lambda$$

$$d\left(\cos\theta\right)\Delta\theta = m\Delta\lambda$$

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\left(\cos\theta\right)}$$

Gratings: Resolving Power

$$R = \frac{\lambda_{\text{avg}}}{\Delta\lambda} \quad \text{(resolving power defined)}$$

$$\boxed{R = Nm \ \text{(resolving power of a grating)} \quad (36\text{-}32)}$$

Rayleigh's criterion for half-width to resolve two lines

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd\cos\theta}$$

Substituting for $\Delta\theta$ in calculation on previous slide

$$\Delta\theta_{\text{hw}} \to \Delta\theta$$

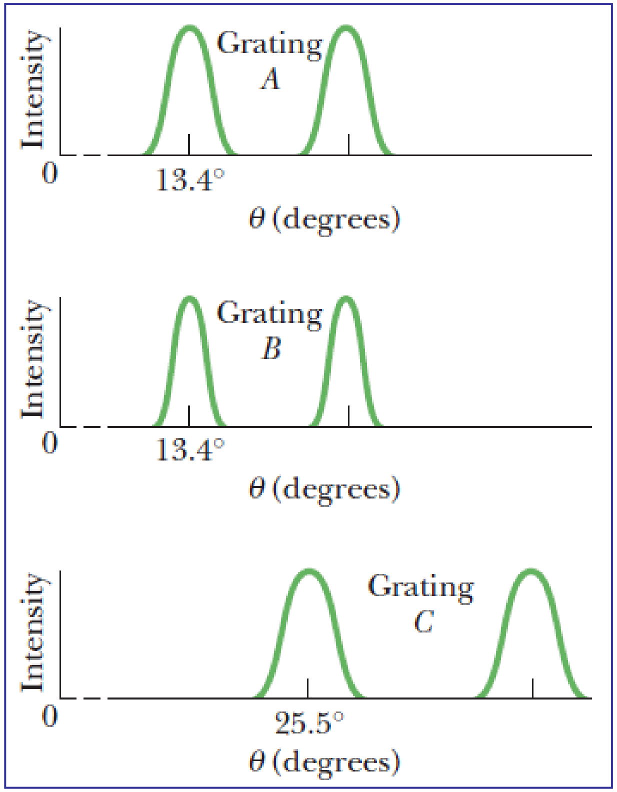
$$\to \frac{\lambda}{N} = m\Delta\lambda$$

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

Dispersion and Resolving Power Compared

Grating	N	d (nm)	θ	D (°/μm)	R
A	10 000	2540	13.4°	23.2	10 000
B	20 000	2540	13.4°	23.2	20 000
C	10 000	1360	25.5°	46.3	10 000

[a]Data are for λ = 589 nm and m = 1.



X-Ray Diffraction

X-rays are electromagnetic radiation with wavelength ~1 Å = 10^{-10} m (visible light ~5.5×10^{-7} m)

X-ray generation

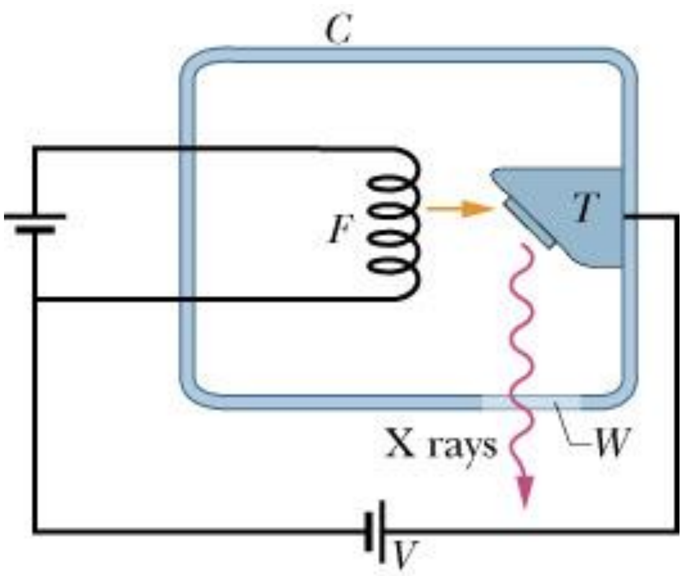
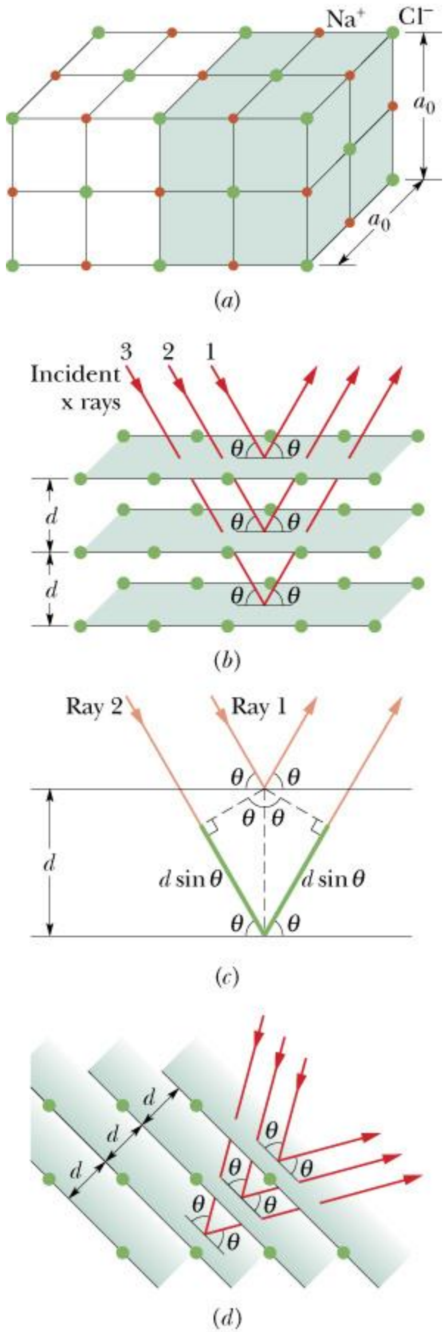


Fig. 36-29

X-ray wavelengths to short to be resolved by a standard optical grating

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(0.1\text{ nm})}{3000\text{ nm}} = 0.0019^\circ$$



Diffraction of x-rays by crystal

d ~ 0.1 nm

→ three-dimensional diffraction grating

$$\boxed{2d\sin\theta = m\lambda \quad \text{for } m = 0, 1, 2\ldots \quad \text{(Bragg's law)}}$$

Fig. 36-30

X-Ray Diffraction, cont'd

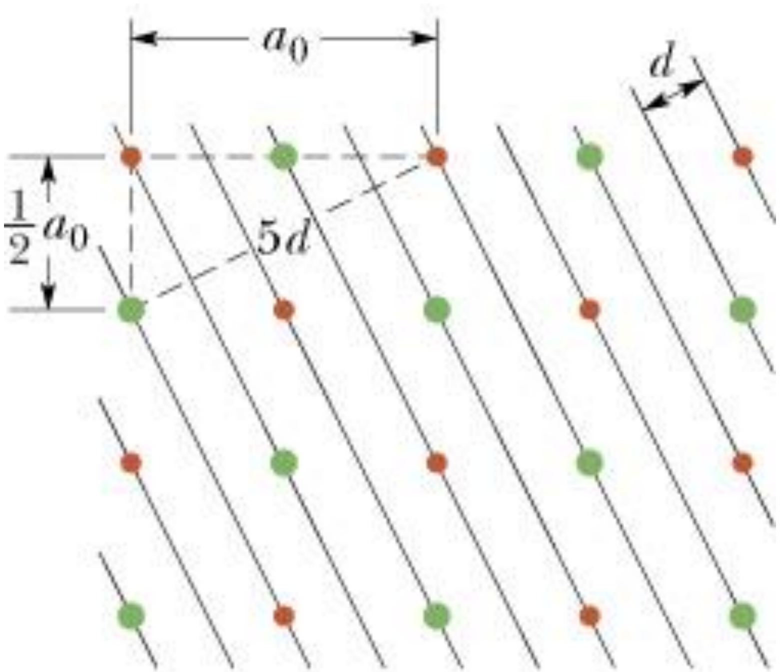
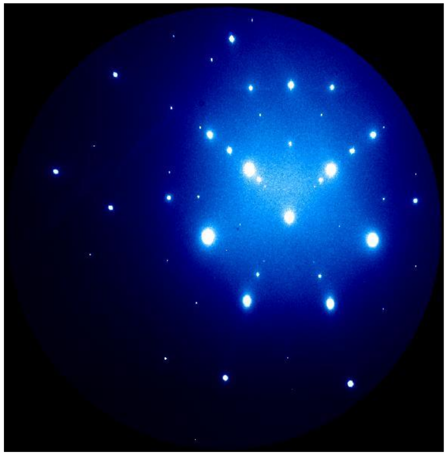


Fig. 36-31

$$5d = \sqrt{\tfrac{5}{4}a_0^2} \quad \text{or} \quad d = \frac{a_0}{20} = 0.2236a_0$$



Structural Coloring by Diffraction