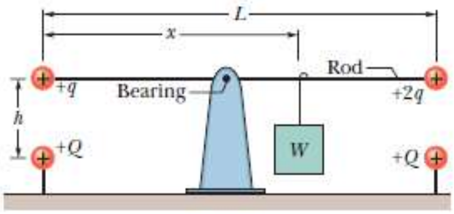
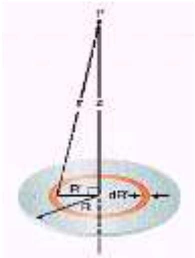
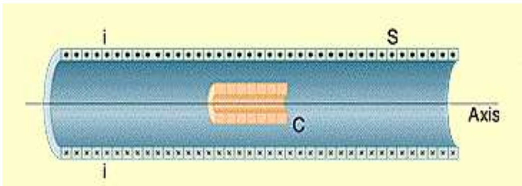
『北區高中物理科學人才培育』計畫高二物理期末考試卷

1. 

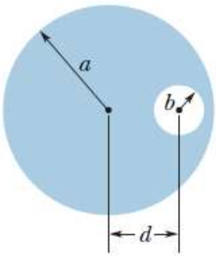
The figure to the left shows a long, nonconducting, massless rod of length L, pivoted at its center and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and 2q, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q. (a) Find the distance x when the rod is horizontal and balanced. (10%) (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced? 。(10%)

2. 

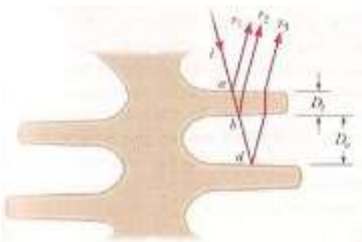
(a) 試求左圖均勻帶電圓盤軸線上 P 點之電位。(10%)
(b) 試由(a)之電位求 P 點之電場。(10%)

3. 

左圖螺線管之電流及圈數分別為 i_s =1.5 A，n_s = 2.2×10^4 turns/m，其內有一線圈之截面積及圈數分別為 A = 3.46 $\times10^{-4}$m^2，N_c = 130 turns，如電流 i_s 在 25ms 內穩定降至 0，試求線圈內之感應電動勢。(20%)

4. 

The figure to the left shows a cross section of a long cylindrical conductor of radius a = 4.00 cm containing a long cylindrical hole of radius b = 1.50 cm. The central axes of the cylinder and hole are parallel and are distance d = 2.00 cm apart; current i = 5.25 A is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (10%) (b) Show that the magnetic field in the hole is uniform. (10%)

5. 

左圖是大藍摩爾蝴蝶翅膀內階梯狀結構的截面圖及光干涉之示意圖。其中 D_t = 63.5nm，D_a = 127nm，折射率 n_t = 1.53，n_a = 1，入射波 i 與反射波 r_1、r_2、r_3 可視為垂直介面。請問我們看到的藍綠色(～450nm)蝶翼是來自 r_1 及 r_2 或 r_1 及 r_3 之干涉？(20%)

1. (a) Since the rod is in equilibrium, the net force acting on it is zero, and the net torque about any point is also zero. We write an expression for the net torque about the bearing, equate it to zero, and solve for x. The charge Q on the left exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\,(qQ/h^2)$, at a distance $L/2$ from the bearing. We take the torque to be negative. The attached weight exerts a downward force of magnitude W, at a distance $x-L/2$ from the bearing. This torque is also negative. The charge Q on the right exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\,(2qQ/h^2)$, at a distance $L/2$ from the bearing. This torque is positive. The equation for rotational equilibrium is

$$\frac{-1}{4\text{p}\varepsilon_0}\frac{qQ}{h^2}\frac{L}{2}-W\left(x-\frac{L}{2}\right)+\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2}\frac{L}{2}=0.$$

The solution for x is

$$x=\frac{L}{2}\left(1+\frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2W}\right)$$

(b) If F_N is the magnitude of the upward force exerted by the bearing, then Newton's second law (with zero acceleration) gives

$$W-\frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2}-\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2}-F_N=0.$$

We solve for h so that $F_N = 0$. The result is

$$h=\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{3qQ}{W}}.$$

2. (a)

$$\boxed{\begin{aligned}
&dq=\sigma(2\pi R')(dR')\\
&dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{(z^2+R^2)^{1/2}}\\
&V=\int dV=\frac{\sigma}{2\varepsilon_0}\int_0^R\frac{R'dR'}{(z^2+R'^2)^{1/2}}\\
&=\frac{\sigma}{2\varepsilon_0}(\sqrt{z^2+R^2}-z)
\end{aligned}}$$

(b)

$$\boxed{E=-\frac{\partial V}{\partial z}=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)}$$

3.

$$\boxed{\begin{aligned}
&B_i=\mu_0 in=4.15\times10^{-2}\,\mathrm{T}\\
&\Phi_{B,i}=BA=14.4\,\mu\mathrm{Wb}\\
&\mathcal{E}=N\Delta\Phi_B/\Delta t=75\,\mathrm{mV}
\end{aligned}}$$

4. (a) The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current *i* which is uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

at a distance *r* from its axis, inside the cylinder. Here *R* is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi\left(a^2 - b^2\right)}$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_1}{2\pi\left(a^2 - b^2\right)}.$$

The current in the cylinder that fills the hole is

$$I_2 = \pi J b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

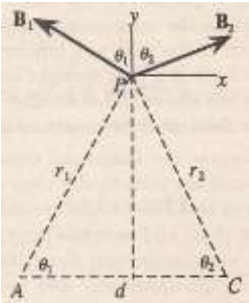
$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi\left(a^2 - b^2\right)} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(5.25\,\text{A})(0.0200\,\text{m})}{2\pi[(0.0400\,\text{m})^2 - (0.0150\,\text{m})^2]} = 1.53\times10^{-5}\,\text{T}$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b)



$$B_x = B_2 \sin\theta_2 - B_1 \sin\theta_1 = \frac{\mu_0 i r_2}{2\mathrm{p}\left(a^2 - b^2\right)}\sin\theta_2 - \frac{\mu_0 i r_1}{2\mathrm{p}\left(a^2 - b^2\right)}\sin\theta_1$$

$$= \frac{\mu_0 i}{2\mathrm{p}\left(a^2 - b^2\right)}[r_2 \sin\theta_2 - r_1 \sin\theta_1] = 0$$

$$B_y = B_2 \cos\theta_2 + B_1 \cos\theta_1 = \frac{\mu_0 i r_2}{2\mathrm{p}\left(a^2 - b^2\right)}\cos\theta_2 + \frac{\mu_0 i r_1}{2\mathrm{p}\left(a^2 - b^2\right)}\cos\theta_1$$

$$= \frac{\mu_0 i}{2\mathrm{p}\left(a^2 - b^2\right)}[r_2 \cos\theta_2 + r_1 \cos\theta_1] = \frac{\mu_0 i d}{2\mathrm{p}\left(a^2 - b^2\right)}$$

5. r_1 及 r_2：

$$\boxed{\lambda = \frac{2n_2 L}{m + \frac{1}{2}} = \frac{2n_2 D_t}{m + \frac{1}{2}} = \frac{2(1.53)(63.5\text{nm})}{m + \frac{1}{2}} = \frac{194\text{nm}}{m + \frac{1}{2}}}$$

依此條件，$m = 0$ 時，$\lambda = 388\text{nm}$ → 紫外線

r_1 及 r_3：

$2D_t$ 及 $2D_a$ 長度內之波數

$$\boxed{N_t = \frac{2D_t}{\lambda_n} = \frac{2D_t n}{\lambda}, N_a = \frac{2D_a}{\lambda}}$$

r_1 及 r_3 同相之條件

$$\boxed{\begin{array}{l} \dfrac{2D_t n}{\lambda} + \dfrac{2D_a}{\lambda} = m, \quad m = 1, 2, 3\cdots \\ \lambda = \dfrac{2(63.5\text{nm})(1.53) + 2(127\text{nm})}{m} = \dfrac{448\text{nm}}{m} \end{array}}$$

依此條件，$m = 1$ 時，$\lambda = 448\text{nm}$ → 藍綠光